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DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

DESIGN AND CONSTRUCTION OF SAN GABRIEL DAM NO. 1

By Paul Baumann,[1] M. Am. Soc. C. E.

Synopsis

Dam sites distinctly favorable for masonry structures, particularly of the massive type, are rapidly being exhausted the world over. Hence, engineers are faced, more and more, with the problem of design and construction of safe and economical dams for dam sites far from ideal, relative to topography, geology, and geography. The latter is particularly important with regard to earthquake belts, such as are well established, and to the latitude and elevation at which the dam is to be located.

In the design and construction of the San Gabriel Dam No. 1 in California, it was necessary to consider all of these features. The solution of the problems thereby encountered, and particularly the one connected with stability and watertightness during major earthquakes, is believed to be novel. For the dam proper it expresses itself in the combination of compacted earth with loose rock—and particularly with compacted rock (that is, with artificial stone). The manufacturing methods of the latter and its physical qualities established by tests prior to, and during, construction are believed to be of special interest. Hence this paper treats this subject in particular detail. It also deals with features of special interests, incorporated in appurtenant works.

Introduction

The 4,000 sq miles of surface area of Los Angeles County, California, may be divided into four principal sub-areas from a standpoint of flood control: Mountain area (catchment, erosion) with 1,500 sq miles; foothill area (debris cones) with 500 sq miles; valley area (inundation) with 1,700 sq miles; and desert (arid) occupying the remainder.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **January 15, 1942.**

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The San Gabriel Mountains form the principal range and the San Gabriel River is its principal drain. Its drainage area includes 220 sq miles above the mouth of the canyon with a maximum range in elevation of nearly 10,000 ft.

Particularly interesting is the nearly straight alinement of the two main forks, east and west, which approach each other in opposite directions and the confluence of which is a distinct landmark known as "The Fork."

San Gabriel Dam No. 1 is located approximately $2\frac{1}{4}$ miles downstream from the Fork. It is the principal one of an originally proposed system of three dams, two of which (Dam No. 2 and Dam No. 3) were to control the West and East Forks, respectively. Although Dam No. 2 on the West Fork was completed in the spring of 1935, Dam No. 3 on the East Fork was abandoned for reasons of economy.

The Original Design

In order to comprehend, fully, the revised or final design of Dam No. 1 as constructed, it is necessary to refer back to the original design that was prepared by the Los Angeles County Flood Control District prior to 1933 when the original contract was let.

The original design called for a rock-fill dam chiefly consisting of large, hard, and strong rock with voids of corresponding size. This rock was to be placed dry—that is, without the aid of sluicing water. The dam was to be provided with a laminated, shotcrete water slab with a substantial layer of packed rock directly beneath.

The rock was to be produced by approved methods in what is known as Quarry 10 which is located approximately one-half mile downstream of the dam site (described subsequently in connection with Fig. 28). It is a cut in a ridge with its floor at El. 1390 and an average slope of 1 to 1. It consists chiefly of a granitic diorite, sheared and crushed due to faulting and crossed by numerous intrusions of varying magnitude and composition. The quantity, quality, and size of the quarried rock was specified for the various parts of the dam.

In the spring of 1933 the work was started under contract. After excavation of the stream-bed material down to bedrock and the construction of a section of the cutoff wall along the base of the upstream contact, the placing of the rock fill in the excavated stream bed was started.

For the purpose of producing the rock to be placed in the fill, the specifications provided for the elimination of fines and spalls by means of a screen, commonly known as a "grizzly" in western construction parlance. This "grizzly" had clear slot openings of 6 in. The material passing through these openings was wasted, and the material passing over was acceptable for the fill, provided it satisfied the requirements regarding weight, strength, and soundness.

However, as the quarrying of rock progressed, it became quite evident that rock, satisfying the specifications, could not be produced economically in sufficient quantity in that for each acceptable cubic yard approximately three had to be wasted. Under the contract the waste had to be paid for at 40¢ per cu yd or $53\frac{1}{3}\%$ of the acceptable rock and the waste had been estimated at less than one third of the latter. It was obvious, therefore, that completion of

the dam inside the estimated cost was out of the question. Consequently, construction was suspended in October, 1934, at which time some 450,000 cu yd out of an estimated total of 5,562,000 had been placed in the dam and studies for a revised design were commenced.

The original design had been approved by consulting engineers and geologists of national repute.

The Revised Design

The Dam Proper.—The knowledge regarding relative quantities and characteristics of the rock, gained during quarry operations, led to the conclusion that the design of a feasible dam would have to include the material originally classed as waste—that is, principally the material passing through the "grizzly" openings. Hence, in principle it was necessary to adapt the design to existing conditions, which conditions, in turn, called for a change in the design from a true or homogeneous rock-fill type with a small, unavoidable percentage of fines to a mixed or heterogeneous rock-fill type with a large and, in fact, predominant percentage of fines. Hence, it called for a change to a type of fill whose physical characteristics (particularly in relation to stability and permeability) were of a radically different nature from the original type. This recognition was given further emphasis through the indispensable requirements regarding earthquake resistance of major structures in California.

The inclusion of a large percentage of fines in the fill naturally called for the flattening of the upstream and downstream slopes relative to the original design. Clearly, this requirement was primarily governed by the behavior of such a fill when saturated, particularly when under the influence of lateral mass forces in addition to gravitational forces during earthquakes. The open-type rock fill, as originally designed, was not subject to saturation above the stream bed and hence was not affected by this important phenomenon.

The flattening of the slopes as well as experience gained during the construction of San Gabriel Dam No. 2 (which was of substantially identical design as to type as the original of Dam No. 1) called for a change in the watertight membrane. This change was encouraged by the discovery of a large deposit of clayey sand or loam of unusual uniformity approximately 1¾ miles upstream from the dam site. It was an old homestead, known as Persinger Flat, situated inside the reservoir area which the Los Angeles County Flood Control District had acquired. The character of this material suggested its use as a relatively thin, impervious layer or zone whose plasticity (and with it, flexibility) would be assured so long as it was protected against drying out. The intimate contact of this zone with the existing part of the cutoff wall, as well as with the ultimate extension of the cutoff wall, was naturally a pertinent feature. To enhance the sealing quality of this zone a 6-in. shotcrete blanket was provided along its contact area with bedrock. This was to prevent the intrusion of water along paths shorter than the full normal thickness of the zone. A watertight connection was provided between this blanket and the cutoff wall.

These studies in connection with the revised design were accompanied by extensive tests on the available rock material particularly for the purpose of establishing suitable methods of compaction or petrifaction of the waste

material which, as previously outlined, formed the bulk of the quarried rock. The purposes of these tests were to establish:

(1) The maximum size of rock which, if embedded in finer material, could be rolled with standard sheepsfoot rollers;

(2) The necessary roller weight to produce adequate compaction;

(3) The necessary number of roller passes;

(4) The most favorable proportion between fines, defined as particles $\frac{1}{4}$-in. or less in size, and rock, defined as particles greater than $\frac{1}{4}$ in. in size;

(5) The breakdown of the rock;

(6) The moisture content of the fines which would lead to adequate compaction with least work;

(7) The permeability of the compacted fines;

(8) The plastic and the elastic volume changes of the compacted fines;

(9) The penetration pressure of plasticity needles for the compacted fines; and

(10) The shearing strength of the compacted fines and its relation to plasticity.

Previously, tests had been conducted on loose rock and further tests were now made on mixes of loose rock and fines for the purpose of establishing: (*a*) Settlement under load for dry and wet material; (*b*) breakdown under load; (*c*) change in unit weight due to settlement; and (*d*) permeability.

Figs. 1 and 2 show the revised design in plan and section. The cross section of the dam is divided into six zones as follows:

Zone 1, consisting of quarry-run material varying from very fine material next to Zone 2 to relatively coarse material at the upstream slope. This zone may be considered largely a protective zone.

Zone 2, consisting of a clayey sand compacted to a dry weight of at least 115 lb per cu ft.

Zone 3, which may be considered the backbone of the dam, or the core, consisting of quarry-run material having passed 6-in. by 9-in. "grizzly" openings, compacted by rolling and tamping to a dry weight of the fines (particles smaller than $\frac{1}{4}$ in. in size) of at least 120 lb per cu ft.

No gradation of material between the upstream and the downstream slope was proposed for either Zone 2 or 3.

Zone 4, consisting of loose quarry-run material with the finest material next to the downstream slope of Zone 3 and gradually becoming coarser and more porous toward the downstream end of Zone 4.

Zone 5, consisting of relatively large rock forming a free draining mass and a protection against erosion to the zone below.

Zone 6, consisting chiefly of rock originally placed in the stream-bed excavation and moved under the revised design into its new position. This rock is relatively hard.

A statical analysis of equilibrium was made of a theoretical maximum vertical dam section of unit width under the influence of simultaneous full

hydrostatic water pressure and lateral mass forces due to earthquake. The latter are based on the acceleration of one tenth that due to gravitation—that is, on 0.1 g of the mass $\frac{V}{g}$ of the dam, as well as of the so-called body of inertia of water next to the dam.[2] The resultant force H of all horizontal loads was

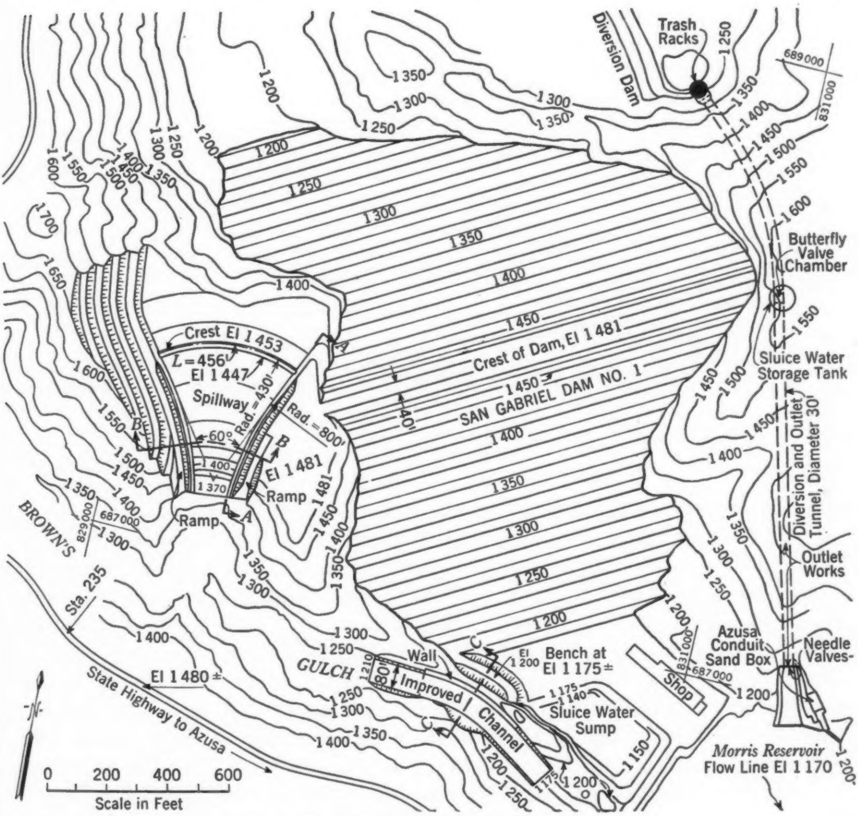


FIG. 1.—PLAN VIEW, SAN GABRIEL DAM NO. 1

assumed to be resisted along a horizontal plane at El. 1120, which coincides approximately with the bedrock level. Due to the protective rather than constructive nature of Zone 1 and the plastic condition of Zone 2 as previously described, no resistance was credited to Zone 1 and Zone 2. Hence, because of their mass and acceleration these zones acted entirely as a load.

The analysis shows a necessary average shear coefficient $\frac{H}{V}$ of 0.37. Test results outlined in the course of this paper will show this value to be well within the actual shearing strength of the critical material—namely, the saturated fines in Zone 3.

[2] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 418.

The safety against sliding due to shear is further enhanced by the configuration of the dam site (see Fig. 1).

The Rock and Soil Tests

With few exceptions the physical tests on materials for the dam were conducted in the field, either in the laboratory or in the open. The latter prin-

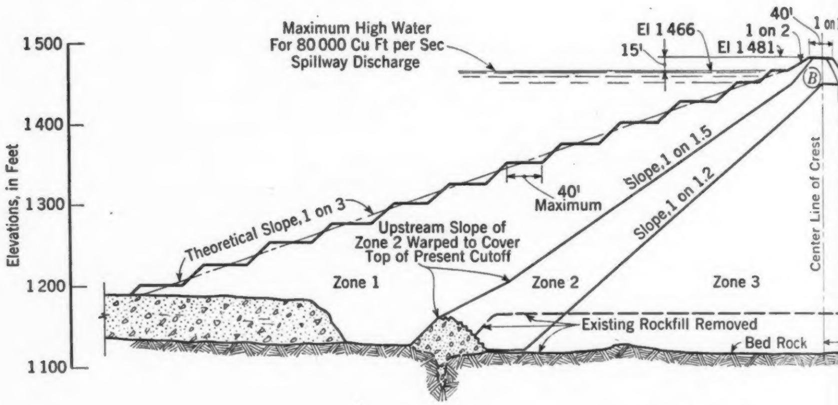


FIG. 2.—COMPOSITE MAXI

cipally included full-scale rolling and tamping tests for the purpose of establishing practical construction methods for Zones 2 and 3. Hence they differ from the relatively small-scale laboratory tests which served to establish the intrinsic properties of the materials by means of soil mechanics research. They will, therefore, be treated separately. The former fall into three basic groups—namely, those on rock only, those on rock and fines, and those on fines only.

The first group of tests represents open rock fill, placed dry or wet, such as constituted the bulk of the original design and as was placed in the upstream extremity of Zone 1 and in all of Zones 5 and 6. The second group of tests represents a mixture of rock and fines, varying between "mostly rock" and "mostly fines," such as was placed in the greater part of Zone 1 and all of Zone 4. The third group of tests represents fine material such as was placed in Zone 2 and such as forms the governing part of Zone 3.

Whereas the first two groups principally served to establish relations between load and settlement as well as permeability of the loose material, the third group served to establish all of the pertinent properties of compacted sand and soil.

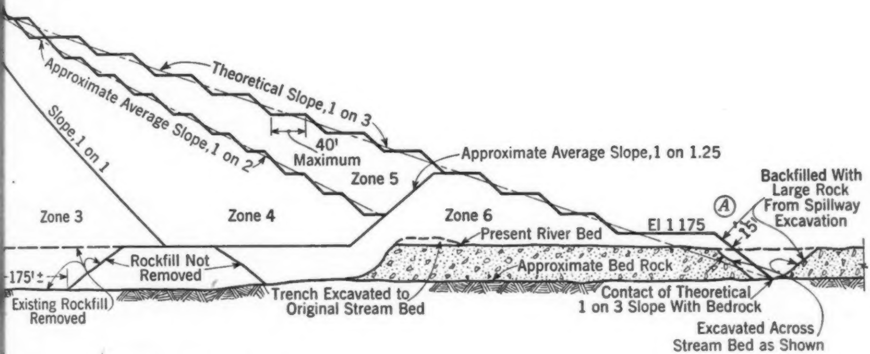
The discussion of all of the tests conducted in each group would require far more than the space available. Therefore, it will be limited to those, the results of which are believed to be typical.

First Group of Tests.—Tests to establish the settlement under load of representative rock samples were conducted in the field laboratory by means of cylinders. Two sets of three cylinders, each 24 in. and 30 in. in diameter and 24 in. and 30 in. high, respectively, were used. Pressures as great as 325 and

200 lb per sq in., respectively, could be applied by means of hydraulic jacks. Apparatus for the purpose of sluicing the rock was provided overhead. Samples of variable gradation of igneous rock were used.

Fig. 3 shows the test result of typical rock samples from Quarry 10. In Tables 1 and 2 test data are shown in detail.

The influence of sluicing, such as that due to a torrential rain, is evidenced in the sudden change in the settlement trend in Cylinders 2 and 3 upon the application of water. Cylinder 1 contained a sample from the Devil's Canyon



Section, San Gabriel Dam No. 1

Quarry which supplied the rock for Dam No. 2. It is a granitic gneiss of excellent quality. The corresponding results are shown in Table 1 for the purpose of comparison. They are omitted from Fig. 3.

These tests combined with the experience gained during the construction of San Gabriel Dam No. 2 prompted the provision, in the revised specifications, for the application of approximately 2 cu yd of water to 1 cu yd of loose rock at the time of placing.

The breakdown due to the settlement under load (Table 2) is reflected in the screen analysis made at the end of the test as compared to the original one and particularly in the respective fineness moduli.

Second Group of Tests.—Samples used in these tests included fines varying between 20% and 60% by weight. Three 24-in. by 24-in. cylinders were used. The following tests are typical for this group:

A mixture of 80% rock and 20% fines was placed in Cylinder 1, 60% rock and 40% fines in Cylinder 2, and 40% rock and 60% fines in Cylinder 3. Uniform initial dry weight (placing density) was striven for. The cylinders were equipped with porous disks at the top and bottom of the sample to permit the passage of percolating water. A small initial vertical load was applied to balance the pressure due to water percolating in an upward direction until the rate of percolation had become constant.

The load was then increased in steps of 25 lb per sq in. until an ultimate load of 325 lb per sq in. was reached. Before the application of each load increment, movements due to settlement had diminished to negligible values. The total settlement and the rate of percolation were observed for each load

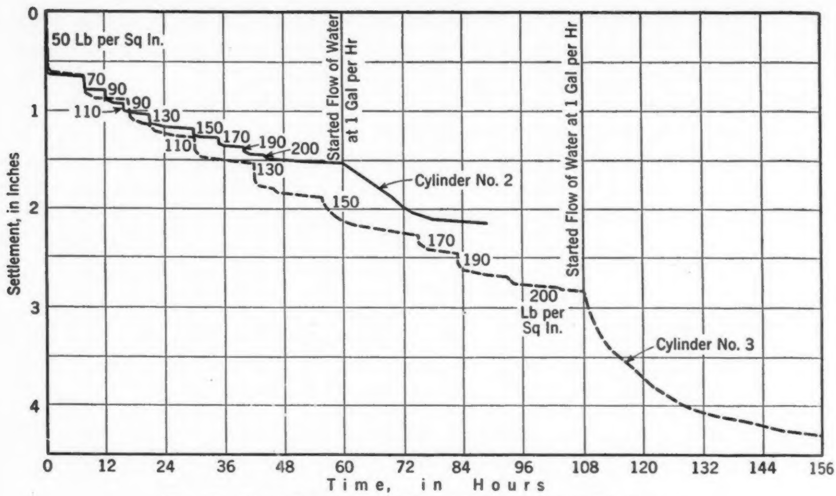


FIG. 3.—ROCK SETTLEMENT TESTS; SAMPLES FROM QUARRY 10

TABLE 1.—CONSOLIDATION, SAMPLES FROM QUARRY 10

Load (lb per sq in.)	CYLINDER 1			CYLINDER 2			CYLINDER 3		
	Hours at the given load	Height of rock (in.)	Total settlement (%)	Hours at the given load	Height of rock (in.)	Total settlement (%)	Hours at the given load	Height of rock (in.)	Total settlement (%)
0	0	29.000	0	0	29.000	0	0	29.000	0
50	7	28.160	2.90	8	28.344	2.26	8	28.343	2.27
70	6	27.960	3.59	4	28.203	2.75	9	28.107	3.08
90	16	27.632	4.72	4	28.074	3.19	13	27.744	4.33
110	4	27.472	5.27	5	27.960	3.59	12	27.470	5.28
130	5	27.261	6.00	9	27.822	4.06	14	27.127	6.46
150	5	27.075	6.63	5	27.720	4.41	19	26.739	7.80
170	8	26.858	7.39	5	27.633	4.71	8	26.553	8.44
190	6	26.704	7.92	4[a]	27.550	5.00	20[a]	26.208	9.63
200	5[a]	26.612	8.24	16	27.479	5.25	5	26.174	9.75
200	27	25.077	13.53	29	26.864	7.37	49	24.710	14.80
Total	89	....	....	89	....	....	157	....	....

[a] Water added at the end of this period.

TABLE 2.—PERCENTAGE GRADATION, SAMPLES FROM QUARRY 10

Description	CYLINDER 1		CYLINDER 2		CYLINDER 3	
	Before loading	After loading	Before loading	After loading	Before loading	After loading
Particle Sizes (In.):						
$1\frac{1}{2}$ to 3	28.5%	24.3%	27.6%	25.4%	100.0%	66.3%
$\frac{3}{4}$ to $1\frac{1}{2}$	28.5%	24.5%	20.5%	19.6%	0	18.9%
No. 8 to $\frac{3}{4}$	40.0%	40.4%	29.5%	31.0%	0	11.5%
Passing No. 8	3.0%	10.8%	22.4%	24.0%	0	3.3%
Volume (cu ft)	12.180	10.53	12.174	11.28	12.174	10.38
Unit weight (lb per cu ft; dry)	106.7	123.4	121.4	131.1	89.1	104.5
Fineness moduli	7.35	6.85	6.35	6.20	9.00	8.29
Average rate of flow through rock	1.15 gal per hr		1.01 gal per hr		1.0 gal per hr	

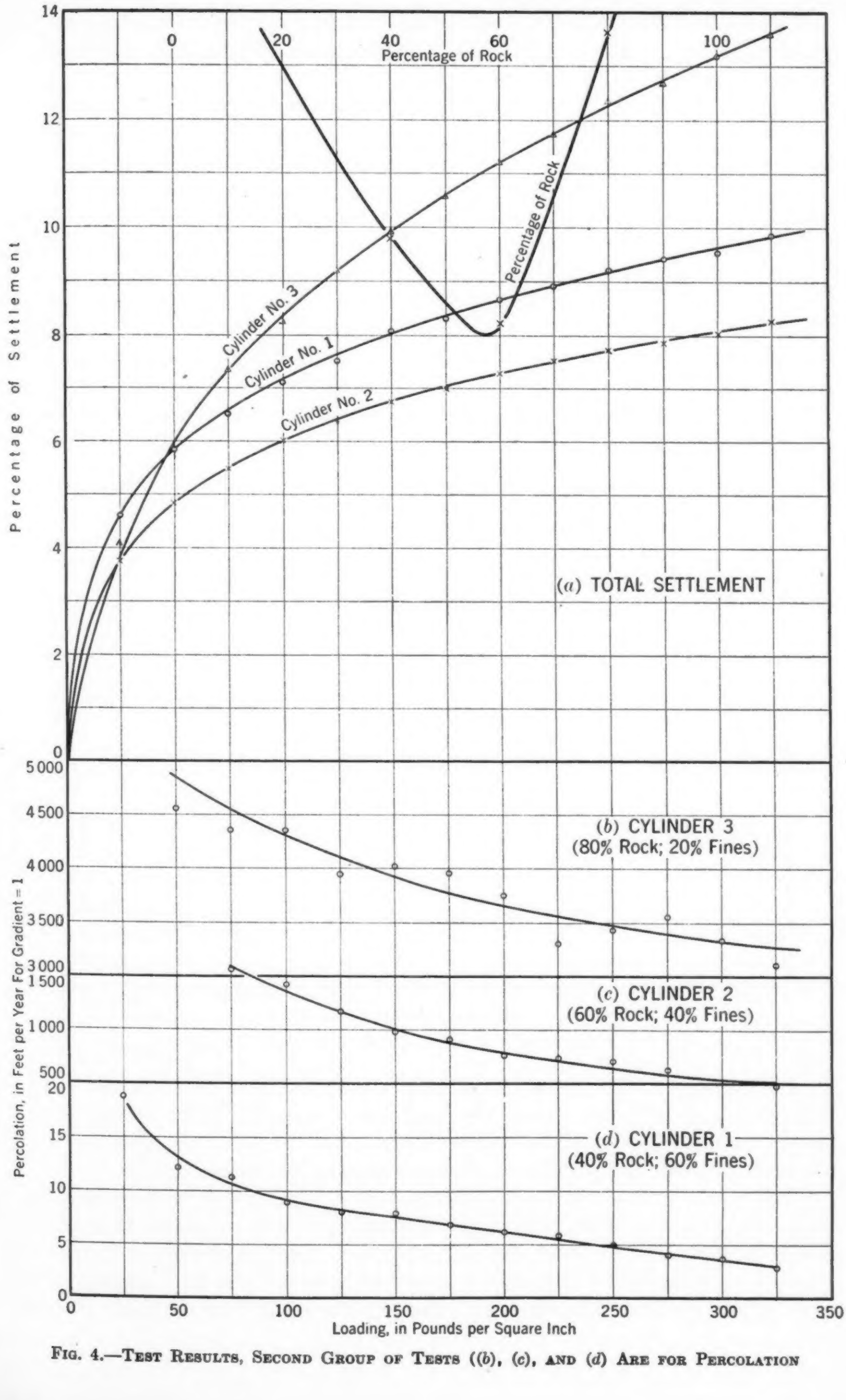


FIG. 4.—TEST RESULTS, SECOND GROUP OF TESTS ((*b*), (*c*), AND (*d*) ARE FOR PERCOLATION

increment. As indicated by the heavy line in Fig. 4(*a*), a mixture of 55% rock and 45% fines would tend to show minimum settlement. Percolation rates are shown in Fig. 4(*b*).

Thus far no attempt had been made to ascertain the effect on the settlement of "arching" of the material in the cylinders. For this purpose the bottoms of two of the three 24-in. by 24-in. cylinders (Cylinders 1 and 3) were separated from the base plate by means of set screws and wedges, respectively. The perforated disks supporting the material were blocked up about 6 in. from the base plates to make ample allowance for the settlement of these cylinders. Cylinder 2 was set up as before; that is, it rested directly on the base plate.

A mixture of approximately 65% rock and 35% fines was placed as uniformly as possible in each of the three cylinders with the aid of sluicing. The top disks were then placed and a vertical load applied in 25 lb per sq in. increments to a maximum of 325 lb per sq in. as before. Percolation of water through the samples was not practicable because of the separation of two of the cylinders from the base and consequent lack of a water seal.

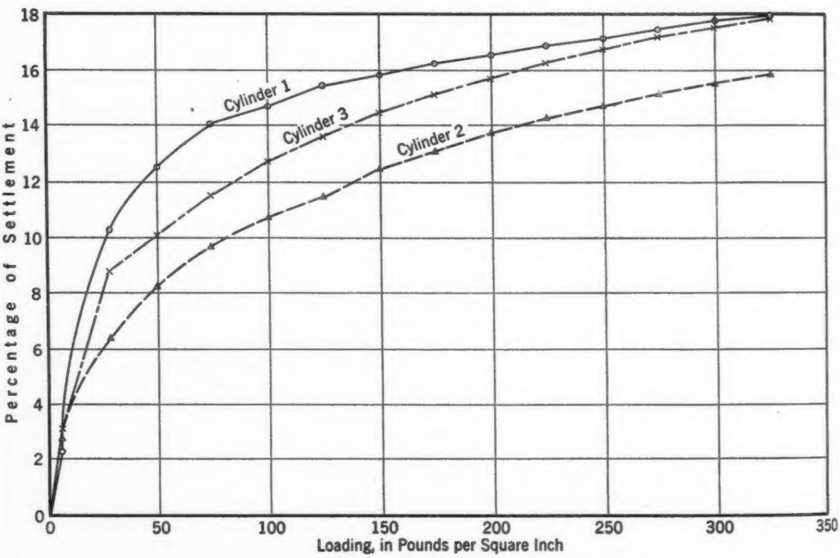


FIG. 5.—ARCHING EFFECTS, SECOND GROUP OF TESTS

Immediately after the application of the initial load of 25 lb per sq in., Cylinders 1 and 3 were completely freed from base support by raising the set screws and removing the wedges, respectively. Hence the cylinders were supported by friction between their inner surface and material only.

The result of this test is shown in Fig. 5. It shows that with cylinders of these dimensions arching is an important factor. It is expressed through the excess in settlement of the material in Cylinders 3, and particularly Cylinder 1 relative to Cylinder 2. The set screws served to raise Cylinder 1 slightly before each load increment. Furthermore, the cylinder walls were tapped. Arching

to the walls was thereby not only reduced but substantially eliminated. Fineness moduli resulting from these tests are given in Table 3.

TABLE 3.—Fineness Moduli, Samples Tested for Arching Effect

Description	Cylinder 1		Cylinder 2		Cylinder 3	
	Before	After	Before	After	Before	After
Percentage of fines	35.3	43.1	35.3	43.0	35.9	39.9
Fineness moduli	6.01	5.61	6.01	5.68	5.96	5.82

Before proceeding with the third group of tests (which is concerned with the fines only), the field tests will be discussed.

The Rolling Tests.—The average quarry material, passing either over or through the 6-in. by 9-in. "grizzly" openings showed a typical gradation based on dry screening, as shown in Table 4.

TABLE 4.—Typical Gradation of Quarry Material, Based on Dry Screening

Description	Large Particles[a] (29% > 6 In. by 9 In.)		Smaller Particles[b] (71% < 6 in. by 9 in.)				
			43%		50%		7%
	From	To	From	To	From	To	
Size range, in millimeters	150	500	6	150	0.10	6	<0.10
Classification	Rock		Rock		Fines		Fines

[a] Passing over. [b] Passing through.

Hence the material, subject to compaction in Zone 3, contained an average of about 43% rock and 57% fines. Obviously the stability and permeability of such a fill are governed by the density of the fines. The latter, in turn, is governed by the breakdown of the rock due to compaction, because unless breakdown occurs, pressure is transmitted from rock to rock through point contact and relatively little pressure is transmitted by fines in the interstices.

Test fills of quarry materials, having passed through "grizzlies" with openings varying from 3 in. by 9 in. to 6 in. by 9 in. in the clear were placed in 9-in. to 12-in. layers and compacted by means of a single-drum sheepsfoot roller. The drum was 58 in. long and 42 in. in diameter. It had eighty teeth and feet, each 8 in. long, and a bearing area for the feet of approximately 8 sq in. The weight of the roller was varied between 4.75 and 12.1 tons by filling the drum with suitable materials. A total of twenty-four test fills were rolled, each about 80 ft long, 12 ft wide, and 3 ft high. The test fills were divided into four sections, *A*, *B*, *C*, and *D*, roughly 12 ft by 20 ft. Six layers were spread, between 9 in. and 12 in. thick, and compacted by means of from

ten to forty roller passes. The moisture content was varied between the four sections in intervals of approximately 2%—namely 7%, 9%, 11% and 13% more or less. Immediately after completion of each fill, twelve samples were taken (three from each section) to ascertain the degree of compaction. For this purpose a shallow pit about 4 ft by 4 ft in size was first dug to remove the top layer and its floor leveled off. A metal ring 12 in. in diameter was then set and plaster of Paris placed around it. A hole or pocket was then dug within the ring by suitable hand tools and the excavated materials placed in a can, sealed, and weighed. The volume of the hole was determined by filling it with fine, dry beach sand of known unit weight. The unit weight, inclusive of moisture, of the compacted fill was thereby established.

Next, the rock was separated from the fines by means of a No. 4 screen, spread out on burlap, dried, and weighed. Its apparent specific gravity and volume were then determined through immersion. Subtracting the rock volume from the total volume gave the volume of fines. After determining the average moisture content of the latter, the dry density of the fines resulted. The dry density served as the criterion of compaction as it proved to be the governing factor in determining the intrinsic properties of the fill.

For one and the same roller weight and number of passes, the dry density of the fines was principally governed by: The relative hardness, the percentage of rock, and the moisture content. The respective typical gradation before and after rolling is shown in Fig. 6. The breakdown due to rolling is evident.

Based on the rolling tests and particularly on the screen analyses before and after rolling, a relation between roller weight and breakdown of the average quarry rock was obtained. This is shown in Fig. 7. It is based on equal percentages of fines; that is, 40% ± before rolling. Guided by this result, the District proceeded with the design and construction of five double-drum roller units as shown in Fig. 8 and one single-drum unit. The dead weight of each double-drum unit was 14.50 tons and averaged 24.75 tons, or 12.375 tons per drum when filled with saturated sand.

TABLE 5.—TEST LOADS IN TONS TO DETERMINE ECONOMICAL SIZE OF ROLLERS

Load	Roller Unit[a]			
	D_1	D_2	D_3	S_1
Unloaded	14.50	14.50	14.50	7.00
Water loaded	19.00	19.25	17.25	9.25
Water and sand loaded	25.50	24.25	24.50	11.00

[a] D = double drum and S = single drum.

Studies were made to ascertain the most economical combination of rollers that would satisfy the requirements regarding the compaction of the material in Zone 3. They were accompanied by tests. Three double-drum units and one single-drum unit were used unloaded, partly loaded and fully loaded, with weights in each case as given in Table 5.

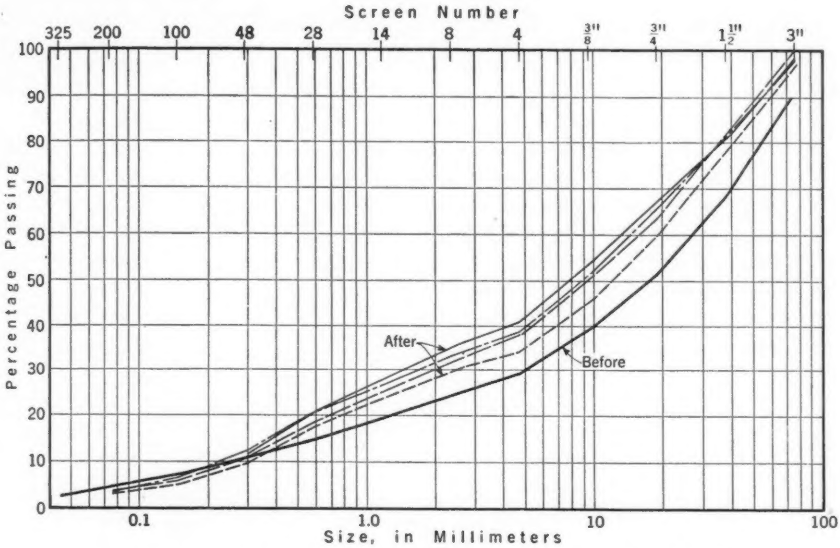


FIG. 6.—GRADATION OF ROCK FROM QUARRY 10, BEFORE AND AFTER ROLLING

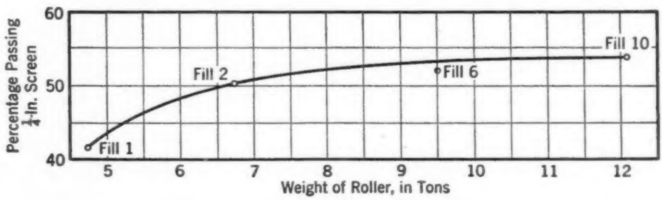


FIG. 7.—BREAKDOWN DUE TO ROLLING

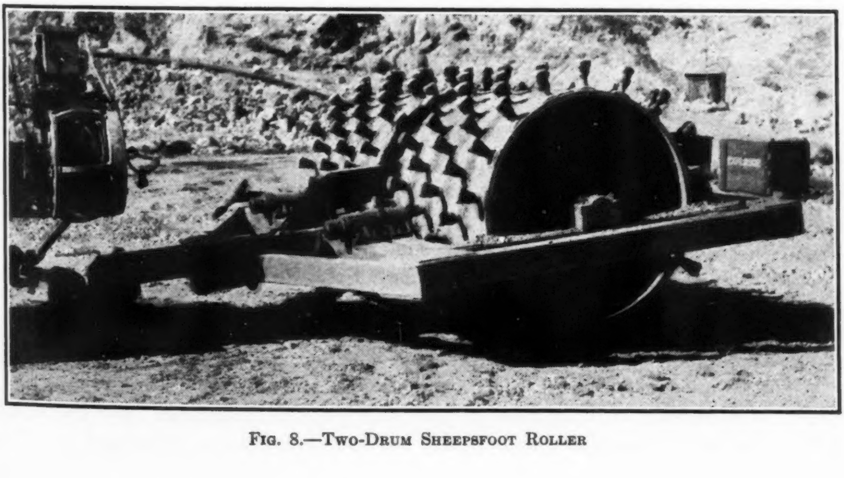


FIG. 8.—TWO-DRUM SHEEPSFOOT ROLLER

The rollers were used in the following combinations:

Combination of rollers	Number of passes	
One double unit, water and sand loaded and one double unit unloaded in tandem	8	
Two double units, water loaded in tandem	8	
Two double units, unloaded in tandem	10	
One double unit, water and sand loaded	12	
One double unit, water loaded	12	
One double unit, unloaded	16	
One single unit, water and sand loaded	12	(24)
One single unit, water loaded	12	(24)
One single unit, unloaded	16	(32)

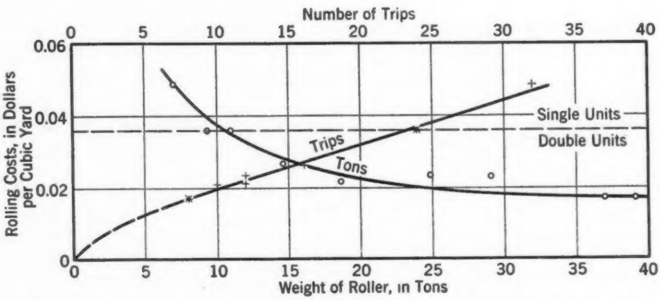


FIG. 9.—RELATION BETWEEN ROLLER WEIGHTS AND NUMBER OF TRIPS FOR CERTAIN UNIT COSTS OF ROLLING

The result is shown in Fig. 9. The field cost, without overhead, in dollars per cubic yard of rolled materials, is based on the following charges:

Equipment	Rate, in dollars per hour
One tractor with bulldozer	3.75
Tractor operator	0.8625
Double-drum unit roller	0.50
Single-drum unit roller	0.30

The Tamping Tests.—Next to the abutments a strip, 2 ft to 3 ft wide, had to be compacted by means of hand tampers because the rollers could not operate there. For this purpose, tamping tests were conducted with electric, pneumatic, and gasoline tools. It was found that satisfactory compaction could not be obtained if the rock exceeded about 40%.

Relatively few field tests were conducted on the material in Zone 2. Its typical characteristics are reflected in Figs. 10, 11, and 12, showing wet and dry screen analyses and compaction data. This material compared favorably with the best of the soil on record.

Tests were also made on samples taken from the abutments, mainly on the right or west side to determine its characteristics as compared with those of the materials in Zone 2 and Zone 3. It was found that even the softest of rock

encountered after stripping showed dry densities and corresponding stability and permeability which were at least as satisfactory as those pertaining to the fines in Zone 3.

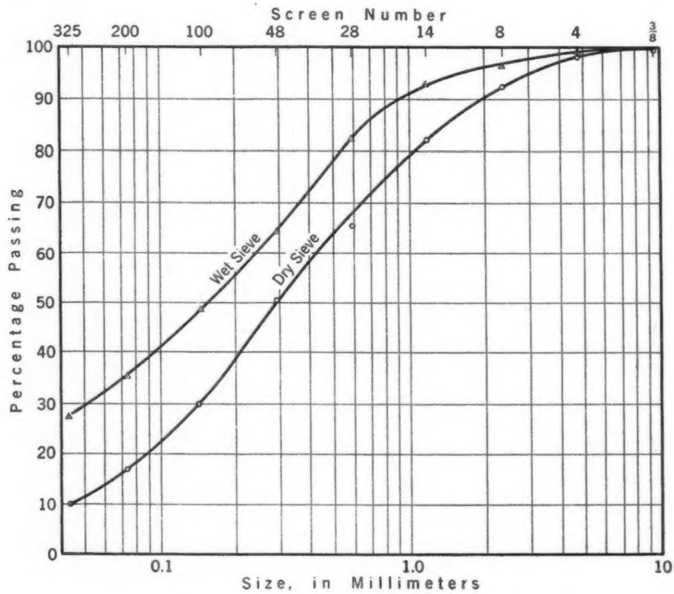


FIG. 10.—SIEVE ANALYSIS OF COMPOSITE SAMPLE, ZONE 2

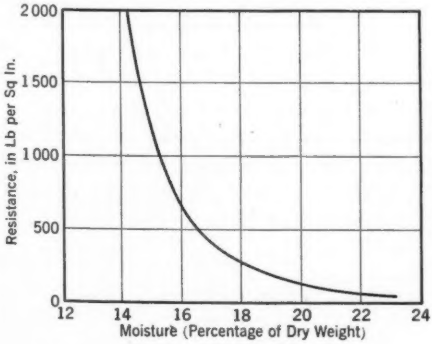


FIG. 11.—PLASTICITY-NEEDLE PENETRATION RESISTANCE; ZONE 2

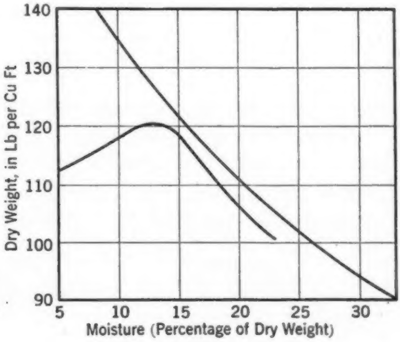


FIG. 12.—DRY WEIGHT; ZONE 2

Samples from the natural stream bed taken at various depths and distances downstream from the proposed toe of Zone 3 showed satisfactory stability and permeability under surcharge. The latter had to be such as to freely drain the flow due to percolation through the dam on the assumption of a full reservoir for an indefinite period of time.

THE THIRD GROUP OF TESTS

All of these tests were conducted in the laboratory. They served to determine the intrinsic properties of the fines in Zone 2, and particularly in Zone 3, and hence were the most significant tests, from the standpoint of experimental soil mechanics. The importance of Zone 3 relative to the stability of the dam called for special attention to the fines of that material, particularly when saturated.

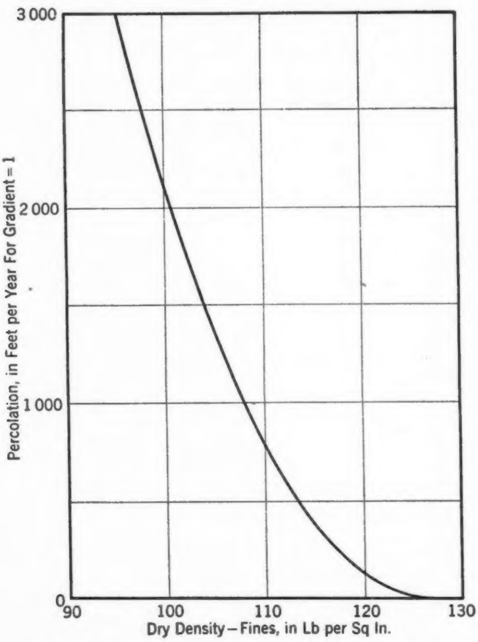


FIG. 13.—RELATION BETWEEN DRY DENSITY OF FINES AND RATE OF PERCOLATION

Permeability.—Percolation tests, conducted by means of 8-in. cylinders (fines only) showed an average rate of percolation of 1.5 ft per yr for a dry density of 125 lb per cu ft for Zone 3 material and 0.50 ft per yr for a dry density of 116 lb per cu ft for Zone 2 material. These values refer to a gradient of 1 on 1. They were obtained by reducing the test values, which resulted from a gradient of 80 on 1, in the same proportion.

Percolation tests conducted on composite and mixed materials of Zone 3 in the 24-in. cylinders substantially confirmed the results. They are shown in Fig. 13. They clearly reflect the importance of adequate compaction of the fines (see Fig. 13).

Compaction and Breakdown.—Fig. 14 shows the relation between compaction and breakdown for variable moisture contents. Isopenetration resistance in pounds per square inch is shown by broken lines. The results were obtained by compacting typical fines with a $5\frac{1}{2}$ lb rammer as follows:

Curve (Fig. 14)	Number of drops	Force
A	5	4-in. drops
B	15	4-in. drops
C	15	Medium tamps
D	10	Hard tamps
E	25	Hard tamps

Shearing Strength.—The tests to establish the shearing strength of the saturated lines in Zone 3 led to results that are believed to be most significant so far as the intrinsic properties of this material are concerned. They were based on the fundamental concept that the shearing strength is the only

criterion of safety of a saturated, granular mass in a dam fill in so far as structural stability is concerned, because, on the one hand, such a granular mass has no tensile strength and, on the other hand, it cannot fail due to com-

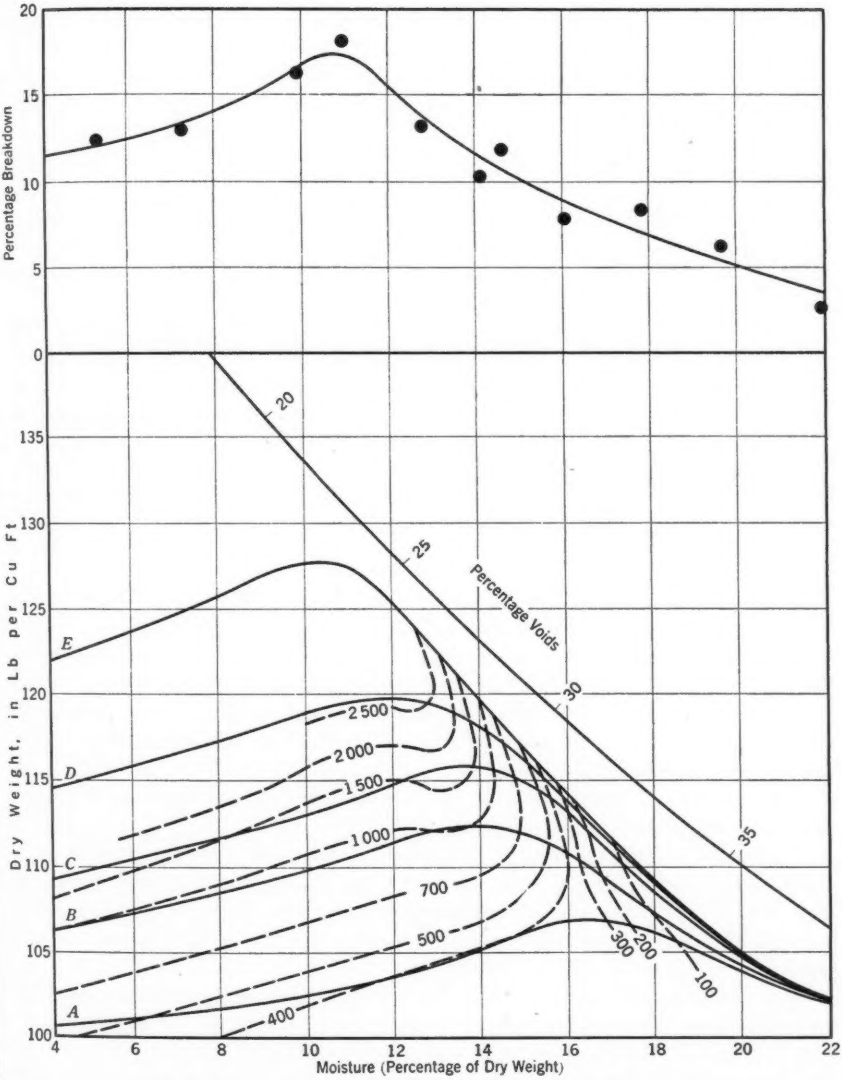


FIG. 14.—COMPACTION, BREAKDOWN, AND PENETRATION RESISTANCE

pression as long as its lateral movement is restricted—that is, as long as it is confined. This material placed in a cylinder, for example, could be subjected to any pressure without the danger of failure, as long as the cylinder successfully resisted lateral pressure. Failure of the cylinder, however, would elimi-

nate the outer resistance to lateral pressure and would permit particles to move laterally and relative to each other unless restricted by internal friction; that is, by shearing strength.

Fig. 15 shows the apparatus specially designed for the shearing tests. It consisted of a steel cylinder, machined and chromium plated inside, with a 0.5-in. wall, 4 in. internal diameter, and 11 in. high. A slip ring (point 1) reinforced with a heavy band, 2 in. high, fitted snugly between the upper (point 2) and lower (point 3) parts of the cylinder. The inside and the edges of the slip ring were chromium plated and highly polished. There was a clearance of about 0.001 in. between the slip ring and the upper and lower parts of the cylinder.

By means of a counterbalanced (point 4) lever (point 5) arrangement, horizontal pressures could be exerted on the slip ring to cause horizontal movement. The ratio between vertical action and horizontal reaction was 1 : 14.40.

The soil to be tested was placed in the cylinder at the desired density, with the slip ring at its initial position—namely, flush with the inside of the cylinder. The material rested on a perforated seat and vertical pressures were applied to the soil by means of an hydraulic jack (point 6), the load being transmitted to the soil through a stem (point 7) and perforated disk.

Before any vertical load was applied (other than enough to balance the water pressure) the soil was subjected to percolating water. This was continued until the percolation rate became substantially constant.

Test Procedure

Two distinct methods of procedure were used in producing failure of the soil under shearing stress—namely, the slow test and the quick test.

In the slow test the desired vertical load was applied and allowed to bring the soil to its final settlement before the horizontal load was applied. The horizontal load was then applied and the resulting horizontal movement allowed to come to rest before the next load increment was added. This procedure was continued to failure.

The quick tests differed from the slow tests principally in the method of load application at the end of the lever.

The quick test loading was accomplished by means of running water through a calibrated meter, into a tank suspended at the end of the lever (point 5, Fig. 15). The horizontal loading and displacement were read at the same instant, at intervals, until the soil was ruptured. The time required for the quick test was from 2 min to 10 min.

Upon the completion of the test, the following data were obtained: (*a*) The penetration resistance of the material within the slip ring; (*b*) the moisture content of the soil; (*c*) the weight of soil, to determine its density, and (*d*) the specific gravity of the material, to determine the relative saturation of the soil under test conditions.

Soil Characteristics

Gradation.—Enough of typical Zone 3 material was secured to run all of the tests before they got under way in order that there would be no appreciable variation in the material to influence the results.

The soil used in the tests could be classed as silty sand. When wet the soil was more plastic than elastic. Periodic gradation analyses were made on the soil to check its variation as the series of tests progressed. The fineness

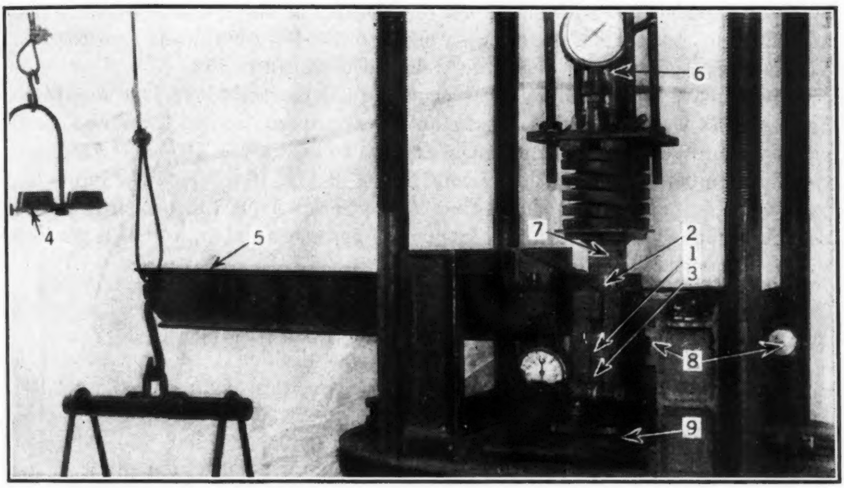


FIG. 15.—SHEAR TEST APPARATUS

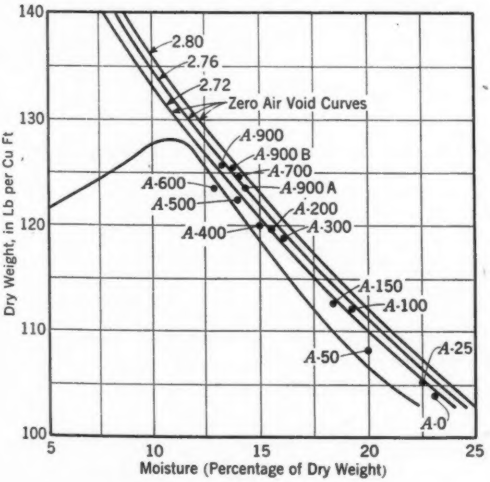


FIG. 16.—DRY WEIGHT; ZONE 3

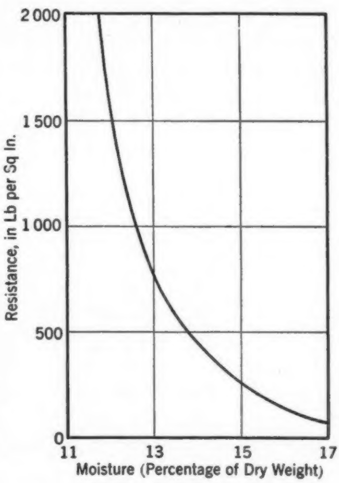


FIG. 17.—PLASTICITY-NEEDLE PENETRATION RESISTANCE; ZONE 3

modulus ranged from 1.81 to 2.07. The mean effective size (size for 10% passing) averaged 0.080 mm, with the uniformity coefficient (ratio of size at 60% passing to size at 10% passing) being about 6.5. The foregoing values

are for the material screened dry. When the material was screened wet, the average fineness modulus dropped to 1.45 with a corresponding drop in the effective size and increase in the uniformity coefficient.

Compaction and Penetration Resistance.—An average of six compaction tests made on the soil showed a maximum dry density of about 128.2 lb per cu ft at an optimum moisture of 11.0% by weight, or 30.4% by volume (see Fig. 16). The moisture-penetration resistance relation is shown in Fig. 17.

Specific Gravity.—The soil was ground in a mortar to various degrees of fineness. It was found that a satisfactory approximation of the true specific gravity could be had by grinding the dry soil to pass a No. 28 sieve (0.59 mm).

The numbers shown at the various points of Fig. 16 indicate the individual test; for example, A-900B shows that it was Series A (105 lb per cu ft placing density), 900 is the vertical load in pounds per square inch, and B is the particular test with this vertical load.

Test Features

Placing Density.—The series of tests was divided into four groups of different placing densities. The placing dry densities of fines used were 105, 110, 115, and 120 lb per cu ft.

Vertical Load.—The vertical loadings were 0, 25, 50, 100, 200, 300, 500, 600, 700, 900, and 1,200 lb per sq in. The 1,200-lb load was slightly greater than that recommended by the hydraulic jack manufacturer. Consequently, all tests at 1,200 lb per sq in. vertical load were dropped from consideration.

Rebound.—At the completion of each individual test the dry density of the material within the slip ring at removal was corrected for rebound from the density at the time of incipient failure. This was the density used in determining the degree of saturation at the time of incipient failure.

Incipient Failure.—It was quite simple to determine the point of incipient failure for the quick tests. The relation between time of loading and horizontal displacement was plotted on logarithmic paper (Fig. 18). It is seen that there appear to be two distinct trends, one being plastic displacement with respect to time and horizontal loading and the other being beyond stability. Therefore, the point of intersection can be called the time of incipient failure.

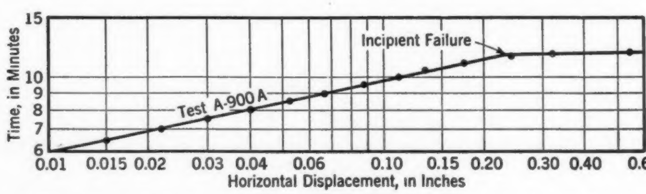


Fig. 18.—Location of Points of Incipient Failure for Quick Tests

It was not always possible to determine the point of incipient failure in this manner for the slow tests. However, failure of the material generally resulted shortly after the horizontal displacement had passed 0.2 in. At this

point a small increment of horizontal load caused failure. The relation between horizontal and vertical pressures at that instant was used in determining the shear coefficient.

Horizontal Load.—The shearing stress was determined by dividing the horizontal load, applied by the lever, by the shear area, after the latter had been corrected for reduction due to the horizontal displacement. No correction was made in the compressive stress due to the vertical load because of the horizontal displacement.

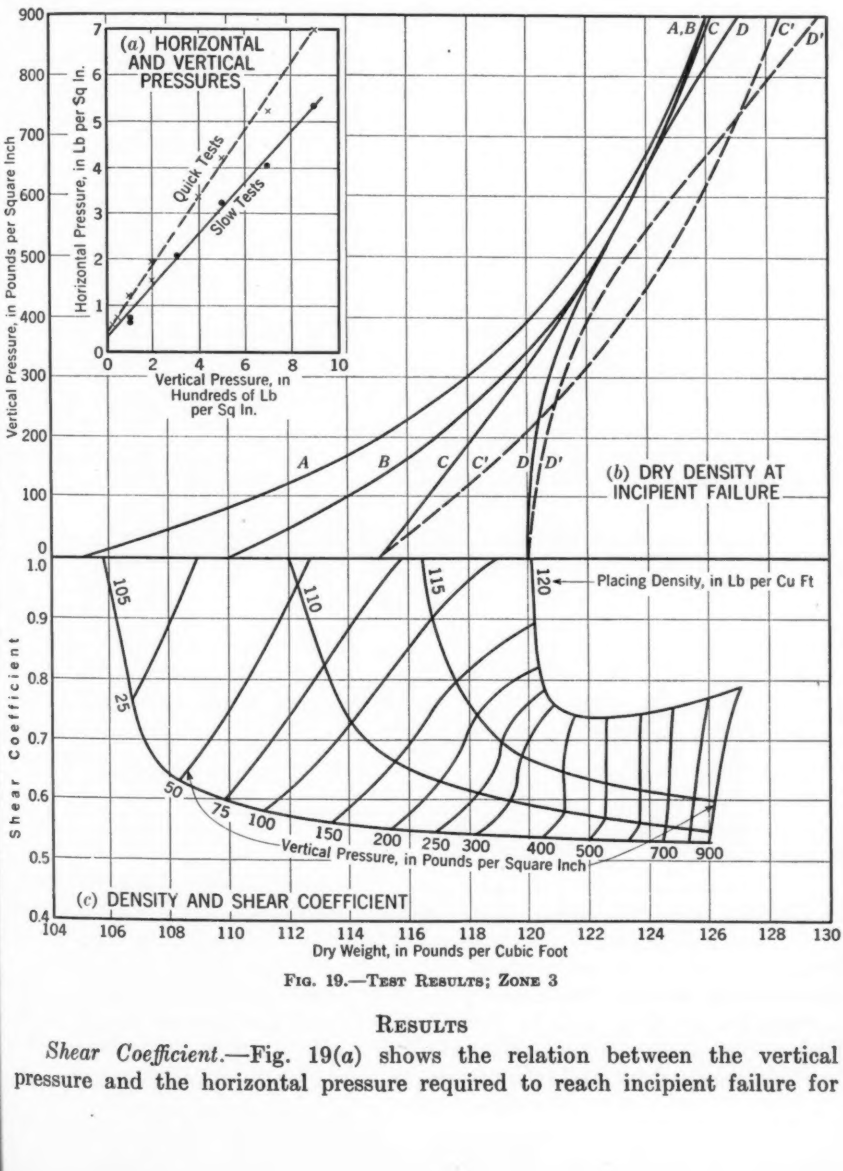


FIG. 19.—TEST RESULTS; ZONE 3

RESULTS

Shear Coefficient.—Fig. 19(*a*) shows the relation between the vertical pressure and the horizontal pressure required to reach incipient failure for

the series having a placing density of 115 lb per cu ft. This relation is typical of the four series conducted. The shear coefficient is the ratio of horizontal pressure to vertical pressure.

Consolidation.—Fig. 19(*b*) shows the density attained by the soil at incipient failure under the various vertical pressures. There was no appreciable difference between the slow and quick tests for the two lower placing densities ($A = A'$, $B = B'$). However, there was a noticeable increase in density for the quick tests over the slow tests for the two higher placing densities ($C' > C$, $D' > D$). Obviously the influence of the placing density decreases with an increase in vertical pressure.

Density and Shear Coefficient from Placing Density-Load Relation.—From Fig. 19(*c*), for a given placing density and vertical load, the shear coefficient for incipient failure may be obtained. For example, for a placing density of 117 lb per cu ft and a vertical pressure of 300 lb per sq in. the shear coefficient is 0.70 at incipient failure.

These results confirm the conclusion already drawn from the field and the percolation tests that 120 lb per cu ft placing density will result in a fill of low permeability and high stability. The results of the quick tests were omitted as they substantially agree with those of the slow tests.

Penetration Resistance Relations

Features of Determinations.—Four readings, that is one from each quadrant, were obtained from each sample. The highest value was always in that quadrant adjacent to the applied force; in the quadrant opposite it was considerably lower. The readings taken in the two quadrants at right angles were generally equal and between the high and low values mentioned.

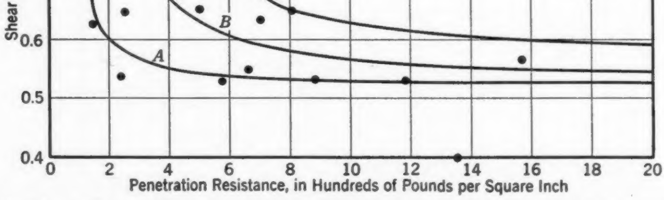


Fig. 20.—Relation Between Penetration Resistance and Shear Coefficient

Shearing Strength Relation.—A relation between shearing strength and penetration resistance may be derived. The result is shown in Fig. 20. Although it is purely theoretical it shows a characteristic similar to the dry-weight versus shear-coefficient relation curves.

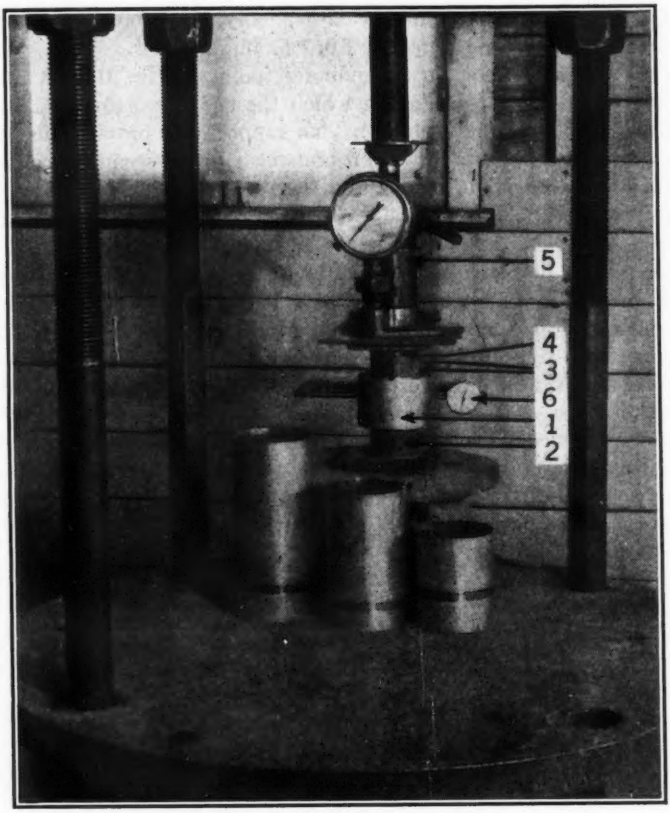


FIG. 21.—LATERAL PRESSURE TEST

The heavy dots represent actual penetration needle readings for 105 lb per cu ft placing density only. They show too wide a spread to check the theoretical relation. Hence, it would appear a hopeless task to predict the shearing strength and with it the stability of similar materials based on the penetration resistance.

LATERAL PRESSURE

The shearing strength may also be determined if the lateral pressure produced by vertical loading is known. With this in mind the following series of tests was conducted:

Method.—The determination of the lateral pressure was accomplished by measuring the change in diameter of a thin cylindrical shell, produced by various vertical loads applied to the soil. By knowing the change in diameter and the modulus of elasticity of the shell material, the value of the lateral pressure can be determined.

Apparatus.—The apparatus (Fig. 21) consisted of four brass cylinders, 1/16 in. thick, 4 in. internal diameter, and of various heights. The object of the

different heights was to determine the effect, if any, for different height-diameter of soil ratios. Those used were 0.5, 1.0, 1.5, and 2.0.

The soil was compacted in the cylinder (point 1, Fig. 21). A perforated disk was fitted loosely inside, upon which the soil was supported. This, in turn, rested upon the stem (point 2). No support was provided for the brass cylinder other than friction between it and the soil. A second perforated disk (point 3), was placed upon the soil and the vertical load transmitted through the stem (point 4). The vertical load was produced by an hydraulic jack (point 5).

The change in diameter of the brass cylinder was measured to the nearest 0.0001 in. by an Ames dial gage (point 6). The cylinder wall was divided into several rings of points spaced at regular intervals vertically. Six diameters were used at each ring in obtaining an average change in diameter.

To determine the effect of vertical loads, the test was performed under five different pressures—200, 400, 600, 800, and 1,000 lb per sq in., respectively.

Shear Coefficient.—With the change in diameter and the modulus of elasticity being known the lateral pressure created by the various vertical loads could be calculated. For active thrust with the surface horizontal and for frictionless walls, Coulomb's equation

$$\frac{E}{P} = \tan^2\left(45 - \frac{\phi}{2}\right) \qquad (1)$$

was used, in which E is the lateral pressure, P is the vertical pressure in lb per sq in., and ϕ is the angle of internal friction. The tangent of ϕ would then be equal to the shear coefficient. For example, since

$$\frac{E}{P} = \tan^2\left(45 - \frac{\phi}{2}\right) \qquad (2)$$

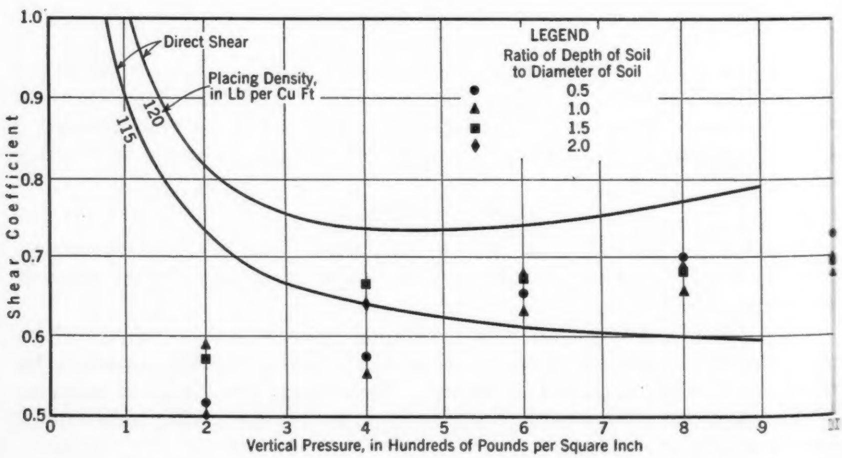


FIG. 22.—COMPARISON OF SHEARING STRENGTHS AS DETERMINED BY DIRECT SHEAR AND LATERAL PRESSURE

it follows that

$$2 \tan^{-1} \left(\frac{E}{P}\right)^{0.5} = 90 - \phi \dots\dots\dots\dots\dots\dots (3)$$

The angle of pressure transmission is

$$\theta = \tan^{-1} \left(\frac{E}{P}\right)^{0.5} \dots\dots\dots\dots\dots\dots (4)$$

Then $\phi = 90 - 2\,\theta$; and $\tan \phi$ = coefficient of shear.

The comparison between the coefficient of shear as determined by the lateral pressure tests and by direct shear is shown in Fig. 22.

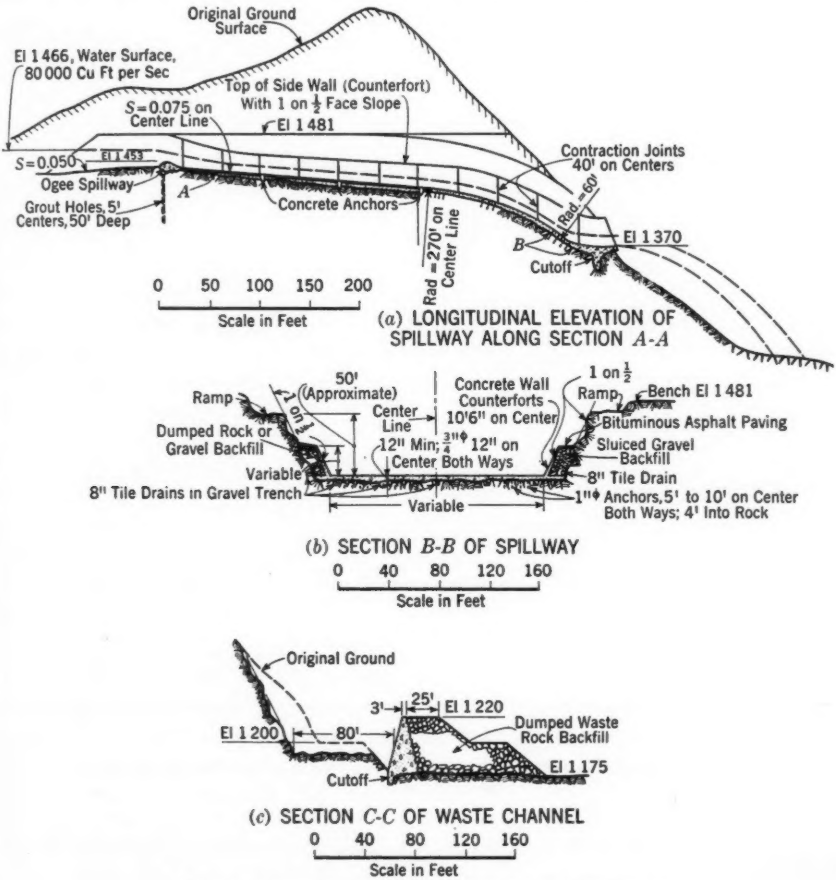


FIG. 23.—SECTIONS, SAN GABRIEL DAM NO. 1 (SEE FIG. 1)

THE SPILLWAY

As shown in the plan view of the project (Fig. 1), the spillway cuts through the ridge forming the right or west abutment of the dam. The original con-

tract had no provision for this work. The revised or supplemental contract, however, included the provision for the rough excavation of the spillway cut as an auxiliary quarry, known as Unit 2 of Quarry 10. This rough excavation was estimated at 1,200,000 cu yd above El. 1400 and the contractor agreed to quarry this rock and place it in Zone 1 at the regular unit cost of 75¢ per cu yd for rock fill. Without this saving in cost of excavation the project would not have been feasible.

Fig. 23 shows three sections indicated in the general plan view (Fig. 1). At point *A*, Fig. 23(*a*), the concrete floor is 18 in. thick with ¾-in. round rods, 12 in. on centers both ways. This floor is laid on a minimum of 8 in. of gravel, with tile underdrains. Contraction joints are approximately 40 ft apart, with copper water seals. At point *B*, the minimum depth of floor is 12 in., being laid on rock. It has no contraction joints, but 1-in. round anchors 5 ft to 10 ft on centers.

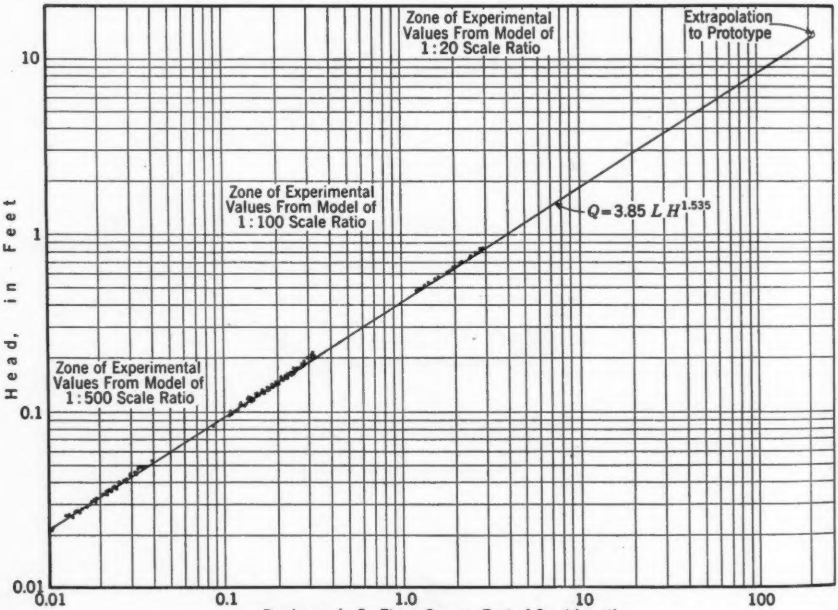


FIG. 24.—RELATION BETWEEN HEAD ON SPILLWAY CREST AND DISCHARGE PER UNIT LENGTH OF CREST

Hydraulic tests were conducted on spillway models. They served to determine:

(1) The discharge coefficient of the overflow section;

(2) The best shape and slope of the spillway channel from a standpoint of both hydraulics and economy; and

(3) The discharge conditions in Brown's Gulch.

Fig. 24 shows the result of the coefficient tests which were conducted on models of three different scales—namely, 1 : 20, 1 : 100, and 1 : 500. It shows that the discharge for all models and probably also for the prototype can be expressed by the equation

$$Q = 3.85\, L\, H^{1.535} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Hence the weir coefficient $\mu = 0.74 =$ constant because $\frac{1}{1.535}\, 2\, g\, \mu = 3.85$.

This parabola satisfies only those tests that were not affected by backwater. Adhering to the conventional head exponent of 1.50 the same tests are satisfied if the weir coefficient is varied between 0.61 and 0.80. This exponent, however, strictly applies to free discharge only—that is, for atmospheric pressure above the upper and below the lower nappe. This was not the case here and very seldom is with ogee sections in general. Hence, there seems to be no reason why a correction of the pressure head exponent should not be more logical than the variation of the weir coefficient.

Based on these model tests it is believed safe to assume that at least 290,000 cu ft per sec would be discharged through the spillway before the dam were overtopped. This happens to be about three times the peak flow into the reservoir during the flood of March 2, 1938, and more than six times the peak flow on record prior to that date.

The heterogeneous character of the rock, in place, called for a spillway channel with side walls independent of the rock cut. Hence counterfort walls with a 1 on 0.5 face batter and backfilled with gravel against the rock were provided as shown in Fig. 23(*a*) and 23(*b*).

For the same reason the floor slab was placed on a gravel mat in the upper (that is, relatively flat) part of the spillway channel. Stops, in the form of shallow concrete anchors on circular lines, were provided for the stability of the slab as shown in Fig. 23(*a*). A system of drains was provided under the slab and the backfill behind the side walls.

Fig. 23(*c*) shows a section through the waste channel.

The Outlet Works

The outlet works were incorporated in the diversion tunnel which directed the stream flow through the east or left abutment during construction of the dam. Fig. 25 shows the general arrangement. The cross section of the dam and its position relative to the outlet works are indicated in Fig. 25. The valve chamber at Station 9, Fig. 25, contains four units: Two 123-in. butterfly valves; one 96-in. butterfly valve; and one 51-in. butterfly valve.

At Station 21 there are: One 51-in. butterfly valve; two 51-in. by 39-in. needle valves; one 39-in. by 75-in. energy absorber; one 24-in. by 60-in. energy absorber; one 30-in. by 24-in. needle valve; one 30-in. butterfly valve; one 12-in. by 9-in. needle valve; one 102-in. by 90-in. needle valve; and two 129-in. by 117-in. needle valves. Section *A-A*, in Fig. 26, shows the diversion or outlet tunnel to be circular in cross section and 30 ft in the clear. The capacity of the

outlet works was based on the regulation of a so-called 50-yr capital flood, with a peak of 118,000 cu ft per sec to a maximum outflow of 50,000 cu ft per sec.

The necessary outlet capacity was thereby found to be 14,000 cu ft per sec with the water surface at the spillway lip.

A cylindrical trash-rack tower, circular in cross section, was provided at the intake end. Its principal design features are evident from Fig. 27. The four heavy columns, diametrically opposite, serve to make the structure earthquake resistant. This feature will become increasingly important with the anticipated future raising of the tower in keeping with the siltation of the reservoir.

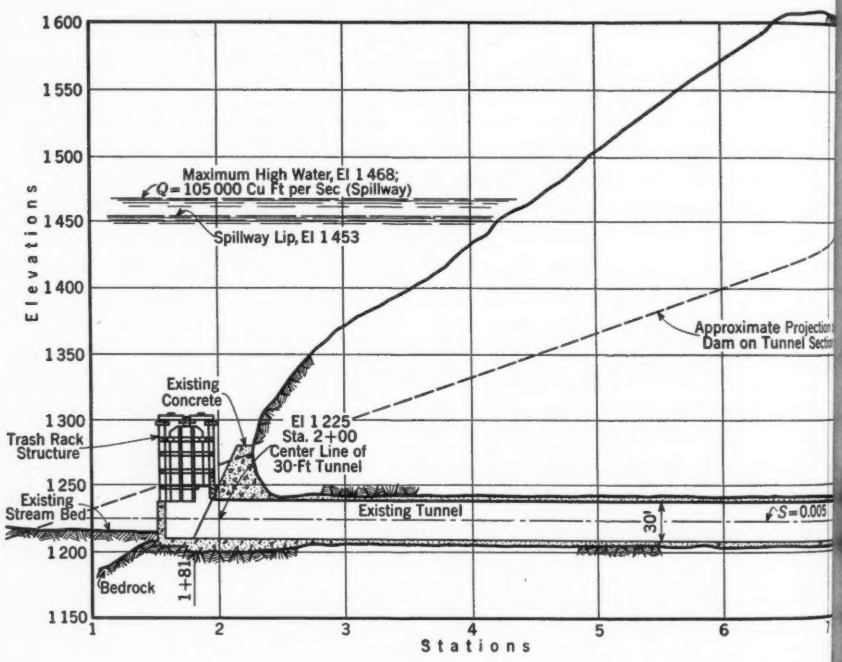


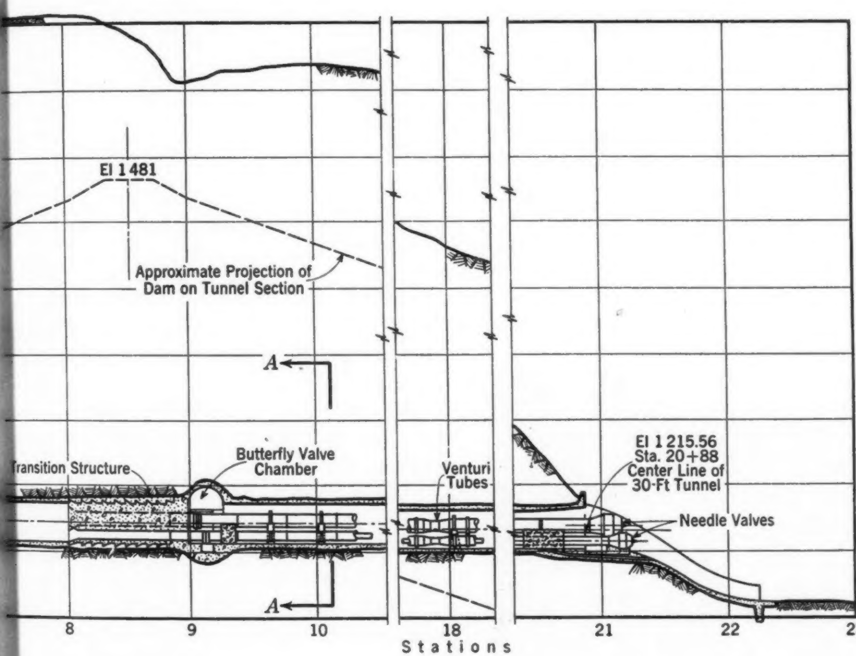
Fig. 25.—Longitudinal Section Center

voir. In this connection the design provided the blocking off by means of solid plain concrete of one or more lower tiers of trash-rack panels and the adding on top at least an equal number of tiers or stories of new trash-rack panels.

The gross area of trash racks initially provided was 5,920 sq ft or a net area of 4,950 sq ft after deduction is made for the bars. The velocity through the racks for the maximum discharge is 3.0 ft per sec.

Although the solid cylinder shell of the tower, including its base slab, was designed for pressures due to silt and water up to the spillway crest, the metal bars and ring girders supporting them were designed for pressure due to the water surface 50 ft above the top of the tower.

Approximately in line with the axis of the dam (Fig. 25) a concrete plug 100 ft long was provided with a chamber roughly ellipsoidal in shape immediately downstream from it. It was naturally desirable to place the plug as far downstream as possible in order to shorten the lengths of the penstocks. Obviously the feature governing the location was the allowable unbalanced internal pressure in the tunnel above the plug. Due to its seamy character, the rock was expected to carry some seepage water unless special provisions were made to stop it. As previously described, the cutoff wall and grout curtain along the upstream contact of Zone 2 were used to prevent this seepage.



Center Line of 30-Ft Tunnel

Although this provision, combined with the 6-in. shotcrete blanket under Zone 2, was believed to be an effective seal against water percolating around the ends of the dam and under it, no artificial seal was provided to prevent water from percolating through the left abutment due to direct contact with reservoir water as the overburden there was considered highly impervious and the path of percolation relatively long. Hence a conservative assumption had to be made as to the probable effective gradient of water percolating through the left abutment more or less parallel to the tunnel. This led to the maximum allowable excess internal pressure based on the strength of the existing tunnel lining.

The anchorage of the plug was provided by means of saw-tooth recesses in the existing tunnel lining. It was designed to safely resist full reservoir

pressure combined with the maximum pull due to thermal contraction of the penstocks. Facilities for the grouting of the contact zone between the plug

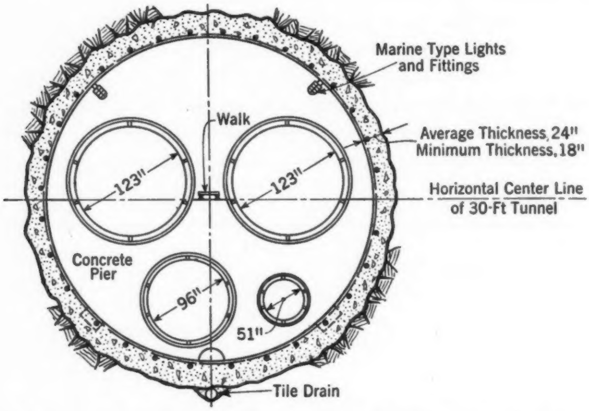


FIG. 26.—SECTION OF 30-FT TUNNEL (*A–A*, FIG. 25) SHOWING ARRANGEMENT OF OUTLET PIPES AND SUPPORTS

and tunnel lining, particularly in the upper half, were provided in the design. At the upstream end of the plug a ring curtain of grout was provided by means of radial grout holes extending about 20 ft into the rock.

FIG. 27.—TRASH-RACK TOWER

The chamber immediately below the plug was to house the butterfly valves forming the upstream control of four penstocks (Station 9, Fig. 25). The design allowed these butterfly valves to be operated by means of hydraulic cylinders, the latter being actuated by means of centrifugal oil pumps, capable of producing a static pressure of 3,000 lb per sq in.

At the lower end of the penstocks, needle valves were provided as well as certain diversion facilities for low-water flow. The penstocks were designed on the ring-girder principle with the pier supports 40 ft apart. The design further provided fixed ends with initial tension.

The penstocks were to be fabricated—that is, machine welded in 40-ft lengths in the shop, with provision for field-riveted joints and welded seams. An internal enamel coating approximately $\frac{1}{8}$ in. thick was to be applied by a centrifugal process. The joints were to be treated by hand application of the enamel in the field.

The hydraulic gradients were computed upon the following assumptions:

Entrance losses—

Trash rack (through the bars)	$0.5 \dfrac{V^2}{2g}$
Outlet tunnel (in the tunnel)	$0.065 \dfrac{V^2}{2g}$
Plug (in the plug)	$0.10 \dfrac{V^2}{2g}$

Friction losses (Manning's n)—

In the tunnel	0.013
In the plug	0.011
In the pipe	0.011

Valve losses—

Butterfly valves (in the pipe)	$0.32 \dfrac{V^2}{2g}$
Needle valves (at the outlet and including velocity head)	$1.562 \dfrac{V^2}{2g}$
Venturi tube losses (in the pipe)	$0.02 \dfrac{V^2}{2g}$

Approximately 300 ft upstream from the needle valves (Station 18, Fig. 25) venturi tubes were provided in all of the penstocks for the purpose of measuring the flow. Remote controlled flow-indicators and recorders were provided at the needle valves and in the control house, respectively.

Construction of the Revised Project

On August 12, 1935, the revised design was approved by the California state engineer, and on August 13 construction work was resumed.

Rock-fill material underlying Zones 2 and 3 was excavated and moved to Zone 6 as indicated in Fig. 2. Simultaneously the contractor started work on a large "grizzly" plant in Quarry 10, Unit 1 (original quarry) with a peak capacity of screened materials of 1,600 cu yd per hr. The plant arrangement was perhaps one of the boldest ever used. It is shown in Fig. 28. At the toe of the existing, nearly 600 ft high quarry face, a concrete tunnel was built to protect the "grizzly" mechanism and the loading trucks from falling rock. The excavated quarry material was dumped from the edge of the cut into a chute, blasted out of the quarry face, where it partly slid and partly fell down into a receiving crater, formed by a loose rock rim immediately above the "grizzly."

With the "grizzly" in motion the material would "crawl" along the slanting "grizzly" bars, pieces greater than 6 in. by 9 in. moving over the bars and into a waiting 24-yd truck and pieces smaller than 6 in. by 9 in. including fines falling through the openings into hoppers below the floor of the tunnel. The hoppers fed this material on to a belt conveyer which lifted it into two storage

bins of about 400 cu yd capacity each, shown in the foreground of Fig. 28. From these bins it was loaded into 24-yd trucks and hauled to the dam.

Immediately after completion of excavation of the old rock fill under Zones 2 and 3, the placing of compacted materials was begun. Specially designed sheepsfoot rollers were used, each two-drum unit (as previously stated) weighing about 25 tons. The minimum bearing pressure of each foot was 375 lb per sq in.

FIG. 28.—VIEW OF QUARRY 10

The cutoff wall across the canyon bottom had already been completed prior to the shut-down. Its extension, first on the east side and afterward on the west side, was now begun. The depth of the cutoff was estimated at 50 ft at the base and 25 ft at the top of the dam. It was excavated to sound rock either as a continuous trench or a series of shafts.

Special excavation was made where faults crossed the cutoff wall. According to specifications the cutoff wall had to be extended along these faults to a depth equal to one half the static head measured at the intersection of the fault with the top of the cutoff wall.

Grout holes varying from 1 in. to 8 in. in size and 100 to 150 ft in depth were drilled along the cutoff in two rows, 5 ft apart. The holes were spaced a maximum 10 ft on centers and staggered. The smaller holes, those 2 in. in size or smaller, were drilled with diamond core drills whereas the 8-in. holes were drilled by blast hole drills. The former were pressure grouted in the usual way and the latter served to apply the circulation method of grouting.

The principal excavation of the abutments was done in the conventional manner by benching from the top down and the rock surface in contact with the rock fill was stripped by means of a dragline which worked from the fill level to a height somewhat in excess of the next lift. The rock fill was placed in 25-ft lifts.

Special attention was given to the stripping of the rock surface under the shotcrete blanket, and a reasonably careful stripping job was done along the contact with Zone 3. Fig. 29 shows stripped areas and grouting operations.

To produce the water necessary for the sluicing of the loose rock fill and the sprinkling of the compacted rock and earth fill two 400-hp turbine pumps were installed in a sump below and one 150-hp turbine pump above the dam.

Monitors were used for the sluicing of the loose rock in Zones 1, 4, 5, and 6 and sprinkling trucks were used to supply the necessary moisture for the compaction of Zones 2 and 3.

The placing density of the fines in Zones 2 and 3 was checked by means of tests throughout the job. A total of 2,250 tests were taken in Zone 2 and 5,221 in Zone 3. This amounts to one test for every 500 and 620 cu yd, respectively, of materials placed. The average placing density was 117.3 and 122.20 lb per cu ft for Zones 2 and 3, respectively. Hence the actual average placing density was more than 2 lb greater than the specified one. The average unit dry weight of Zone 3 material, including rock and fines, was 144 lb per cu ft, when placed. It was probably raised to an average of 150 lb or more due to self-compaction. It assumed the properties of good concrete or artificial stone. The average moisture content was 5% of the dry weight of rock and fines combined. The $1\frac{3}{4}$-yd-dragline bucket was unable to "bite" into this mass until it had first been loosened with suitable tools.

Fig. 29.—Grouting and Drilling Operations

An unusual opportunity offered itself to measure the actual consolidation when the fill reached a height of some 70 ft above bedrock. At this level an abandoned power tunnel 4 ft by 6 ft in size, "day-lighted" near the axis of the dam, and had to be sealed by means of a 20-ft concrete plug. As the tunnel afforded access to the back of the plug it was possible to observe the settlement of the Zone 3 fill by means of manometers. These manometers consisted of two calibrated vertical glass tubes $\frac{1}{2}$ in. (inside diameter) connected to $\frac{1}{2}$-in. copper tubes. These copper tubes were installed inside of 2-in. standard pipe

and extended some 75 ft and 150 ft, respectively, into the fill. The copper tubes and 2-in. pipes were bent up at the fill end, the former being open and the latter capped. They were placed on an initial slope of 1%, the fill end being higher than the tunnel end.

By filling the copper tubes with water from the tunnel end it was possible to establish the elevation of the open end of the copper tubes by means of the manometer tubes.

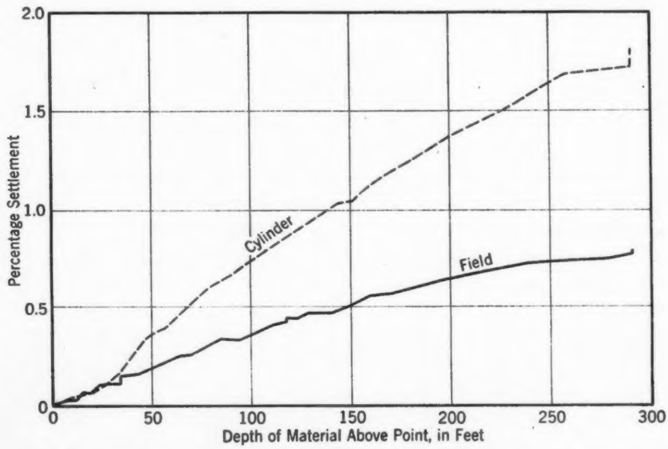


FIG. 30.—SETTLEMENT, ZONE 3, SHOWING RELATION BETWEEN FIELD AND LABORATORY TESTS

Simultaneously Zone 3 material was placed in one of the 24-in. cylinders in the laboratory at the same average density as had been established for Zone 3 up to that level. The cylinder was freed from the base to minimize arching, as previously outlined. Vertical pressures corresponding to those due to Zone 3 fill above the tubes were brought to bear on the material in the cylinder, and the settlements were recorded. The results are shown in Fig. 30.

OVERFILL AND SETTLEMENT (REVISED PROJECT)

Overfill to allow for residual settlement was provided as follows: None at the toe of the upstream slope, with a linear increase to maximum 2 ft at the crest; and none at the toe of the downstream slope, with a linear increase to maximum 3 ft vertically above the toe of Zone 3 and a linear decrease to maximum 2 ft at the crest.

Maximum vertical settlements to July, 1941, were as follows: 0.40 ft at the crest and 1.57 ft vertically above the toe of Zone 3. Under nearly full water load, the dam showed a maximum downstream deflection of 0.11 ft at the crest. With the emptying of the reservoir the crest returned substantially to its initial position.

The rock was quarried in 50-ft lifts by means of blast holes. Approximately 0.6 lb of black powder was used per cubic yard of solid rock. Including waste due to the original design as well as abutment excavation, a total of about 10,000,000 cu yd of solid rock was blasted.

The average production for two eight-hour shifts (6 days per week) was 26,000 cu yd of rock per day, placed in the dam. The maximum daily production was 42,000 cu yd and maximum monthly 975,000 cy yd. An average of 750 men were employed. Fig. 31 shows the construction of the dam completed and the spillway construction in progress.

FIG. 31.—SAN GABRIEL DAM NO. 1, COMPLETED; SPILLWAY UNDER CONSTRUCTION

PERCOLATION THROUGH THE DAM (REVISED PROJECT)

Based on the actual placing densities of material in Zones 2 and 3 and the corresponding permeability conditions, a study was made on the percolation through the dam with the water surface at the spillway lip for an indefinite period.

The results are shown in the following analysis, which is based on Darcy's law of laminary flow through soil:

$$\Delta Q = K\, i\, \Delta A = K \frac{\Delta h}{\Delta L} \Delta D \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which: ΔQ = flow in cubic feet per year per longitudinal foot of dam between two adjacent flow lines; i = hydraulic gradient for any path of flow; K = coefficient of permeability expressed as velocity in feet per year for unit area of flow and a hydraulic gradient of one; Δh = head or pressure interval in feet; ΔL = path interval in feet; and ΔA = area interval per unit length of dam in square feet = $\Delta L \times 1$. The flow net consists of flow lines and equipotential lines. The latter are perhaps better defined as equipiezometric lines. They are normal to each other and for equal head intervals form a system of squares or quasi squares as shown in Fig. 32. The flow lines are spaced such

as to have equal flow ΔQ through any area interval ΔA between two flow lines. Hence, because $\Delta D = \Delta L$, Eq. 6 becomes

$$\Delta Q = K\,\Delta h \qquad (7)$$

The coefficients of permeability K_1 and K_2 for Zones 2 and 3 were found from samples of test holes to be 0.35 and 1.05 ft per yr, respectively.

Continuity of flow through Zones 2 and 3 requires that

$$\Delta Q = K_1\,\Delta h_1 = K_2\,\Delta h_2 \qquad (8)$$

or

$$\frac{K_1}{K_2} = \frac{\Delta h_2}{\Delta h_1} = \frac{0.35}{1.05} = \frac{1}{3} \qquad (9)$$

The flow net was constructed by trial and error. The flow through the dam was determined as follows:

$$Q = n_1\,\Delta Q = n_1\,K\,\Delta h = \frac{n_1\,K\,h}{n_2} \qquad (10)$$

in which: n_1 = number of flow paths and n_2 = number of head intervals. Hence, for Zone 2,

$$Q = \frac{K_1\,h\,15}{16} \qquad (11a)$$

and, for Zone 3,

$$Q = \frac{K_2\,h\,5}{16} \qquad (11b)$$

Let h = 303 ft; K_1 = 0.35; and K_2 = 1.05. By Eqs. 11, therefore, Q = 99.50 cu ft per yr, and

$$V_{\max} = K_2\,\frac{\Delta h}{\Delta L_{\min}} \times \frac{1}{(\%\ \text{voids})} \qquad (12)$$

That is, $V_{\max} = 1.05 \times \dfrac{303}{16 \times 20 \times 0.16} = 6.20$ ft per yr.

The Spillway

The spillway was constructed under separate contract. It was begun in June, 1937, and was completed in May, 1938. The spillway channel proper (that is, the concrete ogee section, walls and floor) was completed two weeks before the flood of March 2, 1938, at which time the spillway carried 57,000 cu ft per sec. Fig. 33 shows typical features of the spillway construction.

The Outlet Works

The installation of valves and penstocks was done under separate contract. The 51-in. butterfly valve and penstock were installed first and served to control the low-water flow of the river. For this purpose the piers were built in sections. The 96-in. butterfly valve and penstock were installed next. This latter work was in progress at the time of the flood. Hence the outlet works

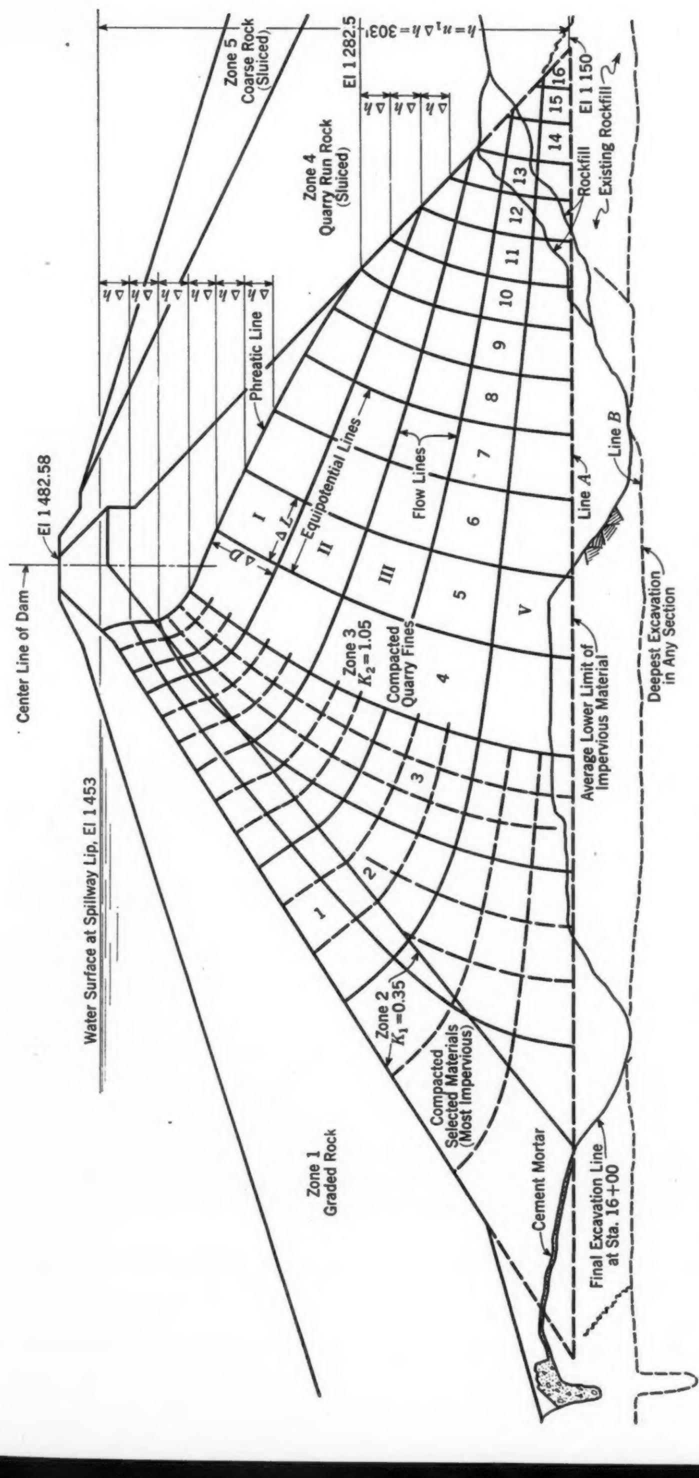


FIG. 32.—TYPICAL FLOW NET DIAGRAM FOR AN AVERAGE MAXIMUM SECTION

were not in an operating condition and no appreciable regulation was possible at that time.

A complete re-survey of the reservoir indicated that more than 10,000,000 cu yd of debris had been deposited in the reservoir during the flood and prac-

TABLE 6.—Pressure Losses in Outlets, Computed from Design Assumptions and from Observed Data

(All Values to Be Multiplied by $\frac{V^2}{2g}$)

Outlet	Total Energy[a]		Total Line Losses[b]			Venturi[c]		Needle Valve Losses[d]		
	De-sign	Ob-served	Design losses[e]	Observed		De-sign	Ob-served	Design losses[f]	Observed	
				n	Losses				c	Losses
51 in.	0.796	0.661	3.82	0.0095	2.87	0.020	....	0.562	0.738	0.555
123 in. (east)	0.535	0.410	1.187	0.0087	0.744	0.020	0.062	0.562	0.680	1.147
123 in. (west)	0.530	0.377	1.193	0.0087	0.746	0.020	0.240[g]	0.562	0.690	0.916

[a] Total energy loss from the reservoir through the butterfly valve in the tunnel, on the basis of the velocity head in the pipe.
[b] Total line losses between the butterfly valve and the needle valve on the basis of the velocity head in the pipe.
[c] Net loss in the venturi tube on the basis of the velocity head in the pipe.
[d] Loss in the needle valve on the basis of the velocity head in the valve outlet (valve 100% open), based on the gross area of the valve outlet.
[e] Manning's $n = 0.011$.
[f] Discharge coefficient $c = 0.80$.
[g] Includes bend loss.

TABLE 7.—Estimated and Final Quantities

Description	Estimated: Prior to Aug. 13, 1935	Estimated: Revised design	Final: Prior to Aug. 13, 1935	Final: Revised design
(1)	(2)	(3)	(4)	(5)
(a) Quantities, in Cubic Yards				
Rock Fill and Earth Fill:				
Zone 1, loose rock fill		2,685,000		2,575,349
Zone 2, earth fill		1,042,000		1,144,220
Zone 3, compacted rock fill		3,273,000		3,211,415
Zone 4, loose rock fill		1,905,000		1,885,563
Zone 5, loose rock fill		1,068,000		1,019,170
Zone 6, loose rock fill		722,000		753,714
Totals	5,562,000[a]	10,675,000	453,659[a]	10,599,431
Stream-bed excavation	1,292,000	110,597	873,403	104,127
Abutment excavation	525,000	559,881	283,119	478,667
Quarry strippings	900,000	150,000	2,201,028	460
Quarry waste	1,800,000	75,000	1,411,892	none
(b) Costs, in Dollars				
Dam proper	10,355,910.00			10,391,834.63
Spillway	478,303.46			376,393.30
Outlet works	804,000.00			1,209,586.33
Sub-total	11,638,213.46		5,023,317.39	11,977,814.26
Grand total (Col. 5 plus Col. 4)				17,001,131.65

[a] Rock fill.

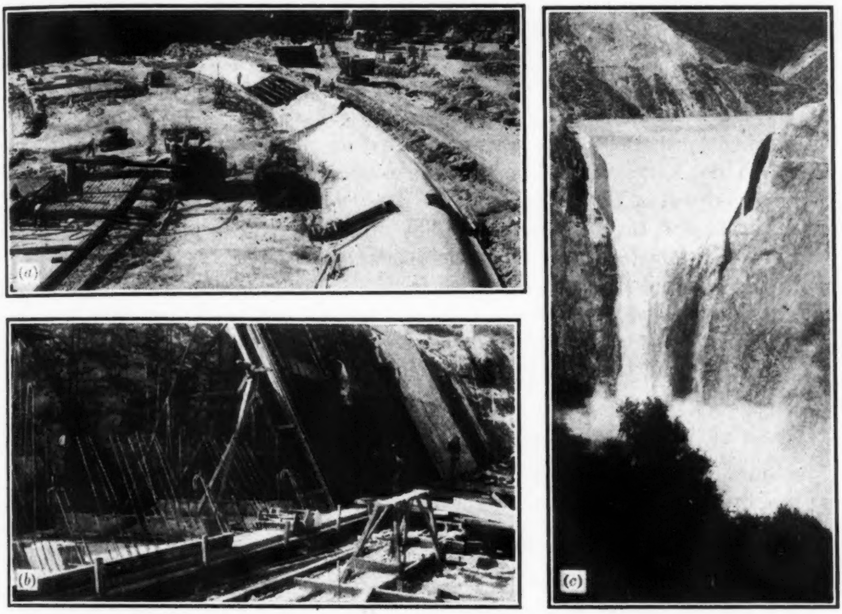


FIG. 33.—TYPICAL FEATURES OF THE SPILLWAY CONSTRUCTION
(a) General View of the Upper Part; (b) Wall Section Showing Steel and Forms; (c) Spillway in Action

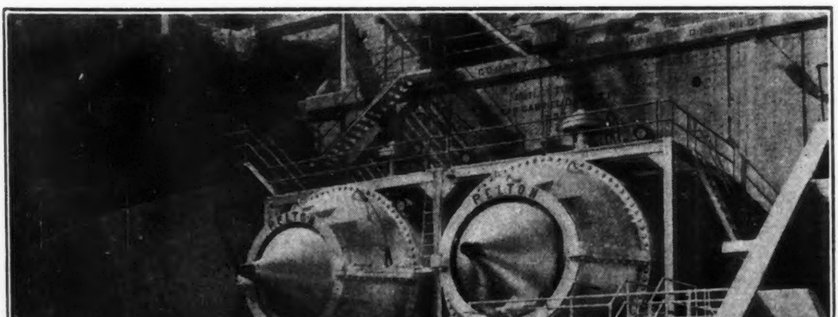


FIG. 34.—TWO 123-IN., ONE 51-IN., AND ONE 12-IN., NEEDLE VALVES, COMPLETE, SHOWING TEMPORARY NOZZLE ON THE 96-IN. OUTLET

tically in the course of one day, approximately 75% of which consisted of sand and silt. This quantity happens to be about equal to the total yardage of rock and earth fill "expeditiously" placed in the dam over a period of two years!

The installation of the outlet works was completed in October, 1938. Fig. 34 shows the installation of the outlet works completed.

During April, 1940, tests were conducted on the penstocks to ascertain the actual energy losses versus the theoretical ones. The initial water surface in the reservoir was at El. 1360±. Hence the head was ±60% of the design head (water surface at the spillway lip). The results are shown in Table 6.

It is of interest to note that these tests were made at very high values of Reynolds' number, which extended from 2×10^6 to about 3.8×10^7 for the highest flows in the large pipes.

The maximum height of the dam above its foundation is 376 ft. Therefore, it may claim the distinction of being the highest dam of its type in the world at this time.

Quantities and Costs

A summary of the estimated and final quantities and their handling costs, is presented in Table 7.

Acknowledgments

Credit is due S. M. Fisher, M. Am. Soc. C. E., chief engineer of the District from September, 1934, to February, 1935, under whose supervision the revised design of the dam was prepared in its essentials.

C. H. Howell, M. Am. Soc. C. E., was chief engineer of the District from February, 1935, to October, 1938. By his untiring efforts, he succeeded in the elimination of seemingly unsurmountable obstacles to make the completion of the project possible in accordance with the revised design, which had been supplemented and perfected under his supervision.

The writer is indebted to H. E. Hedger, M. Am. Soc. C. E., chief engineer of the Los Angeles County Flood Control District, for valuable advice and for making District records available for the preparation of this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

AN INVESTIGATION OF STEEL RIGID FRAMES

Discussion

By A. C. Barrow, Assoc. M. Am. Soc. C. E.

A. C. Barrow,[22] Assoc. M. Am. Soc. C. E. (by letter).[22a]—The data presented in this paper can be tested by applying the formula:

$$f = \frac{P}{A} \pm \frac{P\,e\,c}{I} \qquad (22)$$

in which: f represents a unit stress at either edge of the section, depending on whether the plus or minus sign is used; P represents the force acting on a section, symmetrical about at least one axis through the center of gravity of the section; A represents the area of section under consideration; e equals the eccentricity of the line of action of the force F; c is the distance from the center of gravity of the section to the edge of the section where the value of f is to be determined; and I is the moment of inertia of the section about an axis through the center of gravity and at right angles to the axis through its center of gravity and the point of application of the force P. The flange stresses throughout the rigid frame can be computed by Eq. 22, both for square knees and curved knees.

The Curved Knee.—The knee in these frames (see Fig. 18) may be considered to be loaded or stressed by a moment produced by an eccentric loading, consisting of the horizontal reaction acting through the hinge and the vertical reaction acting through the same point and decreasing the resulting moment of the horizontal reaction.

The writer made a drawing of this frame to a scale of 6 in. equals 1 ft, and the lever arms of these forces were scaled from it. First, a vertical section was passed through point 27, Fig. 18. The moment of the horizontal reaction about the center of this section was obtained. This was decreased by the moment of the vertical reaction of 6,000 lb about the same point. The result was multiplied by the distance of the outermost fiber from the center of the section, and

Note.—This paper by Inge Lyse, M. Am. Soc. C. E., and W. E. Black, Jun. Am. Soc. C. E., was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1941, by C. J. Posey, Assoc. M. Am. Soc. C. E.; February, 1941, by W. J. Eney, Assoc. M. Am. Soc. C. E.; March, 1941, by Messrs. LaMotte Grover, and William R. Osgood; and April, 1941, by Messrs. M. Hirschthal, and Jaroslav J. Polivka.

[22] Cons. Engr., Mount Sterling, Ky.

[22a] Received by the Secretary July 14, 1941.

this result was divided by the moment of inertia of the section about its center and at an angle of 90° with the vertical axis of the plate. The horizontal reaction of 4,350 lb was divided by the area of the vertical section. This latter value was subtracted from the result already obtained to find extreme fiber stress in the outer flanges and added to find the stress in the inner flanges. The gross moment of inertia was used. The author states (see heading "Relation of Test Data to Analysis: Moment of Inertia") that "The effective moment of inertia was slightly less than the gross moment of inertia * * *," and that "If the gross moment of inertia had been used, the error would have been about 3%, for this girder." Therefore, 3% was added to the foregoing results. The inner edge of the knee was at an angle of 45° at point 27 and the value found for the inner flanges was multiplied by the secant of 45°.

TABLE 5.—COMPARISON OF OBSERVED AND COMPUTED STRESSES

(In Kips per Square Inch)

Description	Point (See Fig. 18):														
	3	4	5	6	7	14	15	16	17	18	27	28	29	30	31
Eq. 22[a]	17.5	18.9	16.4	12.5	10.5	8.2	11.4	15.2	17.6	16.2	13.8	15.1	18.4	19.9	18.2
Fig. 18	18.5	18.0	16.0	14.0	11.0	10.0	13.0	16.0	18.5	19.0	14.0	15.5	18.5	22.0	22.5

[a] Computed values rounded out to nearest 100 lb per sq in.

Table 5 shows the results of the values thus determined compared with the observed values as shown in Fig. 18.

A different procedure is required for finding the flange stress in the vertical leg of this frame. It is obvious that the flange stress at point 26 is the same as that at point 28, except that it carries its proportion of uniformly distributed force, producing the vertical reaction of 6 kips, over a horizontal section through point 26. Fig. 18 shows the unit stress at point 26 to be slightly more than at point 28. From this consideration there should be a difference of about 1 kip. The difference in the stress at these points increases down the vertical leg, and this is as it should be because the total stress is the same (6 kips) and the area of the section decreases to and including point 23.

For point 27, the agreement is close (Table 5), but for other points on the inner edge of the knee the difference between the computed values and the observed values increases. Eq. 22 is valid only for short increments of length, and it is the elasticity of the frame at these points that causes a greater stress and increased differences. They are given to show how far Eq. 22 may be used to check the observed values.

The Square Knee.—Applying Eq. 22 to the square knee and using a horizontal reaction of 5,280 lb, the stress at the inner corner is 16,571 lb per sq in. and the observed stress is about 17,000 lb per sq in. Just above this point, there is a computed stress of 14,473 lb per sq in. and an observed stress of about 14,000 lb per sq in. (Fig. 16).

In the paper it is stated (see heading "Test Specimens") that

> "At the intersection of the compression flanges at both knees, where tight bearing should be obtained, small gaps existed. It was considered advisable to fill these gaps with shims that were tackwelded in place, but which produced tight bearing only along the outstanding legs of the girder flange angles."

This connection produces another case of eccentric loading, and Eq. 22 determines a stress of 23,055 lb per sq in. at this point. Fig. 16 shows an observed stress of 23,000 lb per sq in. in the horizontal flanges some distance from this inner corner. The stress decreases rapidly from the inner corner and, if this line is produced to a point in the graph directly opposite the inner corner, a stress of 25,000 lb per sq in. would be indicated.

The observed and computed stresses both show clearly the danger of imperfect bearings at the inner corner of the rigid frame with a square knee.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

FORT PECK SLIDE

Discussion

By Frank E. Fahlquist, Assoc. M. Am. Soc. C. E.

Frank E. Fahlquist,[20] Assoc. M. Am. Soc. C. E. (by letter).[20a]—The engineering profession is indebted to Mr. Middlebrooks for his thorough analysis of the causes and behavior of the Fort Peck Dam slide. To the writer, at least, the crux of the entire problem is indicated in the author's discussion of the rock foundations, as stated under the heading "Test and Analysis Prior to Slide: Slides on Shale" (first thirteen lines). Also of particular interest are the first ten lines of the author's concluding paragraph.

During the summer of 1931, the writer had an opportunity to study the geology of several potential dam sites in the Missouri River Valley of Montana. Although the time available for this work was limited, and the area covered was great, sufficient information was obtained to indicate that, before any final selection of sites, designs, or constructions could be made, consideration should be given to several important problems bearing on the geology of the Missouri River Valley and the design and construction of dams. These geologic problems have not been adequately described in the paper nor in any other accounts of the Fort Peck Dam slide that have come to the writer's attention.

The Missouri River Valley, from the North Dakota state boundary line to Great Falls, Mont., is formed in an alternating series of inclined shale and sandstone formations. These rocks are of Cretaceous age and, therefore, in a geologic sense, are comparatively young. As a result, but chiefly because there has been only a slight burden of overlying sediments, subsequently removed by erosion, these rocks have behavior characteristics that are not ordinarily a part of rock formations found in other parts of the United States. The several sandstone members of this sedimentary series, throughout the eastern half of the valley, in general, are poorly cemented and friable. The shale members (that at the Fort Peck Dam being known as "Bearpaw" shale) are only partly consolidated or indurated. However, where this formation

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Note.—This paper by T. A. Middlebrooks, Assoc. M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Jacob Feld, M. Am. Soc. C. E.; April, 1941, by Joel D. Justin, M. Am. Soc. C. E.; and May, 1941, by Messrs. William Gerig, Alfred J. Ryan, and Glennon Gilboy.

[20] Senior Geologist, U. S. Engrs., War Dept., Providence, R. I.

[20a] Received by the Secretary June 10, 1941.

occurs buried beneath an overburden of soil or water, and, further, where it has not been affected by any geologic disturbances, the rock is firm and stable. These shales, however, have two undesirable characteristics or properties: (1) They contain stratifications of bentonite ranging from a fraction of an inch to several inches in thickness, and (2) when uncovered and exposed to the air, they readily disintegrate or slack into innumerable small pieces.

In his reconnaissance of the valley, the writer was impressed at the evidences of landslides of both recent and old age. Several areas were noted where large masses of shale and sandstone had broken off of the inner valley cliffs and slid on to the valley bottom. Some of these landslides were pronounced and easily identified, whereas others, being eroded and probably of greater age, were difficult to recognize. At one dam site, located at a considerable distance above the Fort Peck Dam, but within the reservoir, landslide activity had extended over a large area, causing the writer to recommend elimination of the site from further consideration.

With these extraordinary geologic conditions in mind, the writer, in submitting his report,[21] outlined several broad concepts that should be considered in the investigation, design, and construction of any dams located in the Missouri River Valley:

(1) That the air-slacking shale presented a special construction problem in rock excavation work. It was not anticipated, however, that this property would create any serious difficulties, as the condition could be easily remedied by forced applications of portland cement grout on freshly excavated rock surfaces. Actually, in the construction of the Fort Peck Dam, asphalt was used, which is equally suitable. In addition, the ease with which the shale disintegrated precluded its use as riprap or in rock fills. Obtaining more suitable and durable rock from distant igneous rock areas was recommended. Actually, hard, durable boulders were shipped in to the site from a remote area.

(2) That the occurrence of bentonite in the shale formation and evidence of landslide activity presented the most serious problem. The writer judged that the bentonite stratifications could be, and probably were, the cause of many of the landslide effects. Because the Bearpaw formation extends over such a large area, the writer was led to report that:

(*a*) Landslide activity should be investigated at any sites finally selected;

(*b*) If any landslide blocks of large proportions were found to occur at any of the selected sites, the question of further study and construction or abandonment was a matter of cost versus benefits, compared with other sites; and

(*c*) In the case of all dams in the valley, particular care should be exercised in foundation and abutment constructions. If surface and subsurface investigations revealed occurrence of landslides or other disturbance in the foundation rock, this condition, together with occurrence of bentonite stratifications, would be serious, and probably would warrant

[21] Report to the U. S. Engineers, Kansas City District, Kansas City, Mo., November, 1931.

excavation and removal of all rock affected by these deformations. This precautionary measure would be necessary to assure founding the dam in the abutment areas on undisturbed bedrock.

The author's discussion of the rock foundations, under the heading "Tests and Analysis Prior to Slide: Slides on Shale" (particularly his description of conditions disclosed by later investigation), leads the writer to believe that the 30-ft thick zone of blocky weathered shale that gradually passes into a subfirm and firm shale is quite likely an eroded remnant of an old landslide block. The observations as disclosed by detailed investigations tally exactly with conditions observed by the writer in the field. The author's conclusion that the solution to similar problems may be found by obtaining representative undisturbed samples does not go far enough. The very first essential in a problem of this kind, or in any foundation or abutment problem, is an understanding of the geologic history of the area or region. Such an understanding, no matter how meager the evidence is at first, will help one to formulate a method of attack for accumulating more pertinent information and to separate the more serious and important problems from those of less importance. Undisturbed samples are fine and necessary, as is also the analysis of stress and strain, but the writer can easily imagine cases in which any amount of undisturbed sampling and mathematical analysis could not fully develop a complete geologic picture. For example, consider the blocky weathered condition described by the author. In this case there is a disrupted mass of rock, a shale which disintegrates and slacks readily, which contains stratifications of bentonite, and which, when weathered and wet at the surface, forms a sticky and slippery mass of soil. When such conditions are recognized, the only adequate exploration to confirm or disprove preliminary geologic conceptions is twofold: (1) A few well-selected and carefully executed borings to outline the extent of such geologic conditions, and (2) real life-size excavations to explore the condition fully, in area and in depth. When this exploratory work is completed, all testing done, and all factors of safety computed, one should bear in mind that there may still be some worse condition not yet disclosed, and then one should consider carefully whether it would not be best to remove the entire questionable mass and build on undisturbed material.

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DISCUSSIONS

PLASTIC THEORY OF REINFORCED CONCRETE DESIGN

Discussion

By Messrs. O. G. Julian, and Charles S. Whitney

O. G. Julian,[71] M. Am. Soc. C. E. (by letter).[71a]—In 1904 Arthur Newell Talbot, Past-President, Am. Soc. C. E., stated:[72]

> "The principles governing the strength and action of reinforced concrete construction have not been fully established and opinions and theories presented by engineers are somewhat conflicting."

That statement is as true today as it was 37 years ago. This is evinced by the discussion by sixteen authorities from eight different countries[3] regarding Prof. Rudolf Saliger's method published in 1937, by numerous articles regarding the effect of plastic flow published within the past fifteen years,[73] and by the discussion on The Joint Committee Report[74] published in 1940.

Concrete is not an elastic material but has characteristics not unlike those of a viscous fluid as defined by James Clerk Maxwell:[75]

> "If the stress when it is maintained constant causes a strain or a displacement in the body which increases continually with the time, the

Note.—This paper by Charles S. Whitney, M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by Messrs. L. E. Grinter, and Basil Sourochnikoff; March, 1941, by Messrs. R. W. Stewart, George C. Ernst, Homer M. Hadley, and Robert W. Abbett; April, 1941, by Messrs. Paul Andersen, and R. A. Caughey; May, 1941, by Messrs. Roberto Contini, and A. A. Eremin; and June, 1941, by Messrs. Jaroslav J. Polivka, and Paul W. Abeles.

[71] Chf. Structural Engr., Jackson & Moreland, Boston, Mass.

[71a] Received by the Secretary August 6, 1941.

[72] *Bulletin No. 1*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1904.

[3] "The Modular Ratio—A New Method of Design Omitting *m*," by K. Hajnal-Konyi (and discussion by various others), *Concrete and Constructional Engineering*, January, February, March, May, June, July, August, September, October, 1937.

[73] "Flow of Concrete Under Sustained Compressive Stress," by Raymond E. Davis, *Proceedings*, Am. Concrete Inst., 1928, p. 303 *et seq.*; "Plain and Reinforced Concrete Arches," by Charles S. Whitney, *Journal*, Am. Concrete Inst., Vol. 3, 1931–1932, p. 485 *et seq.*; and "Studies in Reinforced Concrete, Further Investigations on the Creep or Flow of Concrete Under Load," by W. H. Glanville and F. G. Thomas, *Technical Paper No. 21*, Building Research Dept. of Scientific and Industrial Research, London, England, October, 1939.

[74] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., June, 1940, Pt. 2; discussion in September, 1940, p. 1351; November, 1940, p. 1710; December, 1940, p. 1846; February, 1941, p. 247; March, 1941, p. 457; April, 1941, p. 727; May, 1941, p. 879; and June, 1941, p. 1087.

[75] "Theory of Heat," by James Clerk Maxwell, London, Longmans, Green, 1899, p. 295; see also discussion by J. H. Griffith of "Plastic Flow in Concrete Arches," by Lorenz G. Straub, *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 688.

substance is said to be viscous. When this continuous alteration of form is only produced by stresses exceeding a certain value, the substance is called a solid, however soft it may be. When the very smallest stress if continued long enough will cause a constantly increasing change of form the body must be regarded as a viscous fluid, however hard it may be."

Messrs. W. H. Glanville and F. G. Thomas state:[76]

"From the tests at the Building Research Station, it appears that if there is such a yield stress for concrete below which no inelastic deformation occurs as a result of loading, the value of this stress is so small as to be negligible."

General practice for reinforced concrete design is based on the Bernoulli-Eulerian theory that plane sections remain plane and that stresses follow Hooke's law. Experimental evidence indicates that, neglecting the effect of shearing detrusion, plane sections remain plane but that Hooke's law does not apply. If, as was the advocated practice prior to 1924, working stresses are limited to about one third of the ultimate unit strength of test cylinders, and if the variation of the equivalent modulus of elasticity with the duration of the loading is taken into account, the order of magnitude of the actual stresses and deflections may be computed on the basis that Hooke's law is valid. On account of the number of factors involved, the evaluation of the equivalent modulus of elasticity of concrete, even for low stresses, is highly speculative. As allowable stresses are raised, a point is reached where the action of the concrete is not even approximately elastic. At this point the application of Hooke's law and the use of the concept, equivalent modulus of elasticity, should be abandoned. It is at least questionable if this point has not been passed when working stresses equal $0.45\, f'_c$ for ordinary loads ($0.60\, f'_c$ with the effect of wind loads included) as is advocated at present. A proved method of design that does not involve the highly speculative quantity E_c should be accepted as a boon to the profession. Such a method, which is also simple, easily understood, and frankly recognizes concrete for what it is—a viscous or plastic material rather than an elastic material—is presented in the author's paper. For those cases in which static loads only are involved the method appears rational and amply proved empirically. The paper is comprehensive in as far as members subjected to unilateral flexure and such flexure combined with direct stress are concerned. However, it does not explicitly cover the important case of bilateral flexure and direct stress. Practically all columns of reinforced concrete frames and especially corner columns are subjected to such moments and direct stresses. It is hoped that the author will cover explicitly this important case in his closing discussion.

It has been proved conclusively by independent tests[77,78] that, in the case of reinforced concrete columns subjected to axial forces only, the ultimate strength is independent of the elastic moduli, plastic flow, shrinkage, and temperature

[76] "Studies in Reinforced Concrete, Further Investigations on the Creep or Flow of Concrete Under Load," by W. H. Glanville and F. G. Thomas, *Technical Paper No. 21*, Building Research Dept. of Scientific and Industrial Research, London, England, October, 1939.

[77] "Fourth Progress Report on the Column Tests Made at the University of Illinois," by F. E. Richart and G. C. Staehle, *Journal*, Am. Concrete Inst., Vol. 28, 1932, p. 279 *et seq.*

[78] "Fourth Progress Report on Column Tests at Lehigh University," by Inge Lyse and C. L. Kreidler, *loc. cit.*, Vol. 3, 1931–1932, p. 317 *et seq.*

effects. For such columns, provided they are adequately reinforced laterally, the ultimate strength is a function of the yield point strength of the steel and the ultimate strength of the concrete only. In such a member the concrete, being viscous, gradually flows away from the load and in so doing transfers stress to the steel. When the load becomes equal to the yield strength of the steel, the latter becomes plastic, yields, and transfers stress to the concrete. Upon application of further increments of load the steel continues to carry load equal to its yield point strength, the remainder, of necessity, being carried by the concrete. When the concrete becomes stressed to its ultimate strength, it and the column fail. The ultimate load, P, for the column is given by the well-known formula,

$$P = R f'_c A_c + f_s A_s \qquad (61)$$

in which R is the ratio of the ultimate strength of the concrete in the column to f'_c of comparable concrete in test cylinders. For unreinforced columns

$$R = \frac{P/A}{f'_c} \qquad (62)$$

Tests[77] indicate variation in the value of R from 0.73 to 1.00 for members in air storage and from 0.46 to 0.80 for comparable members stored moist. It is somewhat questionable as to whether the value 0.85 used by the author is warranted, unless a high factor of safety is used.

It appears reasonable that much the same process, as described herein for an axially loaded column, takes place in a member subjected to flexure. Eq. 22, which is derived on the basis that there is sufficient compression steel to take all the compression without exceeding the yield point, implies that the concrete flows viscously and in time transfers all its load to the steel. By taking moments about the tension steel the yield point strength of the compression steel necessary to make Eq. 22 valid is found to be

$$A'_s f'_s = -P\left(\frac{2e + d'}{2d'}\right) \qquad (63)$$

in which the negative sign is used to signify compression. The sum of $A'_s f'_s$ and $A_s f_s$, of course, equals P. In this case the concrete contributes nothing toward resisting the load. A basic principle of the theory is that for loads of protracted duration the concrete in time relieves itself of all stress, provided the contiguous steel is not stressed beyond its yield point, and then, provided it is sufficiently strong, the concrete resists that part of the load that is not taken by the steel working at yield point stress. It should be emphasized that, in order to act in this manner, the concrete must have time to flow. This being the case, it appears that the theory may not be applicable to cases involving suddenly applied loads of transitory or cyclic nature such as impact or those resulting from the vibration of high-speed machines. It may be that the stress-strain relation for concrete subjected to very rapidly applied loads is practically linear up to the point of failure and that this point is materially different from that for static loads. Data on these points are not readily available.

The possible exclusion of the theory from the realm of impact loads leaves it a wide field of application, and there appears to be no good reason why it should not be widely used. It is interesting to note that Hardy Cross, M. Am. Soc. C. E., in 1931 stated:[79]

> "The method which we always use in America of computing the stresses in such [compressive] reinforcement on the basis of an assumed value of n is quite naive. * * * It may not be proper to go so far in this case as it is proposed to do in column design and depend upon the steel to carry load up to its yield point, but the tendency is evidently in that direction."

Since, as pointed out by the author, a balanced ultimate strength design requires several times the quantity of steel needed for a balanced design by the straight-line elastic theory, high yield point steel must be used in small beams if the design approaches balance for ultimate strength and the beams are not unduly crowded with steel.

Differentiating M with respect to f'_c in the equation given in Fig. 7, the following is obtained,

$$\frac{\partial M}{\partial f'_c} = 0.456 \left[\frac{p}{p_0} - 0.256 \left(\frac{p}{p_0} \right)^2 \right] b\, d^2 \ldots\ldots\ldots\ldots (64)$$

This quantity is small and practically equal to $0.456 \frac{p}{p_0} b\, d^2$ when $\frac{p}{p_0}$ is small but approaches $\frac{1}{3}\, b\, d^2$ as $\frac{p}{p_0}$ approaches unity. For a beam of given cross section there is little increase in the resistance moment for a given increase in f'_c in case the steel ratio is small; but there is considerable increase when balanced design is approached. It appears, therefore, that there is little advantage in using high-strength concrete with a low steel ratio but that there is considerable advantage in using such concrete with balanced or nearly balanced design.

It is believed that the curved stress diagrams in Figs. 2 and 4 should indicate the maximum stress at the upper extremity of the beam. The diagrams as drawn imply that, in the process of loading the member, the laminæ near the upper edge of the beam were stressed to their ultimate strength and then, instead of failing, were relieved somehow of some stress while the load was still increasing. It is understood that, as the ultimate strength of the member is approached, the laminæ nearest the top of the beam transfer stress to those nearer the neutral plane, and that this process may continue until the stress for a substantial depth of the beam is practically constant—that is, rectangular stress distribution is approached. However, that laminæ stressed to their ultimate strength (in other words, broken) transfer a part of, but not all, the stress they were carrying to adjacent laminæ appears impossible. It is difficult to imagine the stress caused by a given strain being less than that caused by a lesser strain. These stress diagrams appear to require further justification. However, the point is largely academic and in no way invalidates the author's main argument. The exact shape of the stress diagram is of secondary interest.

[79] Discussion by Hardy Cross of "Flow of Concrete Under the Action of Sustained Loads," by Raymond E. Davis and Harmer E. Davis, *Journal*, Am. Concrete Inst., Vol. 3, 1931–1932, p. 265 *et seq.*

Other authorities have assumed the stress diagram to be represented by various curves, including parabolas up to the fifth degree.[80] These may all be replaced by the hypothetical rectangular diagram used by the author and others[80,81] as a mathematical tool. This concept, although it does not indicate the location of the neutral axis, is the simplest and hence the one best adapted for use in practical design work.

It is customary, in design, to refer the moment of the external loads to the centroidal axis of the member. However, in the equation immediately following Fig. 11 and in Eqs. 23 and 31 the quantity M is referred to the axis of the tension steel rather than to the axis customarily used. It would appear preferable to omit M from these equations, as it may lead to confusion.

Regarding factor of safety the author states (see heading "Flexure and Direct Load on Rectangular Sections"), "* * * it is customary to provide a greater factor of safety for axially loaded columns than for beams * * *." Factors of safety for columns, according to current codes,[82] vary from 4.5 for lightly reinforced tied columns to 2.75 for heavily reinforced spiral columns. The actual factor of safety of over-reinforced beams is not $\frac{f'_c}{f_c}$—which, according to current codes, is 2.22—but, as has been shown by the author (Table 7), 1.62 times that amount or 3.60. Furthermore, if full cognizance is taken of the eccentric loading to which practically all columns are subjected, and the author's method of design is used, there appears to be no good reason why comparable columns and beams should not be proportioned for identical safety factors. As stated by the author the first pronounced crack in a beam may occur at about 25% of the ultimate load; the corresponding percentage for a column increases as e decreases. If the appearance of the first pronounced crack is to be taken as a criterion, there appears to be considerable justification for using a higher factor of safety for members subjected to loadings with large eccentricities than for comparable members subjected to loadings with lesser eccentricities. This is the reverse of the author's statement as to customary practice. The writer cannot concur with the author's statement (see heading "Method of Design and Factor of Safety"),

> "Reasonably well-controlled concrete is such a reliable material at present that the use of any factor of safety to provide for sub-standard concrete can be eliminated by using f'_c as the minimum cylinder strength instead of the mean."

On jobs where considerably better than average supervision was exercised, the concrete strength was found to vary 30% either way from the mean. With sympathetic, omnipresent, and omnipotent supervision it has been found that the mean strength may be raised about 36% and the variation from this mean reduced to less than 10%. However, it is doubtful if such supervision will be

[80] Discussion by Inge Lyse of "Design of Reinforced Concrete Members Under Flexure or Combined Flexure and Direct Compression," by Charles S. Whitney, *Journal*, Am. Concrete Inst., Vol. 8, 1936–1937, p. 498–21 *et seq.*

[81] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931, Chap. 24, p. 168; and "Strength of Materials," by S. Timoshenko, D. Van Nostrand, 1930, Chap. 7, p. 237 (for materials with well-defined yield points).

[82] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., June, 1940, Pt. 2, p. 65.

available for all jobs. Miles N. Clair, Assoc. M. Am. Soc. C. E., reports[83] (under the heading "Variations of Compressive Strength of Concrete on Fourteen Projects Representing Approximately 100,000 Yards of Concrete") deviations from average 28-day compressive strength for any one project as follows:

	Plus (%)	Minus (%)
Maximum	63.0	65.6
Average	40.5	34.4
Minimum	18.5	10.1

These tests were made on concrete proportioned on the basis of 1 : 6 dry and loose by volume. The strongest cylinder for any job tested 4,980 lb per sq in., whereas the weakest tested 777 lb per sq in. The designer cannot know the minimum cylinder strength positively until 28 days after the last concrete is installed. Considering the possibility of loads being increased from the design loads, the variations that may occur in the available materials, and the difficulty of consistently obtaining proper supervision, a factor of safety of the order of 4.5 may not be excessive. This is in line with taking $f_c = 0.33 f'_c$ in designing a beam according to the straight-line elastic theory and somewhat higher than that required by current codes. In case the loads are known definitely, satisfactory material is known to be available, and good supervision can be obtained, this factor may be reduced as found expedient.

Charles S. Whitney,[84] M. Am. Soc. C. E. (by letter).[84a]—The questions raised in the discussion have followed quite closely the pattern anticipated by the writer. They have for the most part been covered in the paper, but it is probable that additional consideration will help to clarify the subject.

The position that the present methods are best because they are familiar and have been used many years is not tenable unless they can be brought up to date and adjusted to new developments. Structural design methods have not advanced, in the twenty years since 1921, as much as the design of machinery and mechanical products, which have been practically revolutionized by the results of research. This has been due in part to the fact that the average structural designer is so circumscribed by codes that he thinks in terms of standard methods instead of underlying principles. A large proportion of structural research has been directed at the determination of factors involved in standard formulas without much examination of the meaning of the formulas themselves. The existence of the standard method has a tendency to direct the thinking of both the designer and the teacher and produce great inertia against the introduction of new ideas in practice. This inertia is of great value in so far as it protects against the adoption of unsound ideas, but it should not be permitted to discourage progress.

The plastic theory of reinforced concrete design has not been presented as a perfected standard but as a basis for research leading toward modernization of standard practice. It is the result of an attempt to build a practical design

[83] "Concrete Technology," by Miles N. Clair, *Journal*, Boston Soc. of Civ. Engrs., July, 1940, p. 195.

[84] Cons. Engr., Milwaukee, Wis.

[84a] Received by the Secretary August 4, 1941.

theory consistent with all pertinent facts that have been discovered regarding the nature of the materials, not with a few incomplete characteristics selected for convenience. There is some resistance to the conception of the rectangular stress block to represent the compressive stress in the concrete. Dean Grinter and Professor Caughey suggest that it is so contrary to physically observable phenomena that it is not sufficiently reasonable to be taught to students. The writer was interested in this reaction and has discussed it with a number of students and teachers who expressed themselves as being entirely satisfied with the use of the rectangle. It may well be that it would rouse the students' interest and stimulate them to pay more attention to the reasons for it, whereas the triangular distribution is so easily accepted that it leads to superficial consideration and a lack of understanding of the real conditions.

As a matter of fact, if the proper phenomena are observed, the rectangular block is truly consistent. There is no evidence that a better shape can be devised nor that any amount of research will ever lead to an accurate predetermination of the actual stress distribution under varying conditions. Fortunately, the determination of the actual stress distribution is unimportant if the total stress and its line of action are known. The stress-strain curves of Fig. 1 were given merely to indicate why the assumption of an equivalent rectangle is perfectly rational. Whether or not the maximum compressive stress at failure of the beam occurs at the face or in the interior is also unimportant.

The writer believes that there is a reduction in the stress toward the compressive face but that it may be small in amount and may occur very close to the ultimate load, especially in the case of high-strength concrete. He has observed the failure of concrete cylinders of strengths ranging up to 9,000 lb per sq in. and appreciates the reason for Mr. Hadley's skepticism regarding the descending portion of the stress-strain curve. With low-strength concrete, it is a simple matter to observe that beyond the maximum load the stress drops off gradually as the strain increases. With high-strength concrete, however, the failure appears to be sudden, and it is only at very slow testing speeds that a reduction in load just before failure can be noted. It should be realized that there is a sudden increase in strain that accompanies the sudden decrease in stress, due to the elasticity of the testing machine, and the stress-strain curve does not drop vertically.

In a beam, the concrete at the face is supported by the concrete in the interior, and compression failure does not occur until a sufficient area of the interior concrete is highly stressed. While the interior is being stressed, plastic flow effects a reduction of the stress at the face.

The writer does not feel that the stress-strain relation is sufficiently determinate to justify its direct application as suggested by Mr. Contini.

Since this paper was submitted for publication, the writer has received from Anton Brandtzaeg, M. Am. Soc. C. E., a copy of the report[85] on his tests to determine the strain and stress distribution in concrete members under bending or eccentric loading. The tests were made on heavily reinforced beams

[85] "Der Bruchspannungzustand und der Sicherheitsgrad von rechteckigen Eisenbetonquerschmitten unter Biegung oder aussermittigem Druck," by Anton Brandtzaeg, Norwegian Technical High School, Trondheim, 1935.

and struts about 6 in. deep, of concrete with cylinder strengths ranging from about 1,250 to 5,000 lb per sq in. Mr. Brandtzaeg assumed a stress distribution in the concrete as shown in Fig. 27.

The location of the neutral axis, a, is determined from the maximum strains in the concrete and steel, ϵ and ϵ', assuming the sections to remain plane. At c, where the strain is equal to the maximum strain given by prism tests, the stress is assumed to be equal to the prism strength, f_P, and the variation from a to c follows the stress-strain curve for the prism or an equivalent parabola. From c to g at the compression face, the stress is uniform and equal to the prism strength. Mr. Brandtzaeg suggests that there may be a falling off of the stress along such a line as $f\,g$ as the maximum load is approached but he does not consider it important. Rudolph Saliger reports[86] indications of a falling off similar to that in Fig. 4. Mr. Brandtzaeg reports that the loads and lever arms calculated according to Fig. 27 agreed satisfactorily with the actual measurements.

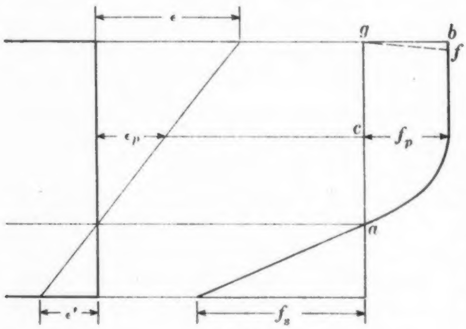

FIG. 27

The value of $\frac{\epsilon}{\epsilon_P}$ for his strongest concrete (equivalent $f'_c = 5{,}000$ lb per sq in.) was 2.13, and for the weakest (equivalent $f'_c = 1{,}250$ lb per sq in.) it was 6.87. The value of prism strength f_P was 0.75 to 0.80 times the strength of 30-cm cubes. If the cube strength is assumed at 1.13 times the cylinder strength, this would be 85% to 90% of the cylinder strength, which agrees very well with the writer's theory. Mr. Brandtzaeg also suggests the use of an equivalent rectangle, but his solution involves the ratio $\frac{\epsilon}{\epsilon_P}$, which is rather erratic although it depends in a general way on the strength of the concrete.

TABLE 14.—COMPARISON OF ACTUAL AND THEORETICAL LOADS FOR THOMAS TESTS

SERIES II; f'_c (AVERAGE) = 1,960 LB PER SQ IN.		SERIES III; f'_c (AVERAGE) = 4,640 LB PER SQ IN.	
Item No.	Ratio[a]	Item No.	Ratio[a]
6	0.919	12	0.778
7[b]	0.993	13[c]	1.059
8	0.922	14[b]	0.787
9	1.007	15	0.845
10	0.827	16	0.875
11	0.905	17	0.754
....	....	18	0.860
....	....	19	0.860
Average	0.930	Average	0.853

[a] Actual load divided by computed load (see Col. 6, Table 10).
[b] Fixed ends.
[c] $f'_c = 5{,}400$ lb per sq in.

Although the writer believes with Professor Ernst that more tests are needed, he does not agree that the tests already made indicate that the equations are not correct for concrete strengths up to 5,000 lb per sq in. In his

[86] "Versuche über Zielsichere Betonbildung und an druckbewerten Balken," by Rudolph Saliger, *Beton und Eisen*, Nos. 1 and 2, 1935.

analysis of the Thomas tests, Professor Ernst used in the formulas the minimum concrete strength and the average steel strength for each series. Therefore, he did not get as true a check on the accuracy of the formula as he would have if correct values were used in each case. If the individual strengths are used for each case, the comparison of actual to computed strengths of the columns is given in Table 14 following the form of Professor Ernst's Table 10.

The writer believes that these results are probably caused by two conditions:

First, inadequate anchorage of the reinforcing steel caused slipping of the bars and splitting of the concrete ends. Contrary to Professor Ernst's statement, the writer believes this would affect the tests with large eccentricity, as well as the others, and Table 14 indicates no consistent trend. There is no doubt but that the efficiency of the specimens was reduced by the inadequate ends.

Second, the control tests were made on 6-in. cubes and the tests on plain concrete columns made at the same time showed the following relations between column and cube strengths:

6-in. cube strength, in lb per sq in.	Column strength divided by cube strength
2,460	0.67
5,780	0.61

This reduction of the ratio of column-to-cube strength for the stronger concrete might be expected to account for a reduction in the strength of the reinforced columns. This may indicate that the standard 6-in. by 12-in. cylinder is a better control specimen than the 6-in. cube because the column tests made at the University of Illinois showed no such trend. The latter are reported in Table 15.[87]

TABLE 15.—Relations Between Column and Cylinder Strengths

Description	Air storage				Moist storage			
6-in. by 12-in. cylinder strength, in lb per sq in.	2,515	4,565	5,705	Average	2,685	4,445	6,510	Average
Column strength divided by cylinder strength	0.92	0.79	0.88	0.86	0.71	0.68	0.75	0.71

Even more conclusive evidence that there is no reduction in the relative capacity of members of concrete with strengths up to at least 5,000 lb per sq in. is furnished by the high-strength specimens of the Slater and Lyse tests (Table 1) and the Brandtzaeg tests. In the range above 5,000 lb per sq in., evidence is lacking. There is no definite explanation for the lack of strength of the Richart and Olson specimens, but it is significant that those under axial load were also deficient.

The writer does not agree with Professor Abbett that the plastic theory is appropriate only for cases of sustained loading. The equations are based on tests made at ordinary testing speeds and should be suitable for general use. They are intended to indicate the strength of the member accurately so that

[87] "Fourth Progress Report on the Column Tests Made at the University of Illinois," by F. E. Richart and G. C. Staehle, *Journal*, Am. Concrete Inst., Vol. 28, 1932, p. 296.

the proper factor of safety can be provided. If they accomplish that, the calculation of theoretical working stresses is of no particular value. The writer believes that the theoretical stresses calculated by the straight-line formula are inaccurate and not satisfactorily indicative of the true factor of safety, especially for the higher percentages of steel. In that case, the straight-line formula indicates steel stresses lower and concrete stresses higher than the actual stresses to such a variable extent that proper interpretation of the results is difficult or impossible.

The significance of the comparison made by Mr. Stewart is questionable because there appears to be no justification for the assumption of a fixed value of $k\,d$. The sections do not have equal strengths, either actually or theoretically, and therefore are not comparable. Using the same bending moment (26,600 in-lb) and unit costs (steel at 5 cents per lb and concrete at $16 per cu yd), the writer has made a comparison of costs for different percentages of steel according to the plastic theory and for balanced reinforcement according to the straight-line formula. This comparison is shown in Table 16.

TABLE 16.—COMPARISON OF COST OF BEAMS

d (inches)	p	A_s, in sq in.	$\frac{c}{d}$	Cost per lin ft
PLASTIC THEORY (f'_c = 3,000 LB PER SQ IN.; f_s = 50,000 LB PER SQ IN.; FACTOR OF SAFETY = 2.5; AND b = 12 IN.)				
12.5	0.010	1.46	0.89	$0.964
10.6	0.0136	1.74	0.865	0.920
9.1	0.020	2.19	0.802	0.904
8.4	0.025	2.52	0.755	0.943
8.2	0.0274	2.70	0.731	0.959
STRAIGHT-LINE FORMULA (f_c = 1,350 LB PER SQ IN.; f_s = 20,000 LB PER SQ IN.; n = 10; AND b = 12 IN.)				
10.6	0.136	1.74	0.866	$0.920

This indicates very little difference in cost of beams of low percentages of reinforcement designed by the two methods. The plastic-theory formula requires a little less steel than the straight-line formula when p is less than 0.01 in. It also indicates a minimum cost at about 2% of steel for the conditions assumed, but several factors may operate to raise the most economical percentage above that value. They are the cost of forms, variation in dead load, and the fact that only a part of the beam requires maximum reinforcement. It may be cheaper to use a higher percentage of steel in that part than to increase the concrete section for the full length. This is especially true at the ends of continuous beams where haunches or deep stems may be avoided. The general use of the maximum percentage of steel will probably not be economical, but it will undoubtedly be desirable in special cases. Mr. Eremin may have had this in mind when he said the formulas require an excessive percentage of steel in deep girders, but the writer does not understand that there is any difference in their application to deep or shallow sections. The designer is left free to determine the most economical percentage of steel in any case, and need not use the maximum.

A treatment of irregular cases such as is suggested by Mr. Polivka should be possible, but the writer believes it is a mistake to base calculations on the full section of the concrete. When the steel on one side is in tension, the concrete between the tensile steel and the outside of the concrete has no effect

on the strength of the member and could not influence the location of the neutral axis nor the compression area.

Mr. Abeles is correct in his statement that further investigation is necessary to determine the proper basis of design with small percentages of high-strength steel. The apparent excess strength as compared with mild steel is no doubt due to the lack of a definite yield point. The writer has adopted what seems to be a conservative definition of yield point (the stress corresponding to a total unit strain of 0.004), and, if it can be proved that the actual useful strength is greater with high-strength steel than the equations indicate, an allowance might be made for it by changing the definition of the yield point, using a greater strain. There is not sufficient data to prove whether or not 0.004 is the best value.

Mr. Julian's discussion of the suggestion that "* * * any factor of safety to provide for sub-standard concrete can be eliminated by using f'_c as the minimum cylinder strength instead of the mean" indicates that the writer has not made his meaning clear. The writer wishes to propose that the value of f'_c be selected so as to provide in itself the proper factor of safety for sub-standard concrete, taking into account the job conditions. The concrete will then be designed for a strength higher than f'_c by an amount depending on the accuracy of the job control that is to be expected. This will have the effect of focusing attention on the concrete strength and simplifying the design by eliminating consideration of that factor from the design formulas. It will help to avoid the application of a standard factor of safety to jobs where it should not be used because of a variation in the concrete control methods.

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DISCUSSIONS

LABORATORY INVESTIGATIONS OF SOILS AT FLUSHING MEADOW PARK

Discussion

BY DIMITRI P. KRYNINE, M. AM. SOC. C. E.

DIMITRI P. KRYNINE,[19] M. AM. SOC. C. E. (by letter).[19a]—An interesting and comprehensive laboratory approach to the solution of a very important engineering problem is given in this paper.

Importance of the Problem.—Material termed "silt," "organic silt," "clayey silt," "silty clay," and "mud," is very common along the Atlantic coast, at least close to the New York metropolitan area. This is organic silt mixed with more or less clay, and sometimes mixed with shells. The thickness of a layer of this material is from a few feet to 100 ft or more. The writer's observations showed this material to be mostly underlain with sand of variable coarseness. At Flushing Meadow this material served as a foundation for the World's Fair; tunnels under the Hudson river pierce horizontally thick silt layers; H-piles supporting the bridges over the Potomac River in Maryland and over the Thames River in eastern Connecticut pierce it vertically to reach the ledge rock or reliable sand. Cutoffs from old roads along the coast often pass through similar materials; and it is very probable that some muddy areas on the shore may sometime become airports, as in Baltimore, Md.

Hence it is very important to correlate different studies of this material in order to draw some general conclusions. From this point of view, Professor Burmister's paper, containing abundant data on the properties of the given soil, is a welcome beginning.

Field and Laboratory Solutions.—No civil engineering problem can be solved definitely in the laboratory. Structures stand in the field and conditions of their stability (particularly properties of the soil) should be studied both in the field and in the laboratory. Laboratory data alone are only indicative—not conclusive. From this point of view, it is highly desirable to take advantage of a situation such as that under discussion. There can be no better large-scale consolidation tests and no better large-scale shearing tests than

NOTE.—This paper by Donald M. Burmister, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper appeared in *Proceedings*, as follows: May, 1941, by Gordon E. Thomas, Assoc. M. Am. Soc. C. E., and M. N. Sinacori, Jun. Am. Soc. C. E.; and June, 1941, by E. J. Kilcawley, Esq.

[19] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

[19a] Received by the Secretary July 3, 1941.

those made by the City of New York in dumping ashes on Flushing Meadow. It is true that the sequence of operations in dumping is not well known and hence the influence of the time-factor cannot be defined very clearly. Some limiting assumptions, however, can be made in similar cases. For instance, the presence of a mud wave means that the shearing strength of the material at a given place is overcome, and vice versa; the material should be considered safe so far as shear action is concerned, if there is no mud wave at a given place. Professor Burmister has made interesting observations on the penetration of the casing (Fig. 1), and it was possible for the construction engineer to correlate his data with the actual pile-driving records since many structures of the World's Fair were on piles driven through the given material to reach resistance.

Driving of the Casing.—A 4-in. casing was driven at Flushing Meadow starting with some 5 or 7 blows per ft (Fig. 1). This number gradually increased to about 100 blows per ft in approaching the firm base. Apparently this fact should be ascribed to the increase in the point resistance of the casing due to the support from below. The number of blows in the sand layer was more or less constant although not the same in the case of borings Nos. 2 and 17. A similar picture was observed by the writer at the various times that he had an opportunity to be present.

A variation of the casing-driving diagram (Fig. 1) would correspond to the situation in which the material at the top is so soft that the casing sinks down by its own weight. The writer has observed this phenomenon on the Thames River, at New London, Conn., when a 4-in. casing, similar to that used in the Flushing Meadow investigation, penetrated in this manner from several feet to 39 ft down through the clayey silt. Actual H-piles sank at an average of 25 ft through such material, the range of fluctuations being perhaps 5 ft. The weight of the casing used was 15 lb per ft, and that of the piles 89 lb per ft. In addition to the possibility of computing the coefficient of friction, such studies perhaps may offer as a by-product an opportunity to obtain a general idea of what percentage of the cross section of an H-pile driven into any soil takes active part in the friction against that soil and what percentage remains neutral.

Shearing Resistance.—According to Fig. 5 the shearing strength of undisturbed samples was greater close to the top and to the bottom of the layer, the average shearing strength at a certain depth being of the magnitude of 0.04 kg per sq cm which equals about $0.04 \times 2{,}000 = 80$ lb per sq ft. A very high shearing strength at the top of boring No. 2 is undoubtedly due to the compressing action of the ash pile. The meadow mat and the deep peat layer apparently acted as reinforcing agents.

Besides the direct laboratory shear test in the silty clay in the Thames River, the writer participated in the following simple field tests in Connecticut: (*a*) Brass cylinders containing undisturbed samples extracted from a depth were carefully cut with a hack saw along two parallel planes normal to the length of the cylinder, without disturbing the sample. A force necessary to pull out the ring thus formed was then measured. This test furnished a shearing strength of the order of 100 lb per sq ft; (*b*) when dredging for a cofferdam, huge undisturbed samples were taken directly from the bucket and tested immediately in a field, direct-shear, apparatus (size of the box 8 in. by 12 in.) designed and

constructed by Philip Keene, Assoc. M. Am. Soc. C. E., soil mechanics engineer for the Connecticut State Highway Department. This test also furnished a shearing strength of about 100 lb per sq ft, or more. The sample tested was taken 12 ft below the bottom of the river, the depth of water being 44 ft.

Slopes.—Slopes of fills in this material should be designed rather flat using results of the quick shearing tests, as Professor Burmister advises. The shearing strength of the remolded material is perhaps $\frac{1}{4}$ or $\frac{1}{5}$ of that of undisturbed samples (Fig. 5). So far as cuts are concerned, this material is able to maintain astonishingly steep slopes. Reference is made to an article[20] that describes the construction of the present highway bridge at New London, Conn. The dredging operations are thus described in that article: "The dredging was therefore performed as an independent preliminary operation, with a clamshell dredge, and carried down 18 to 23 ft. below the river bottom, the material standing on a slope one to one."

Natural Moisture Content.—The writer agrees with Professor Burmister's statement that the natural moisture content is characteristically high for these materials, being about 100% by dry weight, or more. Smaller percentages may be found also.

The writer wishes to call attention to the fact that the moisture content as determined from an undisturbed sample of such clayey silt, extracted by using a sampler, and the moisture content of the same material in a bucket during the dredging may be different, the latter moisture content being somewhat smaller. The writer suspects that undisturbed samples may be oversaturated.

Odor.—According to the writer's observations, this material has the odor of methane (marsh gas) in the upper layers only, perhaps to a depth of 40 or 50 ft. Again, if an odorous sample is left alone for a few weeks, it does not smell any longer.

Pre-Consolidation Load.—Assuming that the procedure used for determining the pre-consolidation load is correct (and the writer believes that it is), Professor Burmister states that either the Flushing Meadow deposit was not fully consolidated at the time of investigations or that there was artesian water in the underlying sand layer. Assuming the former alternative, it should be concluded that, in the case of boring No. 2, the settlement of unloaded cuts should still have proceeded during the existence of the World's Fair notwithstanding the relief in load due to the excavation of the ash dump.

In this connection, it would be very interesting to study some values of settlement at the Flushing Meadow as computed and as actually observed.

Conclusion.—Professor Burmister is to be commended for his sound views on the value of the soil laboratory work. He calls the attention of the reader to the variability of material either in samples or in different borings at the same elevation, which means that absolutely conclusive results cannot be obtained in the laboratory. In making his settlement analysis he proceeds with extreme care and checks the values of the pre-consolidation load by a special shear-test correlation curve. Finally, he tends to simplify the laboratory procedure as much as possible by trying to correlate consolidation characteristics of a soil with such simple tests as the determination of the liquid limit and of the plasticity index.

[20] *Engineering and Building Record*, October 11, 1890, p. 296

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DISCUSSIONS

AN INVESTIGATION OF PLATE GIRDER WEB SPLICES

Discussion

BY MESSRS. C. H. GRONQUIST, CHARLES STRATTON DAVIS, AND BRUCE JOHNSTON

C. H. GRONQUIST,[3] ASSOC. M. AM. SOC. C. E.[3a]—The investigation of stresses in plate girder web splices reported in this paper is most timely and valuable. With the increasing use of plate girders for bridges of comparatively long spans ranging up to 300 ft in length, it is proper that the theoretical bases for the design of these girders should be reviewed and checked by experimental investigation.

A type of plate girder web splice that is very frequently used was not included in this investigation. It is similar to the splices of girders G_1 and G_2, with the exception that the web splice plates are supplemented by vertical flange plates which are placed only over the vertical legs of the flange angles, and are not extended to take a row of rivets in the web splice plates as in the splice of girder G_4. This splice has the merit of providing for the cross-sectional area of the part of the web plate under the flange angles, although it does not provide shear resistance at the horizontal plane at the toe of the flange angles. According to conclusion (10) of the paper, this lack of horizontal shear resistance should not be objectionable, but it would be helpful to have the opinion of the authors in regard to this type of splice.

The flange plates, since they splice indirectly, should be extended, at least on the low-moment side of the splice, a sufficient distance to provide rivets for the transfer of the flange-stress increment in addition to those required for the splice of the web under the flange angles. This was revealed in the authors' original notes and reports, although it does not seem to have been emphasized in the paper.

CHARLES STRATTON DAVIS,[4] M. AM. SOC. C. E.[4a]—The experiments conducted by the authors to secure information concerning the behavior of plate

NOTE.—This paper by J. M. Garrelts, Assoc. M. Am. Soc. C. E., and F. E. Madsen, Jun. Am. Soc. C. E., was published in June, 1941, *Proceedings*.

[3] Associate Engr., Robinson & Steinman, New York, N. Y.

[3a] Received by the Secretary June 24, 1941.

[4] Cons. Engr., Pittsburgh, Pa.

[4a] Received by the Secretary July 7, 1941.

girder web splices under load and to determine the relative values of four types of web-splice design commonly used in modern practice are most interesting and instructive.

The experiments were conducted on girders proportional to those used in practice; it might be desirable also to have some tests made on girders with excess flange strength in order to cause failure in the splices. It appears from Fig. 15 that ultimate failure occurred in the outstanding legs of the flange angles and the cover plates of the compression flanges of all girders tested. Fig. 15 further indicates that the rivet pitch in the flange plates might have been too great in proportion to the thickness of the flange angles and plates. The use of thicker material in the flanges, creating excess flange strength, might demonstrate additional information in regard to the various types of splices.

The desire to secure economical construction is an incentive that has led designers to try to splice webs of plate girders, fully, for both shear and moment and has caused the development of web splices requiring a relatively large amount of material and extra labor. Greater economy may be obtained through the use of simple splices placed at favorable locations. Flange plates are stressed to their full value only at or near their centers. They have excess flange value near their ends. It would therefore be wise to take advantage of this excess flange strength when designing web splices and place splices near the ends of flange plates. In case it should become necessary to splice the web at the center of a girder, short flange plates should be added to equal the value of the web plate in bending. It is believed that the advantages gained in simplicity of construction and resultant saving labor will more than offset all extra flange material required.

BRUCE JOHNSTON,[5] ASSOC. M. AM. SOC. C. E.[5a]—Ever since the need for these tests was first suggested in 1936 by engineers of the Bethelehem Steel Company, the writer has followed the progress of this program with interest. The authors have made a concise and able presentation of their results, and deserve much credit for their perseverance and patience in making the many strain measurements and rosette calculations.

The fact that all four splices showed nearly the same behavior constitutes an important finding applying to design economy. Splice G_1 or G_2 would obviously be the most economical, and it is interesting to tabulate the following data pertaining to the relative economy of all four splices.

Splice	No. of rivets	No. of plates
G_1	44	2
G_2	30	2
G_3	36	6
G_4	26	14

(The 26 rivets in splice G_4 do not include eight rivets in the flange angle.)

[5] Assoc. Director, Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

[5a] Received by the Secretary July 12, 1941.

A number of years ago the writer was prejudiced against splices of the G_3 type by statements in textbooks, as well as by engineers, to the effect that this type of splice is poor design. One argument states that the splice has "four planes of shear weakness." It is interesting to have these prejudices dispelled by the fact that splice G_3 is slightly superior to the other splices on the following evidence taken from the authors' report:

(1) According to Table 1, splice G_3 carried in the working range a greater proportion of both bending moment and shear than any of the other splices;
(2) Table 3 shows that the girder with splice G_3 carried a greater maximum load than any other girder; and
(3) Fig. 10 shows that web-plate rotation at working load across the spliced part was about 14% less for splice G_3 than for any other splice.

On other counts the difference in splice behavior is extremely small, and the very slight superiority of splice G_3 is more than offset by the greater economy of splice G_1 or G_2. It is nevertheless an important fact that splice G_3 is proved to be not as bad as sometimes imagined.

On the basis of Table 2 the writer has calculated the approximate weighted yield-point stress of the flange section, assuming one sixth of the area of the web to be acting therein. Assuming a linear stress-strain relationship, and using the gross moment of inertia, the maximum calculated stress at failure is only about 80% of the weighted yield-point average. The calculated stress at maximum load is very close to the weighted yield-point average if the net moment of inertia is used. It does not necessarily follow that the net moment of inertia should govern the design because there are several non-separable variables involved—for example, local bending in the flange angles at the splice, the lower-than-average yield point of the cover plates which apparently buckled first, and the unsupported length of the compression flange. However, it is true, apparently, that in the case of spliced girders of the type tested, the use of the net moment of inertia as a design basis would not appear to be over-conservative. In the light of current design practice, this is a subject that warrants further study and test.

Reserve strengths more than that calculated by substituting the yield point in a maximum-stress formula are sometimes present in compact steel members, according to the principles of "limit design." Such reserve strength is conspicuously "more than absent" in the case of the spliced plate girders tested by the authors.

In their discussion of results, the authors present design data that are evidently based on the use of net moment of inertia and a maximum fiber stress of 18 kips per sq in. On such a basis the load-factor ratio, or real factor of safety for girders G_1 and G_2, adjusted from a weighted average flange yield point of 36.7 kips per sq in. to the 33 kips per sq in. minimum specified for structural steel, is calculated by the writer to be 1.73. This factor of safety is fairly well in line with the ratio of tensile yield point to working stress—that is, $\frac{33}{18} = 1.83$. If the design had been based on gross moment of inertia and a maximum allowable stress of 20 kips per sq in., the adjusted load factor of

safety would be 1.32, which provides a margin of safety (between 1.0 and the tensile stress ratio of $\frac{33}{20} = 1.65$) of only 50%.

In the foregoing calculations the writer used 90% of the reported girder strength to correct the weighted average of the material yield strength of 36.7 kips per sq in. down to the specified minimum of 33.0 kips per sq in. for structural steel. Tests[6] made in 1941 on a number of different representative heats of structural steel, from different manufacturing plants, indicate that structural steel frequently has a yield point very near or even below the specification minimum of 33 kips per sq in. Such steels may show satisfactory yield points when tested at standard test speeds allowed by the American Society for Testing Materials (A.S.T.M.). Such tests are "material control" or "acceptance" tests and do not represent the minimum behavior to be expected at the slow speed of the test. The standard speed of test affects the reported yield point in the laboratory test and has little relation to the speed of loading a structure. The writer raises these questions because they are of considerable importance if the excellent service record made by plate girders in the past is to be maintained. Data available in the authors' tests have a direct relation to these questions. The subject is one that deserves more attention than has been given to it by the authors.

As indicated in conclusion (9), it is possible that the lower-than-yield-point buckling stresses in the flange angles may have been caused partly by local bending stresses in the flange at the spliced section. The writer suggests that the only way to make a spliced girder equally as effective as an unspliced girder may be by butt-welding the web plates before riveting the flange angles, thus eliminating riveted web-splice plates altogether.

It is obviously impossible on the basis of these tests for the authors to answer all of the questions that could be raised regarding plate girder splices. They have thrown new light on a subject that has been largely a field of speculation heretofore. Tests of this type are shown to have practical value for the engineering profession, and the authors deserve much credit for doing a difficult job in such a satisfactory manner.

[6] "Compression and Tension Tests of Structural Alloys," by Bruce Johnston and Francis A. Opila, 1941 Meeting, A. S. T. M.

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DISCUSSIONS

RIGID FRAMES WITHOUT DIAGONALS (THE VIERENDEEL TRUSS)

Discussion

BY MESSRS. A. A. EREMIN, AND YVES NUBAR

A. A. EREMIN,[27] ASSOC. M. AM. SOC. C. E. (by letter).[27a]—An interesting review of the historical development of a Vierendeel truss is presented in this paper. In a short time the Vierendeel truss has found wide application, and various materials are used in its construction. Analysis of stresses in this type of truss has revealed many interesting problems in the theory of mechanics, and many computation methods have been developed. Such methods are generally based on previously computed or assumed points of contraflexure in the vertical members. The author has given an interesting illustration of the application of the photoelastic method in determining points of contraflexure in Vierendeel trusses.

In conclusion (1), Professor Baes suggests that, for the analysis of stresses, the upper chord can be assumed cut at each panel. Eq. 5 or 6 may also be developed assuming the truss to be cut at the points of contraflexure in the vertical members. Obviously, the simplicity of the computation method depends not upon the development of the basic equations, but upon the type of redundant forces and their application to the structure. Determination of the stresses with Eq. 5 or 6 requires the solution, simultaneously, of a system of equations with three unknown redundant forces. The author's analysis of stresses may be compared with that proposed by Dana Young, Assoc. M. Am. Soc. C. E., in 1936.[2] Professor Young (and the writer in a discussion of this paper) cited equations for redundant forces in a Vierendeel truss with a factor including the summation of the horizontal redundant forces. The simplicity of computation by either of these types of equations varies with the individual ability of the designer. Furthermore, by algebraic summation, the equations with a summation factor may be easily transformed into the author's type of

NOTE.—This paper by Louis Baes, Esq., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1941, by Messrs. Jaroslav J. Polivka, and W. A. Miller.

[27] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[27a] Received by the Secretary July 1, 1941.

[2] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 869.

equation with three unknowns. In conclusion (3) Professor Baes disapproves the method of successive approximations in solving the system of equations for stresses in a Vierendeel truss. However, under the heading "The Modern Method of Analysis for Trusses with Chords Having the Same Reduced Moment of Inertia and with Loads Applied Only at the Joints," he shows that Eq. 5 or 6 may be solved by successive approximations.

The method of successive approximations in analysis of stresses in statistically indeterminate structures has been proved to be very important and has found wide application in the design of structures. Therefore, it scarcely could be considered as evidence that this method is complex. This paper is a valuable contribution.

Yves Nubar,[28] Assoc. M. Am. Soc. C. E. (by letter).[28a]—In illustrating the application of photoelastic analysis to the study of the distribution of bending moments in the verticals of Vierendeel trusses, this paper by Professor Baes is of timely interest. It contains several formulas and a number of references to comparatively recent research in European technical literature.

The writer would appreciate knowing whether the photoelastic test specimens to which the author refers formed a part of the model of a Vierendeel truss, or whether they were isolated specimens subjected to given external axial loads and moments. It will also be interesting to follow the derivation of Eq. 1, in particular whether it was based on further photoelastic tests or exclusively on analytical investigation. In either case, this formula, which appears to have a rather logical structure for cases within the limitations specified by the author, will facilitate greatly the reduction of the general problem to simpler elements, certainly at least for a first approximation. In this connection, it is difficult to understand why Professor Vierendeel's isostatic system of reference (Fig. 8) should not be simpler and of more immediate application than that of the author (Fig. 9), since the points of inflection in the verticals may be assumed located by means of Eq. 1 or other equivalent formulas.

The statement of the author (see heading "Principles of Design for Trusses Without Diagonals: Point of Contraflexure in the Verticals") that "This determination of the location of the points of contraflexure in the verticals * * * is essential" may lead to misinterpretations; it cannot possibly mean what it would convey to the casual reader. Many analyses of Vierendeel trusses have been presented in the technical literature that seem to attach to this determination an undue importance, ascribing to it, explicitly or tacitly, an essential rôle in the solution of such frames. The position taken by Professor Vierendeel himself, as described by the author (see heading "Principles of Design for Trusses Without Diagonals: Point of Contraflexure in the Verticals") illustrates this attitude to some extent. The fact of the matter is that the problem is determinate, and its solution (theoretically, at least, quite simple) requires no further assumptions beyond those commonly accepted in the general theory of rigid frames. The function of a preliminary assumption of the location of points of contraflexure is merely to simplify the mathematical analysis involved.

[28] Chf. Engr., Wilcox and Erickson, Cons. Engrs., New York, N. Y.

[28a] Received by the Secretary July 7, 1941.

More precisely, if N is the number of the panels, it may be shown that the complete solution of a Vierendeel truss of a general type leads to a system of $2(N + 1)$ linear equations of the "six-moment" for (instead of $3(N + 1)$ equations of a more complicated type derived by the great majority of conventional methods of analysis); any assumption as to the location of the points of zero moment in the verticals will simply decompose this system into two other independent systems, each of $N + 1$ equations of the "three-moment" type. This may be brought into formal evidence by means of a different approach to the general problem of rigid frames,[29] summarized partly as follows under four headings, A, B, C, and D:

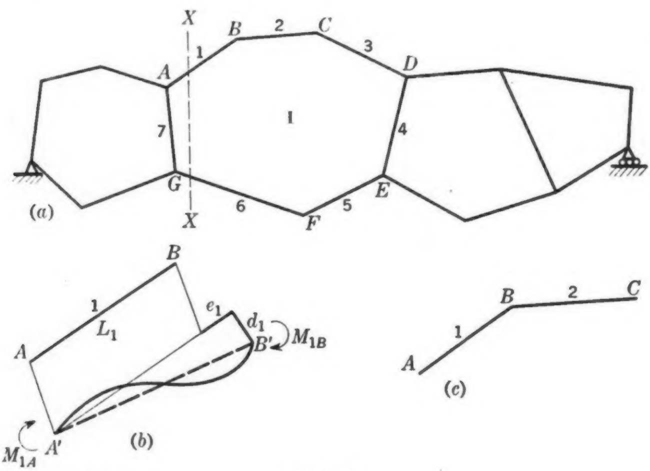

FIG. 16

Consider a Vierendeel truss (or a viaduct tower), of a very general type, composed of single panels of any polygonal shape, joined to each other along a single member (Fig 16(a)) (no claim is here made as to the engineering suitability of such frames, greater generality of results being the only purpose in mind). All symbols and notations entering in the development which follows are condensed in Figs. 16(b) and 16(c). For example, in Fig. 16(b), $A\ B$ is the original position of any member in a given frame; $A'\ B'$ is its position after deformation; M_{1A} and M_{1B} are end moments exerted by the member upon the joints; $R_1 = \dfrac{d_1}{L_1}$; and $S_1 = \dfrac{e_1}{L_1}$. Furthermore (see Fig. 16(c)), let $R_{12} = R_1 - R_2$ and $S_{12} = S_1 - S_2$; and in general R_{pq} and S_{pq}, given by similar formulas, refer to any two adjacent members p and q.

A.—Given any panel I, apply the vectors R_{pq} and S_{pq} at each joint as shown in Fig. 17, these vectors being orthogonal but inclined an arbitrary angle γ_1 to the coordinate axes. Three independent relations may be estab-

[29] See dissertation entitled "Displacements and Stresses in the General Two-Dimensional Framework," presented by the writer to Columbia University, New York, N. Y., in 1941, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

lished and stated as follows:

$$\left.\begin{array}{l}\Sigma(\text{the moment about any point, of vectors } R_{pq} \text{ and } S_{pq}, \\ \qquad\qquad \text{inclined an angle } \gamma_1) = 0 \\ \Sigma(\text{the moment about any point, of vectors } R_{pq} \text{ and } S_{pq}, \\ \qquad\qquad \text{inclined an angle } \gamma_2) = 0 \\ \Sigma(\text{the moment about any point, of vectors } R_{pq} \text{ and } S_{pq}, \\ \qquad\qquad \text{inclined an angle } \gamma_3) = 0 \end{array}\right\} \quad (21)$$

all summations being taken around the complete boundary of the panel. Vectors S_{pq} in Eqs. 21 and in Fig. 17 will drop out if axial deformations are

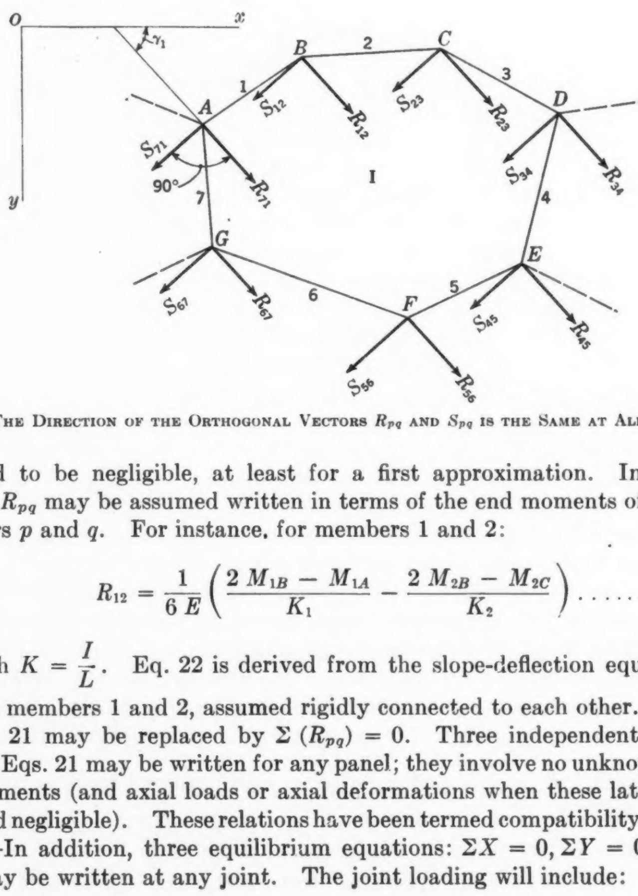

FIG. 17 (THE DIRECTION OF THE ORTHOGONAL VECTORS R_{pq} AND S_{pq} IS THE SAME AT ALL JOINTS)

assumed to be negligible, at least for a first approximation. In addition, vectors R_{pq} may be assumed written in terms of the end moments of the given members p and q. For instance, for members 1 and 2:

$$R_{12} = \frac{1}{6\,E}\left(\frac{2\,M_{1B} - M_{1A}}{K_1} - \frac{2\,M_{2B} - M_{2C}}{K_2}\right) \ldots\ldots\ldots\ldots (22)$$

in which $K = \frac{I}{L}$. Eq. 22 is derived from the slope-deflection equations applied to members 1 and 2, assumed rigidly connected to each other. Any one of Eqs. 21 may be replaced by $\Sigma\,(R_{pq}) = 0$. Three independent equations such as Eqs. 21 may be written for any panel; they involve no unknowns except end moments (and axial loads or axial deformations when these latter are not assumed negligible). These relations have been termed compatibility equations.

B.—In addition, three equilibrium equations: $\Sigma X = 0, \Sigma Y = 0$, and $\Sigma M = 0$ may be written at any joint. The joint loading will include:

(*a*) Loads and moments externally applied at the joint;
(*b*) Axial loads in members meeting at the joint;
(*c*) End shears in these members; and
(*d*) End moments in these members.

Since item (*c*) may readily be expressed in terms of (*d*), it is again seen that these joint equations contain no other unknowns but axial loads and end moments.

C.—Returning now to panel *I* of Fig. 16(*a*), the following relations may be written, assuring the equilibrium and continuity of the frame:

(*a*) Three equilibrium equations for the part of the entire structure on one side of line $X\,X$;

(*b*) Three times three, equals nine, intermediate joint equations for joints B, C, and F; and

(*c*) Three compatibility equations for the panel.

This gives a total of fifteen equations involving the following nineteen unknowns: Five axial loads for members $A\,B$, $B\,C$, $C\,D$, $E\,F$, and $F\,G$; and fourteen end moments for all seven members forming the panel. All of these unknowns may then be expressed in terms of $19 - 15 = 4$ of them. This property is general; all unknowns such as the foregoing may be expressed as functions of four of them, regardless of the type of the panel and the number of its sides. The complete solution of the system is not necessary, at this time at least; choose the four end moments of two consecutive web members as parameters and solve only for M_{1A}, M_{3D}, M_{5E}, M_{6G} in terms of them.

D.—Perform a similar operation for each panel; then write equation $\Sigma(M) = 0$ at each joint of the frame at the ends of the web members, such as A, D, E, and G. This will yield a system of $2(N + 1)$ linear equations, each involving the six end moments of three consecutive web members. The solution of this system is possible, although often laborious. Any assumption relating the two end moments of each web member (that is, any assumption as to the location of the point of inflection in these members) will break this system of $2(N + 1)$ equations into two other independent systems, each containing $N + 1$ equations between the moments at one end of three successive web members. The similarity of the resulting systems to that of "three-moment" equations is obvious. Such an assumption, if reasonably correct, will lead to quite accurate results which, if desired, may be used to determine a second set of values more closely convergent.

The writer has applied this method to the solution of the symmetrical, five story Kinzua Viaduct.[30] For each panel, the equations listed under heading *C* reduce in this case to only two, as follows:

(*a*) One equilibrium equation;

(*b*) No intermediate joint equation; and

(*c*) One compatibility equation.

These two equations contain only four unknowns: Two end moments for the chord member and one more end moment for each web member in the panel. Proceeding as outlined, a system of five linear equations is obtained between the five end moments of the web members, only two or three of these entering into each equation. The solution of this system is greatly facilitated

[30] "Modern Framed Structures," by the late J. B. Johnson, the late C. W. Bryan, and F. E. Turneaure, Pt II, John Wiley & Sons, Inc., New York, N. Y., 9th Ed., p. 404.

by the fact that the coefficient of one of the unknowns in each equation is, on the average, twenty times larger than that of the other one or two unknowns. As a result, some appropriate method of solution by successive approximations yields a first set of values, with errors ranging from 1% to a maximum of 5%.

Eq. 6 given by the author is also of the "three-moment" type and will be found very useful in many cases. However, it appears to be predicated upon the following particular assumptions: A straight bottom chord, rectangular or trapezoidal panels of equal spans, equality of reduced moments of inertia for the two chords, equality of moments of inertia for all verticals, and location of the points of inflection at midheight of these verticals. The foregoing analysis is not subject to these restrictions. It is only fair to add, however, that these assumptions will be found almost entirely verified in the greatest majority of actual Vierendeel trusses, and Eq. 6 has a much wider applicability than might appear at first.

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DISCUSSIONS

DYNAMIC STRESS ANALYSIS OF RAILWAY BRIDGES

Discussion

By R. S. Chew, M. Am. Soc. C. E.

R. S. Chew,[15] M. Am. Soc. C. E. (by letter).[15a]—The profession is keenly interested, at present, in the forced vibratory effect on structures due to wind and earthquake, so that Professor Bernhard's paper treating of the effect on bridges from moving train loads is timely. In his treatment, the locomotive unbalanced wheel load is presumed to build up a vibrational resonance that is more destructive than the dynamic impact that has heretofore been assumed.

In discussing the subject it is necessary to define the terms "dynamic impact" and "vibrational effect," as used by the writer:

(*a*) "Dynamic impact" indicates a blow that, if repeated, recurs at an interval that is widely removed from the natural period of the structure struck; and

(*b*) "Vibrational effect" is the result of a blow that recurs in an interval closely related to the natural period so that the initial effect is accumulative.

In the case of vibration, then, the effect is intimately connected with the period of the structure; but in any problem, including forced vibration, it will be found that the calculation of this period (except for the simplest type, such as a rod) is ambiguous, so that it becomes necessary to test the finished structure in order to obtain a period of any value.

To illustrate, the writer will set up the theoretical equation for the period of a bridge which is excited by the application of recurring movement at regular intervals of the load system shown.

Referring to Fig. 8, let: p_1 = train wheel load; p_2 = locomotive wheel load; w = uniform dead load per linear foot; y = deflection at the point of load from live load, in feet; and T_1 = the period of the bridge. Then,

$$T_1 = \frac{2\,\pi}{\sqrt{g}}\sqrt{\frac{\Sigma\,(w\,p\,y^2)}{\Sigma\,(w\,p\,y)} + \frac{\Sigma\,(p_1\,y^2)}{\Sigma\,(p_1\,y)} + \frac{\Sigma\,(p_2\,y^2)}{\Sigma\,(p_2\,y)}} \quad \ldots\ldots\ldots\ldots (14)$$

Note.—This paper by R. K. Bernhard, M. Am. Soc. C. E., was published in January, 1941, *Proceedings*.

[15] Cons. Engr., San Francisco, Calif.

[15a] Received by the Secretary July 2, 1941.

Eq. 14 indicates:

(1) Certain values of y which include shear and flexure (if y_f = flexure deflection due to live load, and y_s = shear deflection due to live load,

$$y = y_f + y_s \qquad (15a)$$

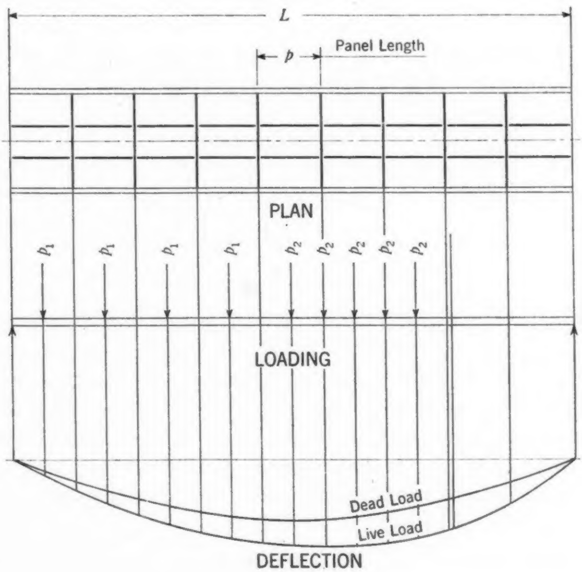

FIG. 8

In order to find deflection values from the elastic curve y_s must be in terms of y_f. If it is assumed that $y_s = R\, y_f$

$$y = y_f\,(1 + R) \qquad (15b)$$

in which values of y_f are determined from statics, with live loads in the position given);

(2) That this value of T_1 is true only for the loads acting in position on bridge as shown and any shifting of loads changes values of y and T_1;

(3) For forced vibration the same loading must recur in the same position during regular intervals; and

(4) The value of T_1 is the period imposed on an elastic structure of known value of $E\,I$ and dimensions, so that the calculated value must assume these characteristics together with R and is thus of little value.

Items 1 to 4 appear to indicate that a moving locomotive and train load will tend to produce a chaotic vibration, which will have an exciting but no accumulative effect.

When the locomotive has passed from the bridge, the structure becomes uniformly loaded and the action of this excited load will tend to produce forced vibration.

For this condition of uniform loading, a simple formula can be written, as follows:

$$\Delta = \frac{5}{384}\frac{w\,L^4}{E\,I} = \frac{5}{384}\frac{W\,L^3}{E\,I} = \frac{5}{8}W\left(\frac{L^3}{48\,E\,I}\right) \qquad (16)$$

in which W is acting through an average deflection of $\frac{5}{8}\Delta$. Therefore,

$$\frac{5}{8}W\,\Delta = \left(\frac{5}{8}\right)^2 \frac{4\,\pi^2\,\Delta^2}{g\,T^2_1}(1+R)\,W \qquad (17)$$

from which the natural period of the bridge is found to be

$$T_1 = 2\,\pi\sqrt{\frac{5}{8}\frac{\Delta}{g}(1+R)} \qquad (18)$$

in which Δ = maximum deflection due to live load. The natural frequency is:

$$f_n = \frac{1}{2\,\pi}\sqrt{\frac{8}{5}\frac{g}{\Delta\,(1+R)}} \qquad (19)$$

If the uniform live load is moving with a speed such that the loads recur at panel points in regular intervals, forced vibration will occur due to an impulse created by the deflection of stringers.

For example, if: V_s = speed of the train, in miles per hour; p = panel length in feet; and T = time of recurrence = period of excitement,

$$T = \frac{0.68\,p}{V_s} \qquad (20)$$

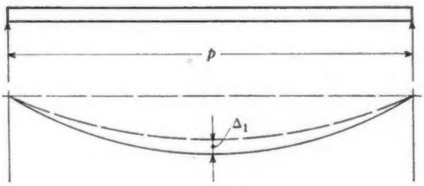

FIG. 9

which gives the period of excitement; and when $T = T_i$ resonance occurs.

In order to find the effect at this impulse (F), let Fig. 9 indicate a stringer carrying a uniform load of w pounds per linear foot moving with velocity of $v = \frac{p}{T}$. Then

$$F = \frac{5}{8}w\,p\,\frac{4\,\pi^2\,\Delta_1\,(1+R)}{g\,T^2} = \frac{0.8\,\Delta_1\,(1+R)}{T^2}w\,p \qquad (21)$$

in which Δ_1 = deflection due to live load. If N = number of impulses, U = length of train, in feet, p = panel length in feet, and L = length of bridge—

$$N = \frac{U-L}{p} \qquad (22)$$

Let V_c = the speed of a train in miles per hour such that $T_i = \frac{0.68}{V_c}$; then

$$T^2 = T^2_i\left(\frac{V_c}{V_s}\right)^2 \qquad (23)$$

which substituted in Eq. 21 gives the forced vibration at the panel in terms of

the panel load:

$$F = \frac{0.8\, N\, \Delta_1\, (1 + R)\, w\, p}{T^2_i} \left(\frac{V_s}{V_c}\right)^2 \qquad (24)$$

This value of F will be limited by frictional damping.

From the foregoing it can be seen that the writer is not in accord with the author in regard to the locomotive unbalanced wheel loads producing accumulative vibrational effect on the bridge as a whole. In reference to the value of z given in Eq. 9 and assuming that it holds, still its application for each impulse is $\pi\, D$ removed, which means that it does not accumulate but rather acts as a dynamic force.

The writer approves of Fig. 3, if z is substituted for z_w.

It appears to the writer that the value z, plus the effects of the sudden application of load, is a measure of the dynamic impact to be used for girders and shear members. If Eq. 24 is adjusted to include the damping decrement K_w, as given by the author:

$$F_d = K_w \left[\frac{0.8\, \Delta_1\, (1 + R)\, (w\, p)}{T^2_i}\right] \frac{V^2_s}{V^2_c} \qquad (25)$$

Two examples illustrating the application of Eq. 25 are given in Table 1,

TABLE 1.—APPLICATION OF EQ. 25

No.	Description	Symbols	Example 1	Example 2
1	Span length, in feet	L	90	200
2	Deflection of bridge, in feet	$\Delta\,(1 + R)$	0.06	0.20
3	Panel length, in feet	p	20	20
4	Stringer deflection, in feet	$\Delta_1\,(1 + R)$	0.01	0.01
5	Bridge period	T_i	0.214	0.395
6	Length of train, in feet	U	4,000	4,000
7	Critical speed, in miles per hour	V_c	63	35
8	F at a speed of V_c	F_d	6.5	1.65
9	Limiting speed, in miles per hour	V_s	31	27
10	F at a speed of V_s	F	1.0	1.0

which is based on the premise that all values of w, p are identical, a condition that is very improbable. However, the formula does emphasize the fact that forced vibration is a factor of the ten items given in Table 1, so that, as in buildings subjected to earthquake, it is impracticable to establish a constant vibrational factor. After completion the bridge should be investigated for the items in Table 1 and from this the factor for the structure should be established.

It may be noted further that Eq. 25, with adjustments, will indicate the vibrational condition on the suspension bridge under wind. If, from any cause, a differential vertical deflection between cables recurs in time T_i, the formula will indicate the forced vibration, in which: $w\, p$ is the load on the suspender; and T_i is the natural period of the bridge.

Then F indicates approximately the condition that existed on the Tacoma-Narrows bridge in Seattle, Wash., when V_s has equaled V_c.

In conclusion the writer believes the author's paper to be of value and trusts that it will receive the attention it deserves.

Corrections for *Transactions:* In January, 1941, *Proceedings*, page 54, Fig. 4, scale (6), change "simplifying" to "amplifying."

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DISCUSSIONS

SOME ECONOMICS OF AIRPORTS

Discussion

BY MESSRS. WILLIAM E. RUDOLPH, AND DONALD M. BAKER

WILLIAM E. RUDOLPH,[9] M. AM. SOC. C. E. (by letter).[9a]—Mr. Pagon has presented most interesting data for determining the rate of growth of air travel and resulting requirements that airports must meet in the future. However, the writer would invite attention to another important factor that is certain to influence the rate of increase of air travel, although not inherently part of airport design—that of speed and comfort of travel between airport and business center.

At present the traveler between Chicago, Ill., and St. Louis, Mo., or between New York, N. Y., and Philadelphia, Pa., actually spends as much time in motor buses to and from airports as he does aboard the plane. Necessarily, these airports are at considerable distances from business centers, with long and not too comfortable trips through crowded city traffic lanes the unavoidable consequence. Whereas air traffic conditions entering and leaving airports may be improved by building longer and wider runways as discussed in Mr. Pagon's paper, surface traffic conditions in crowded city streets will require special and far more expensive remedies. The business center of the future cannot move to the airport as it once encircled the railroad stations of the past—because business requires high buildings, which are not desirable near airports. Hence, there is the obstacle of distance to be surmounted at airports, and the best solution of this problem may be in trunk line motor travel arteries to adjacent business centers, a major construction feat.

The rapid growth of air travel would appear to be predicated upon two major advantages over other methods of transportation—time saving and greater comfort.

From the standpoint of time saving, the future will determine where the line should be drawn between those trips that may better be made by airplane, and those falling within the zone of the railroad and motor vehicle. As the distance that may be advantageously traveled by air becomes shorter and

NOTE.—This paper by W. Watters Pagon, M. Am. Soc. C. E., was published in February, 1941, *Proceedings*.

[9] Executive Engr., Certain-teed Products Corp., Buffalo, N. Y.

[9a] Received by the Secretary June 9, 1941.

shorter, so will the volume of air travel increase by leaps and bounds, because there are far more passenger trips and passenger miles in short journeys than in long journeys. Here the factor of lost time at air terminals looms up as important, inasmuch as its proportion of the total travel time is considerably larger for short trips than for long trips.

From the standpoint of greater comfort, air travelers whom the writer has met agree almost without exception that the airplane is far more comfortable than the railroad or motor car. The one refreshes, the other fatigues. The writer encourages his staff men to use the airways because he considers this to make for higher efficiency; if a man's work is of sufficient value to his employer to justify his time and expenses in travel, it is equally important that this employee be at his best for performing such work upon arrival at his destination. Again, people of the present day are not content with the travel conditions to which their parents were accustomed, and the next generation will look askance at that which is considered comfortable today—another reason why air traffic should grow.

Particularly in mountainous regions does the differential on time saving, and decreased "wear and tear" upon the individual, become apparent. Here the railroad must adopt a circuitous route, whereas the plane flies directly over Nature's obstacles. The railroad pays endless toll in fuel and in carrying charges upon large capital investment in such sections, which places the airplane at a decided advantage. Thus a greater rate of increase might be expected in airplane passenger travel in mountainous regions of dense population than in flat regions where railroad locations and motor roads follow direct courses. In the development of South America, which would seem to be the principal task of the future, air travel should play an important part, in view of that continent's rugged terrain.

Donald M. Baker,[10] M. Am. Soc. C. E. (by letter).[10a]—The use of the type of curve suggested by the author in forecasting future air traffic is based upon a sound theoretical premise, since such traffic will increase in magnitude with population. This soundness is further borne out by examples of the growth of other activities and their similarity in growth pattern to the theoretical curve given in Fig. 1.

Uncertainties as to the probable location of the point of inflection on the air-traffic curve, and consequently of the probable saturation point, can be expected with the relatively small amount of basic data yet available, but the reasoning followed, coupled with the fact that the author calls attention to the danger of attempting to forecast for a period longer than ten years, is sound, and allows a proper safety factor to be used when results of the forecast are used. The formulas presented for computing areas necessary for various facilities are of much value.

The reasoning and supporting data from which the conclusion is reached that a runway 5,000 ft long represents the probable maximum that will ever be required carry conviction, and should be welcomed by those responsible for

[10] Cons. Engr., Los Angeles, Calif.

[10a] Received by the Secretary June 18, 1941.

acquiring and planning airports. Heretofore lengths of runways have shown a constant increase, and no sooner had a port been constructed according to the latest standards than such standards were raised, and inadequacy resulted. From the present time forward the length of runway will control the plane designers, instead of the latter controlling the length of runway. The extremely large proportion of airports with runways much shorter than 5,000 ft is undoubtedly due to the fact that most of the land for airports now in existence was acquired in the late 1920's or early 1930's, when U. S. Department of Commerce ratings for a Class 1 airport stipulated two runways at least 2,500 ft long, at an angle greater than 60°; and for a Class 2 airport one runway 2,500 ft long, in the direction of the prevailing wind, or two runways 2,000 ft long, at an angle greater than 60°. Instrument flying or landing was then practically unknown, and freedom from fog was a very important factor. Planes flew at elevations of from 5,000 to 6,000 ft, landed or took off at an angle of 1 in 7. Two or three transport landings a day were big business. Any flat open area of about 160 acres, even if surrounded by residential development, was suitable for an airport (the term "transport terminal" was unknown), and the owner of such a property, provided he had suitable political connections and did not ask an exorbitant price for his land, was usually able to dispose of it for use as the local airport. Possibilities of future expansion were given little thought.

With present stratosphere flying, instrument landings, greater speed, weight, and size of ships, obstruction ratios increased to 1 in 40, and with reasonably definite information as to ultimate maximum size of fields required, it is possible to consider intelligently the question of the adequacy of existing fields, and to plan for ultimate extensions in the length of runways and the size of fields.

The data given by the author indicate that, in the near future, many fields will have to be increased in size. Additional adjacent land will have to be acquired, or the existing sites abandoned and new and larger sites purchased. When faced by this situation, the agency owning and operating the site must consider the economics of the situation.

Only large metropolitan centers require that the site to be used as a transport terminal have 5,000-ft runways; but, although open unimproved areas of 160 acres (which will accommodate 2,500-ft runways) occur with reasonable frequency at fairly close-in locations in such areas, 640-acre tracts of this character are scarce.

The major factor in encouraging air travel is the time saved in comparison to other methods of travel, usually rail travel. Already fares for air travel have been reduced to a point that, between termini farther apart than overnight trips by rail, a saving can be shown over rail travel if but a nominal value is placed upon the traveler's time.

In rail travel the time spent between the points of origin and destination and the railroad depot is a negligible factor, both because such depots are usually located close to the business district, and also because such travel time is an extremely small proportion of the total time spent on a rail trip. Time spent in ground travel between points of origin and destination and the airport is a much more important factor, particularly when the trip is relatively short.

In 1928 the writer presented a paper before the City Planning Division of the Society,[11] the conclusion of which was that there was little time saving in air over rail travel in distances of much less than 500 miles, when considerable time was required for ground travel at the beginning and end of the trip. This was based upon scheduled air and rail speeds of 100 and 33.3 miles per hr, respectively. It was borne out by the fact that there was then very little "short-haul" passenger traffic.

At present these speeds have increased to approximately 250 and 55 miles per hour, respectively, and "short-haul" passenger traffic has increased materially. When a one-way air trip of 500 miles involved a total time between points of origin and destination of from 6 to 6½ hr, usually with daytime flying, at least two (and sometimes three) working days were spent away from the place of business, whereas overnight rail trips allowed a complete working day at the destination with but one day away from the traveler's place of business. Present-day speeds of 250 miles per hr by air allow a person nearly as much time at his destination, and he can sleep at home. This has greatly increased the "short-haul" traffic.

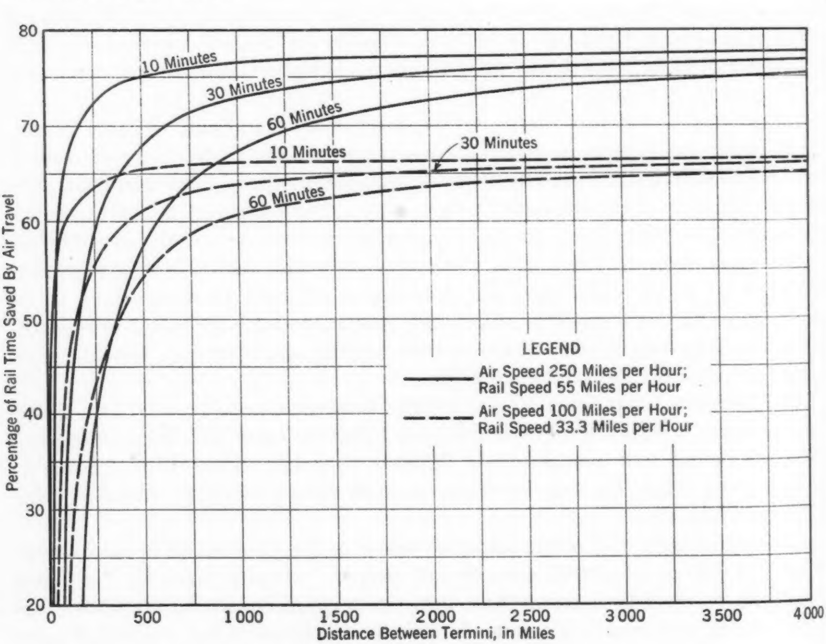

FIG. 4.—TIME SAVED BY AIR OVER RAIL TRAVEL

Fig. 4 shows the saving in time of air travel over rail travel, based upon air and rail speeds of 100 and 250 miles per hr, and 33.3 and 55 miles per hr, respectively, with varying times of ground travel to and from the airport.

[11] "Factors Governing the Location of Airports," by Donald M. Baker, *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 269.

Present higher speeds in air travel increase the percentage of time saved from 10% to 12% over former speeds, but likewise show a greater effect in time of ground travel.

With air speeds of 100 miles per hr, percentage of time saved in air over rail travel increases 9% with a decrease in time distance of from 60 min to 10 min in one-way ground travel for 500-mile trips, and 17% when air speeds are increased to 250 miles per hr. For 1,000-mile trips, the time saved is 5% at 100 miles per hr, and 9% at 250 miles per hr as compared with comparable time periods of ground travel.

Cost of land in metropolitan centers varies inversely with the distance from the business center. Improved land will cost from 4 to 10 times more than unimproved acreage will cost in the same location. If the size of an existing airport must be increased, and such increase is prohibitive in cost because of the price of additional land, the question of acquiring a cheaper outlying site will depend upon the additional cost of maintaining reasonably similar time of ground travel; and this in turn involves a problem of determining the future magnitude of traffic to and from the airport and the business district or higher class residential district, unless the new site can be located near an existing high-speed rail line or highway.

Data given by the author indicate a volume of air traffic in 1940 amounting to 2.2% of the national population, and an estimated volume in 1950 amounting to about 22% of the then estimated population, or a per capita increase of 10 times. Based upon population of metropolitan centers, these percentages will be much greater. The air traffic at the principal Los Angeles air terminal for 1940 was 210,000 passengers, an increase of 80% over that for 1939, and represented between 7% and 8% of the metropolitan population. If such traffic were to increase in accordance with the rate estimated by the author for the nation, air traffic in Los Angeles would be of the order of $2\frac{1}{2}$ millions in 1950. Such traffic would, without question, justify considerable expenditures for improving the accessibility of any major air terminal, these expenditures being in excess of savings made through acquisition of outlying land at a price lower than that for land closer in.

The writer believes that, in the past, inadequate consideration has been given to the economics of the location of airports, but feels that with increasing information and basic data becoming available, such as that contained in the paper, there should be no excuse for such lack of consideration in the future.

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DISCUSSIONS

MOMENTS IN CONTINUOUS RECTANGULAR SLABS ON RIGID SUPPORTS

Discussion

BY MESSRS. RALPH E. BYRNE, JR., AND PIERCE P. FURBER

RALPH E. BYRNE, JR.,[13] JUN. AM. SOC. C. E.[13a]—A comparatively simple method of analyzing a very complex problem is presented in this paper. The method is admittedly approximate, and yet, in the particular case presented in the paper, it produces results that are quite satisfactory for design purposes.

Its most striking feature is that the complicated problem of a continuous slab is reduced to the much simpler problem of a continuous beam. Consequently, the various methods of solving continuous beam probelms are made available for the solution of continuous slab problems. In the theory of continuous beams, methods of distributing moments,[14] and of distributing angle changes,[15] have been developed primarily to avoid the necessity of solving a large number of simultaneous algebraic equations. The latter method (balancing angle changes) is well adapted to the solution of the slab problem as presented by the authors.

Each slab is considered as simply supported and the angles ϕ_s are determined from Fig. 5, or otherwise. At each support in turn, continuity is restored by applying a moment (the same moment to each slab meeting at the support); and the change in angle, θ, for each slab, caused by this moment, is directly proportional to the factor α (Fig. 4) for the corresponding slab. It is assumed that during this process of restoring continuity on one side, the other sides remain simply supported. Consequently, the three remaining sides will be rotated also, the amount of this rotation being equal to θ (the rotation of the side on which continuity has been restored temporarily) multiplied by a carry-over factor. These carry-over factors are seen to be $C_\beta = \dfrac{\beta}{\alpha}$ and $C_\gamma = \dfrac{\gamma}{\alpha}$ for

NOTE.—This paper by L. C. Maugh, Assoc. M. Am. Soc. C. E., and C. W. Pan, Jun. Am. Soc. C. E., was published in May, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1941, by Maurice P. van Buren, Assoc. M. Am. Soc. C. E.

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[13a] Received by the Secretary June 20, 1941.

[14] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[15] "Analysis of Continuous Frames by Balancing Angle Changes," by L. E. Grinter, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1020.

the opposite and adjacent sides, respectively, β and γ being given in Fig. 4.

The process to be followed is thus a well-known one. Continuity is first restored at each side of the various slabs. The angles which, as a consequence, must be carried to the other sides of each slab are determined, and again continuity is restored by the balancing process. This sequence of operations is repeated until the results have converged to within the required limits of accuracy. The only difference between the foregoing process and the well-known one for continuous beams is that it is necessary to consider three carry-over factors, instead of one, for each member at each support.

By this process, the angles θ_{ij} (as defined by the authors) are determined for each support of each slab; and now, to determine the moments, four equations of the type of Eq. 8 must be written, one equation for the angle at each side of a slab panel. These equations are then to be inverted to give the moments as functions of the angles. Let the subscripts a and b refer to quantities relating to the sides of the slab panel of lengths a and b respectively; and let subscripts 1 and 2 further distinguish quantities relating to the sides of the same length. Write

$$\psi = \theta - \phi_s \qquad (17)$$

With this notation, the aforementioned equations of the type of Eq. 8 become

$$\left.\begin{aligned}
a\,\alpha_a\,M_{a1} + a\,\beta_a\,M_{a2} + b\,\gamma_b\,(M_{b1} + M_{b2}) &= D\,\psi_{a1}\\
a\,\beta_a\,M_{a1} + a\,\alpha_a\,M_{a2} + b\,\gamma_b\,(M_{b1} + M_{b2}) &= D\,\psi_{a2}\\
a\,\gamma_a\,(M_{a1} + M_{a2}) + b\,\alpha_b\,M_{b1} + b\,\beta_b\,M_{b2} &= D\,\psi_{b1}\\
a\,\gamma_a\,(M_{a1} + M_{a2}) + b\,\beta_b\,M_{b1} + b\,\alpha_b\,M_{b2} &= D\,\psi_{b2}
\end{aligned}\right\} \qquad (18)$$

Eqs. 18 are easily inverted to give the moments, as follows:

$$\left.\begin{aligned}
M_{a1} &= \frac{D\,(\alpha_b - \beta_b)}{a\,\Delta}\left[A_a\,\psi_{a1} + B_a\,\psi_{a2} + C_a\,(\psi_{b1} + \psi_{b2})\right]\\
M_{a2} &= \frac{D\,(\alpha_b - \beta_b)}{a\,\Delta}\left[B_a\,\psi_{a1} + A_a\,\psi_{a2} + C_a\,(\psi_{b1} + \psi_{b2})\right]\\
M_{b1} &= \frac{D\,(\alpha_a - \beta_a)}{b\,\Delta}\left[C_b\,(\psi_{a1} + \psi_{a2}) + A_b\,\psi_{b1} + B_b\,\psi_{b2}\right]\\
M_{b2} &= \frac{D\,(\alpha_a - \beta_a)}{b\,\Delta}\left[C_b\,(\psi_{a1} + \psi_{a2}) + B_b\,\psi_{b1} + A_b\,\psi_{b2}\right]
\end{aligned}\right\} \qquad (19)$$

The several quantities used in writing Eqs. 19 are defined as follows:

$$\left.\begin{aligned}
\Delta &= (\alpha_a - \beta_a)(\alpha_b - \beta_b)\left[(\alpha_a + \beta_a)(\alpha_b + \beta_b) - 4\,\gamma_a\,\gamma_b\right]\\
A_a &= \alpha_a\,(\alpha_b + \beta_b) - 2\,\gamma_a\,\gamma_b\\
B_a &= -\left[\beta_a\,(\alpha_b + \beta_b) - 2\,\gamma_a\,\gamma_b\right]\\
C_a &= -\,\gamma_b\,(\alpha_a - \beta_a)\\
A_b &= \alpha_b\,(\alpha_a + \beta_a) - 2\,\gamma_a\,\gamma_b\\
B_b &= -\left[\beta_b\,(\alpha_a + \beta_a) - 2\,\gamma_a\,\gamma_b\right]\\
C_b &= -\,\gamma_a\,(\alpha_b - \beta_b)
\end{aligned}\right\} \qquad (20)$$

It is to be noted that Eqs. 19 are exactly analogous to the slope-deflection equations of beam theory. Having determined the moments from these equations,

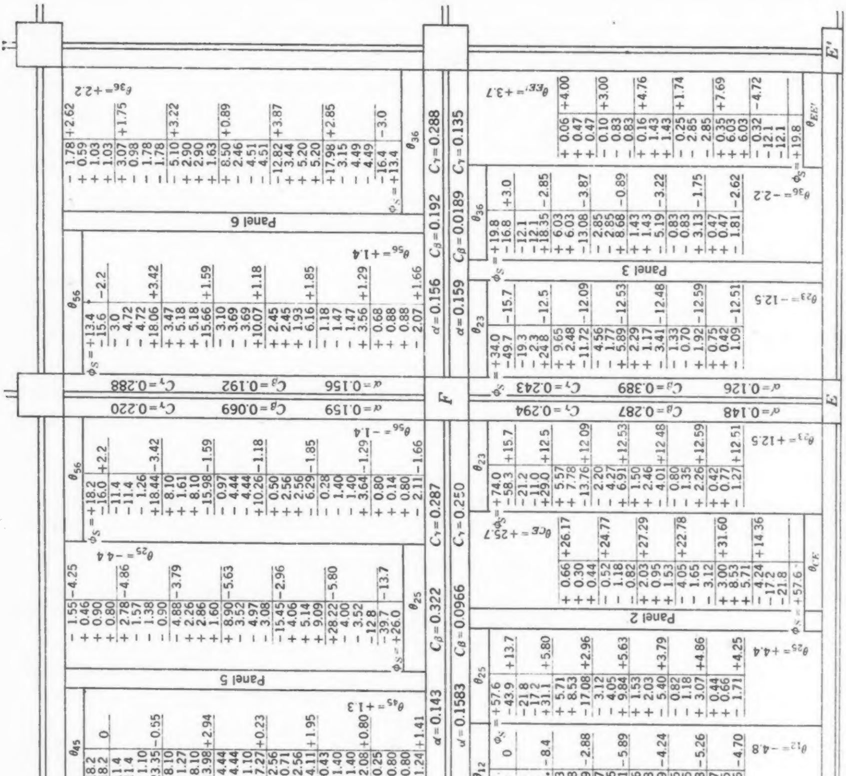

FIG. 9.—DETERMINATION OF THE ANGLES θ BY DISTRIBUTING ANGLE CHANGES

the accuracy of the entire process may be judged by the agreement of moments at the supports as computed from the angles of the two adjoining slab panels, and by the computed magnitude of moments over supports that have been assumed to be simply supported.

By this method the writer has recomputed the example given by the authors, and the results obtained are in substantial agreement with the authors' results as given in Table 2. The computation of the angles θ by the balancing process is shown in Fig. 9. The computations are so arranged that, for each support, the left-hand column shows the angles for simple support, ϕ_s, each carry-over angle, and each balancing angle; the right-hand column shows the cumulative total after each balancing. The convergence in this example is not rapid; and such cumulative totals not only give a means of judging the degree of convergence, but also, when the balancing process is terminated, they give a means of estimating a better value of the angle θ than the final number in the cumulative total column. Thus, the final angles θ to be used in computing moments were estimated from consideration of the sequence of numbers appearing in the cumulative-totals column.

The computation of moments from the angles thus obtained is a matter of straightforward substitution in Eqs. 19; a convenient tabular form will immediately suggest itself to the computer. The results obtained in this manner are shown in Table 5. The last five moments of this table are computed moments over supports that were assumed to be hinged, and are seen to be small compared to the other moments of the corresponding panels. The remaining moments agree substantially with the authors' results (see Table 2).

TABLE 5.—COMPUTATION OF MOMENTS

Moments	Panels						Mean
	1	2	3	4	5	6	
M_{12}	−20.5	−20.45	....	....	....	....	−20.5
M_{23}	....	−12.42	−13.02	....	....	....	−12.7
M_{45}	....	....	....	− 2.55	−2.72	....	− 2.64
M_{56}	....	....	....	....	−4.53	−4.61	− 4.57
M_{14}	−12.0	....	....	−12.56	....	....	−12.3
M_{25}	....	− 9.94	....	....	−9.80	....	− 9.87
M_{36}	....	....	− 3.78	....	....	−3.79	− 3.79
M_{AB}	+ 0.018	....	....	....	....	....	....
M_{AC}	− 0.017	....	....	....	....	....	....
M_{CE}	....	− 0.175	....	....	....	....	....
$M_{EE'}$	....	....	− 0.01	....	....	....	....
$M_{BB'}$	....	....	....	+ 0.21	....	....	....

Theoretically, it would be possible to apply the method of distribution of moments in a similar manner. However, there are several practical difficulties. The starting point of this process is the determination of "fixed-end moments"—that is, the moment at the center of each edge of each slab when the conditions of fixity are satisfied along these edges. Likewise, "stiffness" and "carry-over" factors must be computed, one edge being rotated through a unit angle while the other edges are held fixed, and the corresponding moments at the centers of the four edges determined. The computation of these quantities for the fixed slab is essentially more difficult than the computation of

the corresponding quantities for the simply supported slab. Furthermore, the assumption, in the former case, of a sine distribution of moment along the edges is not as natural an assumption to make as in the latter case. Nevertheless, should these quantities be available for the fixed slabs, the method of distributing moments would be immediately applicable to the problem.

In conclusion, it should be stated that no superiority is claimed for the methods herein suggested over that used by the authors; they are essentially the same. Rather, it has been the purpose of this discussion to point out the analogy between the theory presented in this paper and beam theory, and to suggest other possible methods of attack on the problems that present themselves as a natural consequence of this analogy.

PIERCE P. FURBER,[16] M. AM. SOC. C. E.[16a]—Eqs. 13 have been written incorrectly, thus leading to an erroneous conclusion, which is not a little surprising.

Lateral deformation in a body has long been recognized as of opposite sign to the direct strain. Works on elasticity treat the subject in general. Experimental determinations of the ratio of lateral deformation have been made for many materials, by many men. Poisson was among the pioneers; hence, his name has been connected with it.

The theory of elasticity shows that like strains or deformations at right angles are accompanied by lateral deformations of the opposite sign, in each direction. Consequently, the primary strains are reduced by some amount. In a plate or slab under load, therefore, the extensions and compressions are both reduced, to some extent, by the lateral deformations. The true stresses are less, and hence the true moments of resistance in each direction are less than the apparent moments. Eqs. 13 should be written: $M_x = M'_x - \mu M'_y$, etc.

Engineers of varying opinions have engaged in spirited controversy concerning not only the value of Poisson's ratio for reinforced-concrete slabs, but whether any value at all existed for it. Some seemed to believe that the composite plate of concrete and steel constituted an exception to the general theory of elasticity in that respect. However, scientific explanations for that view have been entirely lacking.

The paper contains the first statement the writer has noticed that the effect of lateral deformations is actually to increase the moments (and consequently the stresses) in a slab or plate supported on four sides. As the paper seems not to limit the equations to reinforced concrete, these formulas must be intended to apply as well to other materials.

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[16a] Received by the Secretary June 30, 1941.

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DISCUSSIONS

VALUE OF PUBLIC WORKS

Discussion

By Messrs. Evan S. Martin, Isador W. Mendelsohn, and W. W. Crosby

Evan S. Martin,[15] Assoc. M. Am. Soc. C. E. (by letter).[15a]—Planning public works to be undertaken primarily to furnish employment during disturbed and disordered economical periods, it is to be hoped, is no longer imperative. There is only one sound and sufficient reason for public works—and private enterprise as well—namely, to serve the public in the way it wishes to be served. Although private enterprise must be attuned to voluntary public demand for its market and success, there is the danger that some public promotion may be forced which does not serve the public to a degree comparable to its cost. Private misadventures are discerned by their financial failure, but not so with public developments. The writer has classed the New York Barge Canal as a public benefit not equal to its cost. Then, too, there are examples of developments resulting in partial losses through arbitrary change of public policy Such are the Grand Trunk Pacific Railway in Canada, which was to facilitate the development of a large new area but suffered because such development was arrested by the Great War and the subsequent drastic restriction of immigration; and the Soo Line Railway, planned to carry international trade, which was estopped by a protective tariff policy.

Major Hallihan sets forth the narrow domain of public works. It is rather the narrow-minded notions held in the past regarding public service. Rivers, harbors, and perhaps mineral deposits, it is generally agreed, are natural resources. Well, what are the forests, soil, and crops of the fields and the inhabitants? Anything that serves the public is certainly public service regardless of its ownership and management. If the use of one's mind is to be hamstrung by precedent, he will continue to be inconveniently handicapped in thinking.

One of the restraints mentioned in this paper is the requirement of self-

Note.—This paper by J. P. Hallihan, M. Am. Soc. C. E., was published in February, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1941, by Clarence W. Post, M. Am. Soc. C. E.; May, 1941, by Messrs. Uel Stephens, William J. Wilgus, Bernard L. Weiner, Albert Ed. Scheible, H. B. Cooley, and Philip W. Henry; and June, 1941, by Baxter L. Brown, M. Am. Soc. C. E.

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[15a] Received by the Secretary June 2, 1941.

liquidation of an investment. During the past two centuries of almost continuous and intensive development, man has accustomed himself to "getting his money out of an investment." This is a phrase totally unrelated to the merits of an investment. One liquidates an investment by means of outright sale or by accumulating a surplus of receipts over expenditures during a term of years. No one can collect more money than he pays out except as others pay out more than they get. This means that investments are capable of being liquidated only when, and to the extent that, others make investments—that is, as development continues.

When the investment dollar stopped circulating in 1931–1932, it did so because further investment seemed unsound. Investment was unsound because a long, active period of development had increased the capacity of national economy much above present standards of living, which provide the market. The proper remedy was, and continues to be, a much higher wage level or lower interest rate—the latter having the same effect. No artificial public works program can give relief for long from making the proper adjustment.

ISADOR W. MENDELSOHN,[16] M. AM. SOC. C. E. (by letter).[16a]—Major Hallihan presents succinctly a timely subject that affects the foundations of the future national existence of this country and that merits, of the engineering profession most of all, sober judgment divorced from partisan politics and personal jaundice. Public works as related to the entire construction field and unemployment and its far-flung effect upon the economic and social structure of the nation has received surprisingly little, and generally superficial, attention. So delineated, the value of public works still remains to be determined.

It is the purpose of this discussion to assist with a number of brief pertinent observations in the equable comprehension of the significance and the rôle of public works in national economy and welfare.

1. Statistical data presented in the paper's two interest-arresting tables concern such terms of widely varying meaning (as "construction," "employment," "national economy," "post-depression," etc.) that it would have aided in their understanding had appropriate definitions been included. Government and private economists and agencies differ in their use of these terms. Statisticians annually revise their construction statistics in the light of more complete and accurate data.

2. There is reason to question the implication of Table 2 that the depression is ended. Unemployment is still prevalent on an extensive scale. Piecemeal construction programs provide piecemeal employment, whether in public works or national defense. Upon completion of a few months' construction project, the same workers are looking for other employment. Entrance into the present war will only postpone the inevitable struggle with this problem. Thorough understanding of the fundamental causes of unemployment and the depression has not as yet been attained because the necessary analysis has not been carried through to completion. When such analysis is made, far-reaching changes in mode of life may be necessary to curb heedless "rugged individualism,"

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[16a] Received by the Secretary June 11, 1941.

whether of man or corporation, in behalf of the better public welfare necessary for enduring democratic national existence.

3. As presented in Table 2, the data do not warrant the presumption of marked correlation between public works construction and employment and national income. Although construction is a significant factor of national economy, there are many others, some of which are of greater importance. A comparison[17] of several major branches of economic activity in the United States during the period 1920–1930 shows the following estimated average annual production—

Estimated average annual production for all:	Billions of dollars
Manufactured products	59.7
Agricultural products	11.2
Retail sales	34.9
Goods marketed at wholesale	74.9
Foreign trade (imports plus exports)	8.8
Construction, of all types, including maintenance and repairs	11.6

The same authority gives the number of workers directly dependent upon the respective industries mentioned for their livelihood (annual averages, in millions of workers) as 11.2, 10.6, and 6.5 for retail and wholesale trade combined; and 3.1 for construction. Fluctuations in national income as presented in Table 2 may very well be due largely to these other factors instead of to construction. In a report[18] of the National Resources Planning Board, the definite statement is made that "present levels of public construction expenditure cannot be considered the major determinant of the level of business activity and national income * * * the determination of the national income is the product of many variables."

4. The term "construction" as used in the paper refers to new construction only. This is verified by comparison of the data with the table on "Estimated On-Site Employment on New Construction, by Sources of Funds, 1925–38." [19] As a matter of fact, there is considerable annual construction for maintenance of fixed works and structures. For the period 1926–1939 inclusive, the estimated value of maintenance construction[20] varied from 1,543 to 3,112 millions of dollars annually, representing 20% to 38% of total annual new, maintenance and work-relief construction, private and public.

5. The paper refers only to on-site employment created by new construction, by its comparison with the table of the reference[21] mentioned in the preceding paragraph. Construction of any kind also provides for employment which is directly attributable to the construction project but which is provided away from the site, primarily in the manufacture and transportation of the materials.

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[17] "Construction Activity in the United States, 1915–37," U. S. Dept. of Commerce, 1938, pp. 27–28.

[18] "The Economic Effects of the Federal Public Works Expenditures, 1933–1938," National Resources Planning Board, November, 1940, p. 36.

[19] *Loc. cit.*, p. 40.

[20] "Survey of Current Business," U. S. Dept. of Commerce, September, 1940, Reprint No. 17685.

[21] "The Economic Effects of the Federal Public Works Expenditures, 1933–1938," National Resources Planning Board, November, 1940.

"On-site and off-site employment together constitute the identifiable effect of public construction on the volume of employment. They do not constitute the total effect of such construction, since additional employment is generated by the expenditure of the wages and incomes received by those engaged in construction or the supplying of materials. These further effects are not statistically identifiable." [22]

6. Public works place less emphasis on on-site employment; much of the expenditure is for materials that provide a considerable volume of off-site employment; a comprehensive volume of heavy construction is undertaken, and, typically, the work is performed under contract. This has been the procedure with PWA projects. The main purpose of work relief is to provide the maximum of direct, or on-site, employment to needy unemployed. Usually this work is done by force account and not by contract. These differences are not too sharply defined, but the broad purposes of public works and work relief make such a distinction necessary.

7. Estimates of the volume of off-site employment created by the various federal construction programs in relation to the volume of on-site employment indicate[23] that the ratio of off-site man-hours to on-site man-hours was 1.0 for PWA Federal projects, 2.0 for PWA non-federal projects, 1.0 for regular federal projects, 0.6 for projects conducted by federal departments under the works program, and 0.16 for WPA projects. These estimates may represent more nearly the maximum than average ratios. The variations in the over-all ratios of the several programs, excepting work relief, result primarily from differences

TABLE 4.—ON-SITE AND OFF-SITE MAN-HOURS PER MILLION DOLLARS OF PWA CONSTRUCTION CONTRACTS AWARDED[a]

(A = On-Site; B = Off-Site)

Description	Buildings				Water Supply		Sewage Disposal		Streets and Roads		Reclamation		Power and Light				Naval Vessels	
	Non-Residential		Residential										Steam		Diesel			
	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
Man-hours[b]	344	657	410	615	262	876	358	667	475	669	402	645	161	620	161	641	491	589
Total,[b] A+B	1,001		1,025		1,138		1,025		1,144		1,047		781		802		1,080	
Ratio, B : A	1.91		1.50		3.34		1.86		1.41		1.60		3.85		3.98		1.20	
Cost per man-hour	$1.00		$0.98		$0.88		$0.98		$0.87		$0.95		$1.28		$1.25		$0.93	

[a] Source: Bureau of Labor Statistics, U. S. Department of Commerce. [b] Thousands of man-hours.

of project composition. For similar types of projects, the off-site ratios are practically identical. In this connection, it is of interest to note in Table 4 the list[24] of on-site and off-site man-hours per million dollars of PWA construction contracts awarded, by types of projects.

[22] "The Economic Effects of the Federal Public Works Expenditures, 1933–1938," National Resources Planning Board, November, 1940, p. 38.

[23] *Loc. cit.*, p. 109.

[24] *Loc. cit.*, p. 54.

8. Public works construction has an important effect upon the materials output of certain industries (see Table 5).[25]

9. In various periods different types of new construction projects are favored. In the period 1934–1940,[20] with public works construction aided by

TABLE 5.—MATERIALS PURCHASED FOR PWA CONSTRUCTION PROJECTS (RATIO OF ORDERS TO TOTAL PRODUCTION, 1934–1937[a])

Type of material	1934	1935	1936	1937[b]
Brick and hollow tile	23.5	26.9	42.7	27.7
Cement	73.6	36.8	16.8	13.2
Structural and reinforcing steel	39.8	35.9	23.8	12.3
Cast-iron pipe and fittings[c]	35.0	30.0	31.8	17.7
Sand and gravel	37.7	26.6	16.6	9.9

[a] PWA orders include certain distribution costs. [b] PWA expenditures curtailed in 1937. [c] Estimates for cast-iron pipe and for structural and reinforcing steel from data supplied by National Resources Committee; all others from Bureau of Labor Statistics.

federal funds, highways, sewage disposal, water supply, public buildings, educational buildings, hospitals and institutions, and social, recreational, conservation, and development projects received more impetus than in the preceding years. In 1917–1918, as at present, military and defense construction projects received major consideration. During 1926–1929, commercial, factory, private residential, and public utility construction were preeminent. According to the National Resources Planning Board report[26]

> "* * * as the proportion of public works increases relative to that of private construction, we tend to get, at any level of total construction, relatively fewer factories, office buildings, public utilities, and housing, and relatively more highways, schools, recreational facilities, and conservation work. Furthermore, the trend of recent years seems to indicate that, within the field of public works, expenditures for conservation, the development of natural resources, and, as yet in only embryonic form, low-cost housing, are increasing in significance."

It is worth considering whether such planless construction as the foregoing, an outgrowth of heedless individualism, works to the best interests of the nation as a whole.

10. Public works in the depression years have been of inestimable value[27] in numerous other ways. For many years practices obstructing best construction have been multiplying, such as the wage "kickback," misclassification of laborers, lax inspection, political or personal favoritism in award of contracts, collusive bidding, unbalanced bidding, restrictive specifications, and inadequate performance bonds by certain contractors. All these have been guarded against and practically eliminated in the strictly public works programs. Public works have raised the standards of communities in public construction by: Insistence upon employment of competent engineers and architects, and prompt payment to them of reasonable fees; the institution of acceptable

[25] "America Builds—The Record of PWA," Public Works Administration, 1939, p. 27.

[26] "The Economic Effects of the Federal Public Works Expenditures, 1933–1938," National Resources Planning Board, November, 1940, p. 23.

[27] "America Builds—The Record of PWA," Public Works Administration, 1939.

accounting practices in small communities; aiding in revamping the financial structure of small communities; improving municipal credit and reduced bond interest; furthering the incorporation of local districts and the creation of public authorities, thus enabling the construction of numerous public works; developing revenue-bond financing, which made possible hundreds of public works projects; and furthering the preparation of thorough project plans and specifications.

11. The letter of transmittal accompanying the report[28] of the National Resources Planning Board to the President of the United States contains pungent paragraphs, on the rôle of public works in construction, which merit the attention of the engineering profession.

12. As mentioned so aptly in the paper, public works have sustained the construction industry during the depression years, preserved contracting organizations, maintained consulting engineer staffs, and revitalized construction material industries. Now in time of increasing emergency, the nation is reaping the benefit of this indirect aid in increasing measure. Many of these consulting engineer firms and contracting organizations have designed and built new Army and Navy camps, airfields, sea bases, and munitions plants, and the supply industries have furnished the vast quantities of defense construction materials and equipment.

13. Engineers, lawyers, statisticians, accountants, clerks, and typists who were employed and trained by federal agencies on public works during the period 1933–1940 are now being employed in the War and Navy departments and other federal defense organizations, applying the skill, technical information, and training acquired in public works construction to serve the nation in defense construction.

14. On June 5, 1941, the President stated:

> "Our defense production is a gigantic assembly line. Transportation is its conveyor belt. If raw materials cannot flow freely to our great industrial plants and the products cannot move continuously to the front, defense breaks down. Bottlenecks in transportation are as serious as shortage of power."

The tremendous federal highway construction programs during depression years will prove timely in moving troops, materials, and supplies in these emergency days. Educational buildings constructed in recent federal programs now (1941) are useful for training technicians and skilled-trades operators, and educating hundreds of thousands of others in defense tasks that are in the making. Hospitals and institutions so sorely needed heretofore to serve civilian populations will provide improved medical facilities for the military wants of the emergency. Waterworks and sewerage systems sponsored by federal aid during the period 1933–1940 have increased public health safeguards as no similar construction work in any 25-yr period in the nation's past, and thereby will indirectly further national defense. Conservation and power projects such as Hoover Dam, Bonneville, Grand Coulee, Santee-Cooper, the All-American Canal, Tennessee Valley Authority projects, the Nebraska proj-

[28] "The Economic Effects of the Federal Public Works Expenditures, 1933–1938," National Resources Planning Board," November, 1940, p. iii.

ects, and a host of others are developing hitherto forgotten sections and resources of the United States and are furnishing power to industries serving national defense.

15. Table 1 shows that federal funds expended for emergency construction exclusive of loans for the period 1933–1939 totaled $10,892,000,000, of which PWA[27] funds were $3,143,000,000. This federal aid has been criticized profusely, at times somewhat vituperatively, as to its advisability, the efficiency of the expenditures, the permanent value resulting therefrom, and whether greater benefits could not have been obtained through other and less costly measures. The Great War, to April 30, 1919, cost the United States $21,850,000,000 of federal funds alone, exclusive of loans to Allies. How many Americans have wondered as to the value of participation in that war, its advisability, the efficiency of expenditures, the permanent value resulting therefrom, and whether greater benefits could not have been obtained through other and less costly measures? Already federal appropriations for the present military emergency amount to $42,000,000,000. How much more will be required is unknown, but the total sum will be stupendous. How many Americans are considering the value of participation in the present war, its advisability, the efficiency of expenditures, the permanent value resulting therefrom, and whether greater benefits cannot be obtained through other and less costly measures?

Must the path to national democratic existence lead through vales of wars and depressions in rapid succession? In view of national experiences since 1917, do Americans feel greater gratification with expenditures for war than for public construction?

16. The United States is facing a war that may test the nation's very foundations. During World War I, for the period April 6, 1917, to July 1, 1919, with combined armed forces of 4,800,000, the total deaths of United States citizens numbered 125,500, of which approximately 43% were due to battle in a period of less than nineteen months.[29] It is within the realm of possibility that the coming war may last five years, may require the combined armed forces of more than 10,000,000, and may entail deaths of more than 1,000,000, both military and civilian. Great as was the dislocation of national life resulting from World War I, it is likely that the aftermath of the coming war will present a far greater cataclysm. It is none too soon to plan and prepare for the expected rehabilitation work. World wars may be necessary in this century to maintain free nations, but of equal if not greater importance for enduring national existence is proper after-war recovery. Planning for utilization of national resources in men, materials, money, and morale in behalf of national interests is a vital necessity. If Germany at war can find time for planning the subjugation of conquered nations, it behooves the United States to plan for maintaining its democratic ideals of national existence after war as well as during war. Public works that have sown the seeds of planning in thousands of communities in depression years should play an important rôle in planning for national recovery and the better American way of life after war.

[29] "The War with Germany," by Col. Leonard P. Ayres, War Dept., August 1, 1919.

W. W. Crosby,[30] M. Am. Soc. C. E. (by letter).[30a]—Utterly allergic, from his breeding, education, and experience, as the writer admits himself to be, to all such hypotheses as "spending for prosperity," "the end justifies the means," and the "survival of the unfit," he nevertheless wishes to avoid in his remarks political argument or even discussion of the political economy involved and to offer only such comment as might be helpful to those of another persuasion who may really and sincerely believe that such postulates can be established as principles of the "American way of life."

Therefore, he now passes without further comment the wastes of "leaf-raking" and "boon-doggling," claimed to have been justified by some of the early "emergencies," to the practical defects as observed by him in the substantial work later attempted to be marketed for the "approval of the people."

The writer enjoys a ringside seat at a relatively small (in area) but active arena where a dozen public, and several different corporate, authorities contend for results from large funds apparently regarded as limitless. The wastes, the lack of cooperation, and the want of coordination evident are reminiscent of a "battle royal." They do not make for "public approval" when the early glamor of the frenzied activities has worn off.

Public discrimination is increasingly keen. Engineers have encouraged its development. They must keep ahead of it. This is not done merely by "making one dollar do the work of two"; nor by "Heath Robinson" or "Rube Goldberg" schemes to effect a temporary result.

Investigation here reveals an apparent cause for the conflicts of efforts and results. It lies in the "tunnel vision" (hemianopia) of the separate authorities. The planners have failed, or been unable, to regard considerations outside of each one's immediate, individual goal. In some cases each one seems to have disregarded consideration of contiguous interests.

The more obvious cases have given the public the idea that the projects were badly planned and wasteful; and the writer believes that the American people are not yet fundamentally wasteful and extravagant, although they may be induced to be at times.

The local public of any locality is inclined to credit the higher authority with greater abilities and perspective. The local planners and executives are too familiarly known and often do have a lack of outside knowledge and experience to offset their familiarity with local conditions. However, it does not follow that the county or state planners prove more competent. Often they show a neglect of points known to be of local importance.

It is when the planning gets into bureaucratic control that the value of the product in the market is often most questionable—if only from the aspects of narrowness of design and uncoordinated response to the needs in the wider field visible to the public. The public has learned that "made in Washington" does not guarantee satisfaction.

If deficiencies in locally made plans could be removed as they went up, and their final approval should signify that the project did actually fit in the

[30] Cons. Engr., Coronado, Calif.

[30a] Received by the Secretary June 16, 1941.

entire picture, the market values would be greater and the demand then might stimulate local support.

Bureaucracy seems too often incapable of cooperation and coordination with others. Whether or not this defect in the setup can be overcome remains to be seen. If all such work is to be done under "emergency," "unlimited emergency," or "super de luxe emergency" conditions, and without a competent coordinating head, the prospect does not seem inviting. It may yet be that an earlier statement, reputed to have been made once by a most eminent authority, to the effect that "this country is too big and too diversified to be run [and, inferentially, to be planned] by one man or one group of men" will be proved and remembered.

The writer means no criticism of the motives of those in authority. Their limitations are generally those of law and environment. However, there is a need for a "catalyst" or some kind of an efficient agent to bring order out of the confusion if the general approval of the public is to endure.

The writer thinks that Major Hallihan has most conscientiously and illuminatingly set forth the case of the proponents of his theory. His recognition of the difficulties of localities in meeting the conditions imposed by higher authority, of the local limitations to action, of the objections found to bond issues by disappointed electorates, and of the belated (if not lacking) establishment of a coordinating or reviewing authority seems to indicate some agreement with the writer's comments.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SIMPLIFIED THEORY OF THE SELF-ANCHORED SUSPENSION BRIDGE

Discussion

By Messrs. C. B. McCullough, Jaroslav J. Polivka, William Bertwell, and A. A. Eremin

C. B. McCullough,[12] M. Am. Soc. C. E. (by letter).[12a]—Little can be added to Mr. Gronquist's most able treatment of the theory of self-anchored suspension spans. His presentation is complete yet concise, and the subject matter should prove of distinct value to every designing bridge engineer.

The self-anchored type presents greater possibilities of economic utilization than are generally realized. This statement finds corroboration in the results of certain studies of short-span highway structures made by the Oregon State Highway Department in 1937 and 1938. The first phase of this latter investigation comprehended the development of designs and estimates for fifteen two-lane highway structures, each having a total length of approximately 750 ft, in groups, as follows:

Group	Type
A	Simple truss spans
B	Cantilever spans
C	Externally-anchored suspension spans (truss stiffened)
D	Self-anchored suspension spans (girder stiffened)
E	Self-anchored suspension spans (truss stiffened)

The various span lengths are given in Table 2. In the self-anchored girder-stiffened suspension spans (Group *D*) the stiffening girders were rigidly anchored to the deck by means of perforated **T**-bars attached to the girder webs with the deck reinforcing passing through the perforations. The deck was also anchored to the stringers by means of **Z**-bar connections spaced approximately 4 ft center to center. The girders and stringers, acting as a composite unit, were designed to resist the stiffening frame moment. This arrangement affords

Note.—This paper by C. H. Gronquist, Assoc. M. Am. Soc. C. E., was published in February, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1941, by A. J. Meehan, M. Am. Soc. C. E.

[12] Asst. Chf. Engr., Oregon State Highway Dept., Salem, Ore.

[12a] Received by the Secretary May 26, 1941.

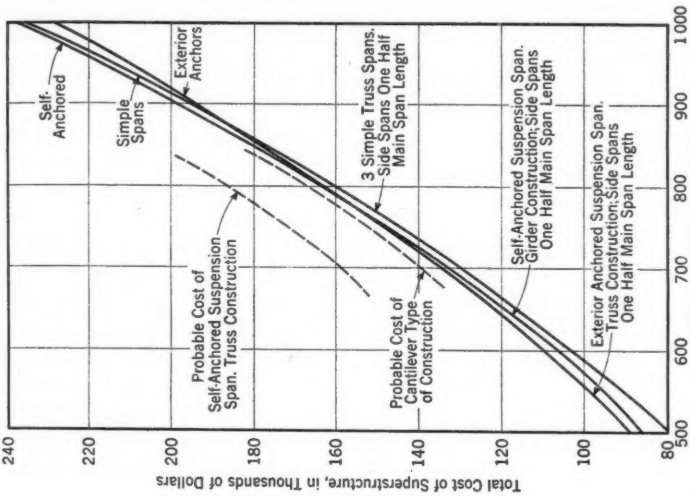

FIG. 8.—TOTAL SUPERSTRUCTURE COST FOR VARIOUS TYPES OF BRIDGES (SIDE-SPAN RATIO = 0.50; 27-FT ROADWAY; AND TWO 5-FT WALKS)

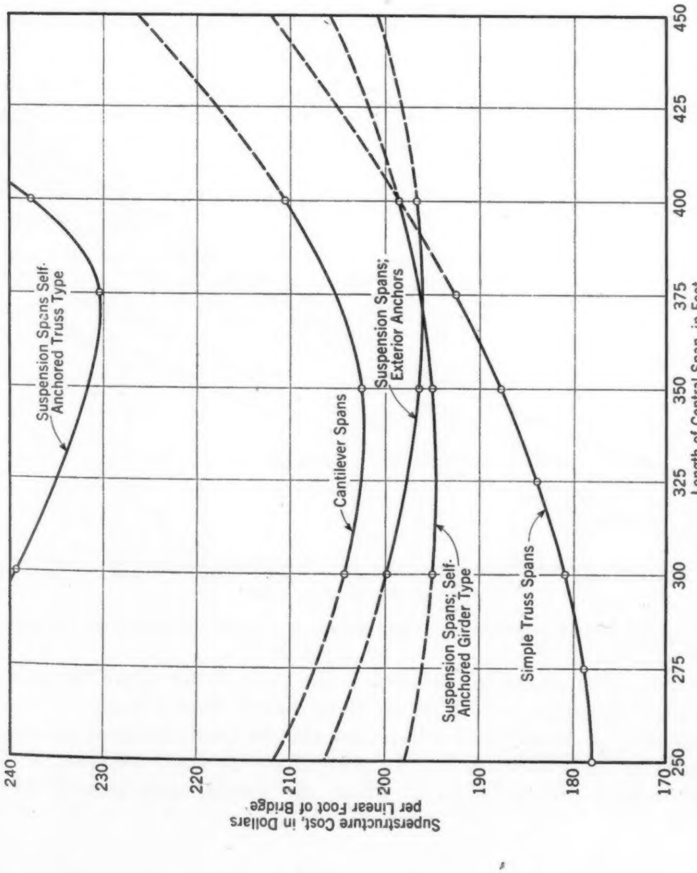

FIG. 7.—COST COMPARISONS PER LINEAL FOOT FOR THE VARIOUS COMPARATIVE TYPES

considerable economy of material and accounts, in part, for the relatively low cost of this type.

Based upon unit prices prevalent at that time (1937 and 1938), the cost per lineal foot for the various design types and span arrangement was as indicated in Fig. 7. (The general trend beyond data points is shown by broken lines.) The comparisons indicated in Fig. 7 were based upon structures having the same total length but with varying side-span ratios. Since some of the span arrangements selected were obviously uneconomical, the investigation was extended to include a comparison of structures of varying total lengths but constant side-span ratio, as indicated in Fig. 8. Since the cantilever type and the self-anchored suspension type using truss construction indicated no economy in the first comparison, they were not included in the second. The probable cost trends for these latter types through the central range of the diagram (Fig. 8) are indicated by dotted lines.

TABLE 2.—STUDY OF SHORT-SPAN HIGHWAY BRIDGES IN OREGON

(Span Lengths, in feet)

No.	GROUP *A*		GROUP *B*		GROUPS *C*, *D*, AND *E*	
	Center span	2 side spans	Center span	2 side spans	Center span	2 side spans
1	250	250	300	225	300	225
2	300	225	350	200	375	187.5
3	350	200	400	175	400	175

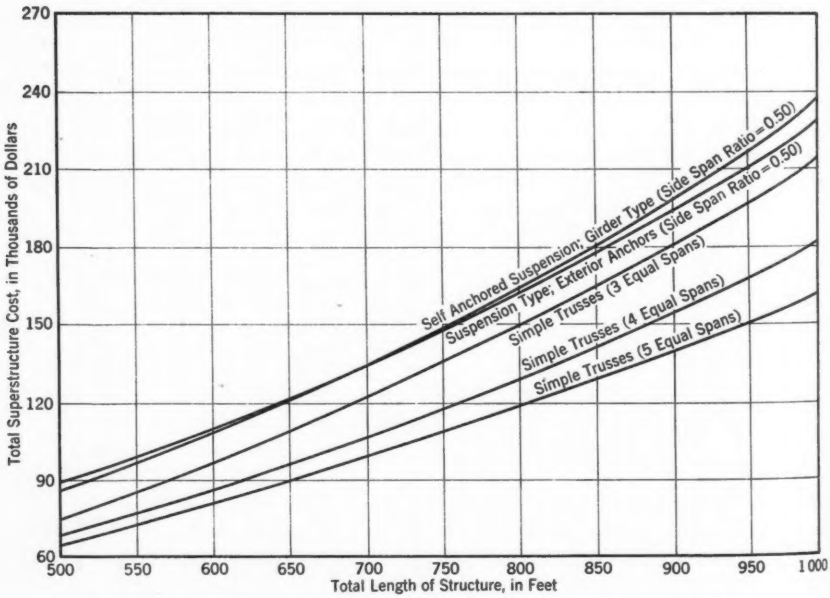

FIG. 9.—COMPARATIVE SUPERSTRUCTURE COST, SUSPENSION TYPES VERSUS SIMPLE TRUSSES

An inspection of Figs. 7 and 8 indicates that the self-anchored suspension design, utilizing the girder system, is quite likely to show economy in first cost for three-span arrangements in which the central span exceeds 400 ft.

This is only true when the central span is the controlling factor. If it is possible to cut the central opening, the simple truss type will show economy in first cost, as indicated in Fig. 9.

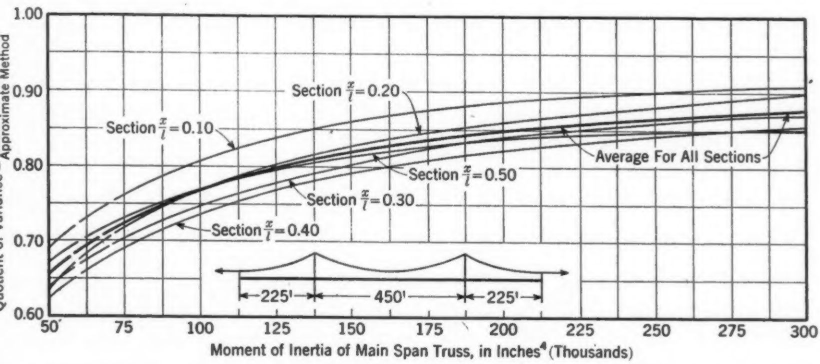

FIG. 10.—QUOTIENT OF VARIANCE OF THE ELASTIC THEORY IN DETERMINING MAXIMUM POSITIVE MOMENTS (450-FT MAIN SPAN; LIVE LOAD PLUS TEMPERATURE RISE = 60° F)

A most important property of the self-anchored suspension type is the fact that the effect of the deflection term is canceled in the moment equation so that a simplified procedure is possible in the analysis. This is nicely developed by the author in his discussion of live loads.

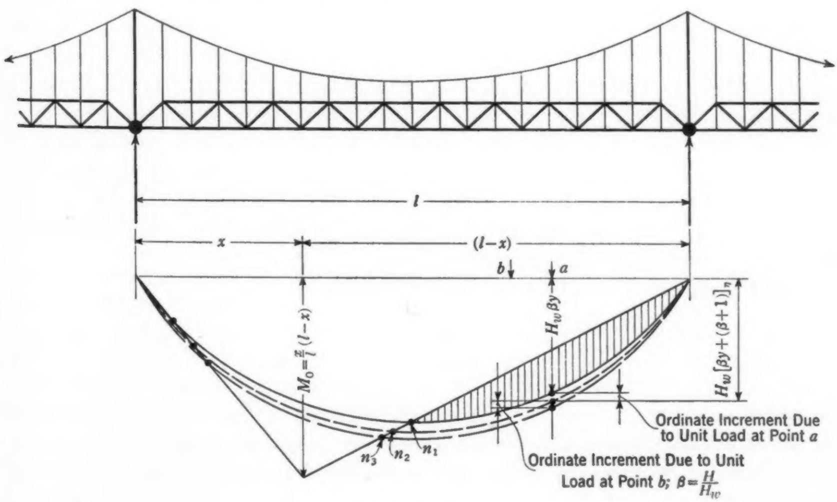

FIG. 11

The effect of this deflection term is considerable even for short spans, as will be seen from an inspection of Fig. 10, which indicates the degree of variance between the exact method and the so-called elastic theory when applied to a

225–450–225 ft, externally-anchored, suspension design, for varying values of the moment of inertia of the stiffening frame. For the more flexible frames the divergence is of material magnitude and this fact (with the further fact that the degree of variance is not constant but varies from point to point along the span) renders the approximate or elastic theory of little value for externally-anchored structures.

Another important advantage in the method of analysis developed by the author for self-anchored spans is the fact that it is possible to develop and utilize influence diagrams. That such diagrams cannot be used for the externally-anchored type is clearly illustrated in Fig. 11. For example, if one attempts to construct an influence line for the frame moment M at section $\frac{x}{l}$, Fig. 1, one immediately runs into difficulties since the deflection of the cable fluctuates with the magnitude and position of the load. In other words, as a unit load moves across the span from right to left, the increasing cable deflection shifts the node point n so that the negative-moment area is a continuously fluctuating quantity dependent upon the position of the load.

In summary, therefore, the self-anchored suspension type not only indicates distinct possibilities for economic utilization but also lends itself to certain simplified design procedures which are not applicable to externally-anchored designs. The author is to be congratulated for presenting, in a most readable manner, a method of analysis that should be of great benefit to the designer.

Jaroslav J. Polivka,[13] M. Am. Soc. C. E. (by letter).[13a]—A valuable contribution to the analysis of the self-anchored suspension bridge is contained in this paper. The arch-like action of the cambered girder may reduce cable and girder stresses considerably, and the economic advantage of such a structure is evident. The economy increases, naturally, with the amount of the vertical camber. Usually the vertical camber is limited by the required grade of the girder, and is relatively small in such a characteristic type of a suspension bridge as is shown in Fig. 1. In this case assumption (3) is justified—namely, that the average moment of inertia of the stiffening girder throughout each span is constant.

This assumption involves the approximation

$$\frac{\cos^2 \alpha}{\cos^3 \phi} = 1 \qquad (42)$$

in which α is the angle of the tangent at the ends and ϕ is the average angle of the structural sections of the arched girder, both with respect to the chord. The condition expressed by Eq. 42 results from the differential equation of the elastic curve,

$$E\, I \cos^3 \phi \, \frac{d^2\eta}{dx^2} = -\, M_x \qquad (43a)$$

[13] Research Associate, Univ. of California, and Cons. Engr., Berkeley, Calif.

[13a] Received by the Secretary June 16, 1941.

and from equations of continuity having the form

$$\frac{\cos^2 \alpha_1}{\cos^3 \phi_1} \theta_1 + \frac{\cos^2 \alpha}{\cos^3 \phi} \theta = 0 \qquad (43b)$$

Eq. 11, representing the continuity of the independent girder, is correct for the varying moment of inertia of the cambered girder following the relationship,

$$I = \frac{I_0}{\cos \phi} \qquad (44)$$

in which I_0 is the moment of inertia of the cambered girder at the crown.

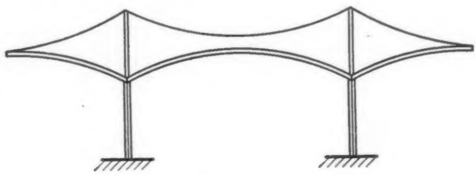

FIG. 12

The theory can be extended and applied for structures as shown in principle by Fig. 12, which represents a new type of structure of great economy, especially for large-span buildings, such as hangars, assembly plants of plane factories, etc.

WILLIAM BERTWELL,[14] ASSOC. M. AM. SOC. C. E. (by letter).[14a]—Although this paper is interesting and valuable, the method of design presented involves the use of fairly complicated formulas. The reduction of computations to substitution in formulas is often useful and desirable, but the writer believes that in this instance a more flexible procedure is preferable.

A clear conception of the action of the structure is obtained through consideration of the geometry of its deflections. If the cable is cut at one connection to the stiffening girder, a unit horizontal tension applied to the cable, and a unit horizontal compression applied to the girder, then the vertical deflections of the girder, divided by the relative horizontal movement of the girder and cable at the cut, yield the influence line for the horizontal component of cable stress. Although this well-known principle is expressed by Eq. 14, the writer believes that if the indicated steps are performed independently the possibility of overlooking a structural peculiarity of a given bridge is minimized, and the work is often simpler. The use of a truss or a girder with varying moment of inertia then results in no more difficulty than in an ordinary continuous structure without a cable.

It also appears that, if the points of actual suspension are chosen for the computation of influence ordinates, the effects of distributed loads might well be determined from those ordinates, rather than by integrations, which assume a continuous supporting medium.

Adopting the author's numerical example, the work would proceed about as follows in determining the influence line for H: The vertical load on the girder is $\frac{8(f + a)}{L^2} = -0.00278$ lb per ft in the side spans, and -0.002571 lb per ft in the main span. The moment at the tower may be determined by moment distribution, for example, and is 23.041 lb-ft.

[14] Regional Office, U. S. Bureau of Public Roads, San Francisco, Calif.

[14a] Received by the Secretary July 14, 1941.

The relative horizontal movement of the cut ends is:
Direct stress in the cable—

$$\frac{L_s}{A_c\,E_c}.$$

Bending in the cable—

$$\int_0^L (y + z + T_1)\,y\,\frac{dx}{E\,I} + 2\int_0^{L_1}\left(y_1 + T_1\,\frac{x_1}{L_1}\right)y_1\,\frac{dx_1}{E\,I_1}$$
$$= \frac{8}{15}\,\frac{f\,(f + a)\,L}{E\,I} + \frac{2}{3}\,\frac{T_1\,f\,L}{E\,I} + \frac{16}{15}\,\frac{f_1{}^2\,L_1}{E\,I_1} + \frac{2}{3}\,\frac{f_1\,L_1\,T_1}{E\,I_1}.$$

Direct stress in the girder—

$$\frac{L'_s}{A_g\,E}.$$

Bending in the girder—

$$\int_0^L (y + z + T_1)\,z\,\frac{dx}{E\,I} = \frac{8}{15}\,\frac{(f + a)\,a\,L}{E\,I} + \frac{2}{3}\,\frac{T_1\,a\,L}{E\,I}.$$

Totaling these terms:

$$\frac{L_s}{A_c\,E_c} + \frac{L'_s}{A_g\,E} + \frac{8}{15}\,\frac{(f + a)^2\,L}{E\,I} + \frac{2}{3}\,T_1\,\frac{(f + a)\,L}{E\,I} + \frac{16}{15}\,\frac{f^2_1\,L_1}{E\,I_1} + \frac{2}{3}\,\frac{f_1\,L_1\,T_1}{E\,I_1}$$
$$= \frac{144}{29 \times 10^6}\,(0.49465 + 0.05890 + 18.52200 - 13.54811$$
$$+ 0.46207 - 0.85091) = 5.1386\,\frac{144}{29 \times 10^6}.$$

The vertical deflections of the girder may be computed by any method the designer prefers. Table 3 shows the vertical deflections $\left(\text{multiplied by } \dfrac{29 \times 10^6}{144}\right)$ of the girder, and the influence ordinates for H, which are obtained by dividing by 5.1386.

TABLE 3.—INFLUENCE ORDINATES FOR H

Description	Fraction of Side Span				Fraction of Main Span				
	0.2	0.4	0.6	0.8	0.1	0.2	0.3	0.4	0.5
Vertical deflections[a]	−0.2694	−0.5467	−0.7426	−0.6611	1.9656	4.6469	7.1795	8.9460	9.5761
Influence Ordinate for H..	−0.0524	−0.1064	−0.1445	−0.1287	0.3825	0.9043	1.3972	1.7410	1.8636

[a] Multiplied by $\dfrac{29 \times 10^6}{144}$.

It is not clear to the writer what the author includes in his correction of H. He states (see heading "Effect of Motion of Ends of Girder Due to Deflection of Bridge") that Δx is the "entire change in horizontal length of girder." The computation he indicates in his numerical example appears to bear out this

definition. However, the writer believes that if this simple interpretation is intended the author has overlooked the fact that the elastic theory determines H by computing, among other values, the change in horizontal length of girder. The error in the elastic theory arises from the fact that deflections change the arms by which angle changes are multiplied to obtain horizontal movements. This error is greatest for the condition of a heavy load in the main span only, which results in deflections that are large compared to the camber of the girder and the sag of the cable. The error will be the difference between the change in horizontal length calculated by the elastic theory, and that which actually occurs. If this correction is made, it should not be limited to consideration of the girder only, as the cable contributes appreciably, particularly in side spans where the sag is small. Using the author's example, for the case of a 500-lb uniform load in the main span only, the value of H was found to be 188,210 lb; the deflection at the center line of the main span was 1.5847 ft; the deflection at the center line of the side spans was − 0.769 ft; and the changes in horizontal length were as shown in Table 4.

TABLE 4.—CHANGES IN HORIZONTAL LENGTH CAUSED BY BENDING

Member	Elastic Theory		Actual	
	Main span	Side spans	Main span	Side spans
Cable...	−0.8240	+0.4096	−0.8288	+0.3237
Girder..	+0.1030	0	+0.0865	−0.0210

Both the cable and girder are actually shorter than is indicated by the elastic theory. (The changes in horizontal length caused by direct stress, which are in addition to those shown in Table 4, are a lengthening of 0.4623 ft for the cable and a shortening of 0.0551 ft for the girder.) The correction in H will be: $\frac{0.0907 - 0.0375}{5.1386} \times \frac{29 \times 10^6}{144} = 2{,}080$ lb. Although this value is not much different from that determined by the author, it would appear that this is a coincidence, and not a result to be expected generally.

A. A. EREMIN,[15] ASSOC. M. AM. SOC. C. E. (by letter).[15a]—A simplified method of analyzing stresses in a self-anchored suspension bridge is presented in this paper. The computations serve to illustrate its simplicity.

It may be of interest to determine the error that may be introduced due to the simplifying assumptions adopted. The author assumed that the moment of inertia of a stiffening girder is constant for the entire length of the girder in each span. Prof. G. G. Krivoshein computed the stresses in a self-anchored suspension bridge with a continuous girder, assuming a constant moment of inertia in the girders and a varying moment of inertia increasing toward the supports. He found that the error due to neglecting the variation of moment of inertia may be more than 15%, and he stated that the method with the constant moment of inertia cannot generally yield suitable results.[16]

[15] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[15a] Received by the Secretary July 18, 1941.

[16] "Simplified Calculation of Statically Indeterminate Bridges," by G. G. Krivoshein, Prague, 1930, p. 204.

In Eq. 2*b*, showing a part of dead load carried by the arched stiffening girder, the author has neglected the effect of bending stresses on the amount of dead or live load carried by the arched girder. Evidently, this effect varies in direct proportion to the arch rise.

The author has shown that the elastic theory applied to the analysis of stresses in a self-anchored suspension bridge gives satisfactory results. In his analysis of stresses in a self-anchored bridge, Professor Krivoshein applied the

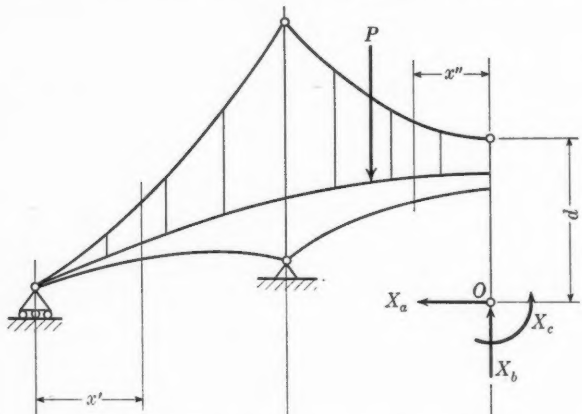

FIG. 13.—REDUNDANT FORCES X_a, X_b, AND MOMENT X_c

theory of elasticity,[16] with similar results, using the same method as that adopted for the analysis of stresses in an arch bridge. He severed the suspension bridge at the center of the middle span and replaced the removed half bridge by three redundant stresses: X_a, a horizontal force; X_b, a vertical shear; and X_c, a bending moment (see Fig. 13), as follows:

$$X_a = -\frac{\int M_q\, M_a \frac{1}{I}\, dx}{\int M^2{}_a \frac{1}{I}\, dx + \int S^2{}_g \frac{1}{A_g}\, dx + \sum S^2{}_c\, s \frac{1}{A_c}} \quad \text{(45a)}$$

$$X_b = -\frac{\int M_q\, M_b \frac{1}{I}\, dx}{\int M^2{}_b \frac{1}{I}\, dx} \quad \text{(45b)}$$

$$X_c = -\frac{\int M_q\, M_c \frac{1}{I}\, dx}{\int M^2{}_c \frac{1}{I}\, dx} \quad \text{(45c)}$$

and

$$d = \frac{\frac{1}{L_1}\int_0^{L_1} z\, x' \frac{1}{I}\, dx' + \int_0^{\frac{L}{2}} z \frac{1}{I}\, dx'}{\frac{1}{L^2_1}\int_0^{L_1} (x')^2 \frac{1}{I}\, dx' + \int_0^{\frac{L}{2}} \frac{1}{I}\, dx''} \qquad (46)$$

in which (in addition to the notation of the paper): M_q = moment at any section of the main statically determinate system (see Fig. 13), resulting from the given external loading P only (X_a, X_b, and X_c being removed); M_a = moment at any section of the main statically determinate system due to loading $X_a = 1$ (all other loads being removed); M_b = moment at any section of the main statically determinate system due to loading $X_b = 1$ (all other loads being removed); M_c = moment at any section of the main statically determinate system due to loading $X_c = 1$ (all other loads being removed); and d = length of stiff arm to which forces X_a, X_b, and X_c applied.

Evidently each redundant force for a given loading may be computed directly, and the influence lines for stresses may be constructed by Eq. 45.

Unlike the formulas in the paper, Eq. 45 cannot be reduced for application to an externally-anchored suspension bridge. This agrees with the author's statement that the self-anchored bridge acts more like a tied arch than an externally-anchored suspension bridge. It is to be noted that the elastic and geometrical qualities of the self-anchored bridge are considered more directly in Eq. 45. However, the author's method of analysis is more simple and requires less time for computation.

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DISCUSSIONS

FORMULAS FOR THE TRANSPORTATION OF BED LOAD

Discussion

By Messrs. A. A. Kalinske, O. G. Haywood, Jr., Samuel Shulits, and John S. McNown

A. A. Kalinske,[18] Assoc. M. Am. Soc. C. E. (by letter).[18a]—Any attempt to rationalize the problem of bed-load transportation by using known basic physical phenomena is most welcome, especially if the expressions developed are correlated with experimental data. Although the author's work in this direction is most enlightening, there are present certain ambiguities and some indefinite statements and assumptions that make the writer question the physical picture of bed-load transportation as presented, and the generality of the final expressions. If these questionings are due to misinterpretations on the part of the writer, it is hoped that the author will attempt to clear up these points.

One confusing item to the writer is in regard to exactly what the author considers as being bed load. In the "Introduction" he states that bed-load movement is confined to those bed particles moving in quick steps and with long rest periods. Does this only include particles that slide and roll or does it also include those that jump up? If a particle rolls along fairly continuously, or with only short rest periods, is it excluded from the author's definition? Under "Derivations" the author states definitely that his treatment excludes "Bed material moving in suspension." However, in his derivations he considers the lifting of particles off the bed and the effect of the turbulence on the particles. What the line of demarcation is between bed load and suspended load is nowhere definitely stated. In a previous paper[19] the author with A. G. Anderson, Jun. Am. Soc. C. E., and J. W. Johnson, Assoc. M. Am. Soc. C. E., gives a definition for bed load as follows: "Bed-load is that part of the total sediment-load composed of all particles greater than a limiting size whether moving on the bed or in suspension, and includes all bed-material in move-

Note.—This paper by H. A. Einstein, Assoc. M. Am. Soc. C. E., was published in March, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1941, by Joe W. Johnson, Assoc. M. Am. Soc. C. E.

[18] Asst. Prof., Univ. of Iowa, Iowa Inst. for Hydr. Research, Iowa City, Iowa.

[18a] Received by the Secretary June 18, 1941.

[19] "A Distinction Between Bed-Load and Suspended Load in Natural Streams," by H. A. Einstein, A. G. Anderson, and J. W. Johnson, *Transactions*, Am. Geophysical Union, 1940, Pt. II, p. 628.

ment." Does this definition apply to the analyses in the present paper? Would it not be better to define or differentiate between various types of sediment transportation by consideration of the different forces causing the movement instead of considering its visual appearance? It seems that this is what must be done if a physically correct analysis is to be made of any type of sediment movement.

The writer cannot agree that the concept of critical tractive force has no meaning in bed-load transportation. Naturally, for any given size of material, there is no single value of the tractive force below which not a single particle moves, and above which all particles of that size do move. First of all, the tractive force is not a constant value but tends to fluctuate about some average value. Second, the dragging and pushing forces tending to move the bed particles depend on the position a particle happens to assume and how much influence its neighbors exert. Nevertheless, statistically a value of the tractive force can be defined such that below it any movement will be practically negligible. To use the author's terminology it is possible to define a tractive force such that the probability of movement of any particular particle is very remote indeed for any practical considerations. Certainly the fine work of A. Shields[2] in regard to the critical tractive force for different sizes and types of particles indicates that this concept has physical meaning and practical usefulness.

The transportation of sediment by the process of saltation, so clearly described and analyzed by R. A. Bagnold[20] for the movement of desert sand by wind, also occurs to a certain extent in water. The physical processes causing such movement are closely related and even similar to those for bed load. In fact, experimentally, separate determination of bed load and saltation load is almost impossible. The author's description and analysis of the movement of bed load seem very much similar to movement by saltation, particularly his statements regarding step movement and the lifting process. Mr. Bagnold differentiates between bed load and saltation load by calling the former those particles which creep, slide, and roll along the bed without any appreciable rise from the bed. For desert sand this differentiation is fairly definite, since the particles moving by saltation rise an appreciable distance above the sand surface, many times as high as 4 ft to 6 ft. In ordinary rivers it is doubtful if the rise of particles off the bed in the process of movement by saltation is more than perhaps a fraction of an inch or a few inches at the most. Therefore, any distinction between bed load and saltation load in rivers and canals becomes difficult.

Physically, the bed load might be considered as those particles which do not have sufficient lift exerted on them to raise them off the bed; instead they can only roll or slide. It appears that saltation occurs when the forces causing the rolling and sliding become large enough to exert a "lift" on the particles and thus raise a particle up into the flowing fluid. The raised particles immediately drop down since there is no force to support them in the fluid. Actually, of

[2] "Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung," by A. Shields, *Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau*, Heft 26, Berlin, 1936.

[20] "Transport of Sand by Wind," by R. A. Bagnold, *Geographical Journal*, Vol. 89, 1937, p. 409.

course, turbulence is present and upward velocity components tend to keep the raised particles up longer than they otherwise would stay. However, it is not considered that transportation by suspension comes into the picture unless vertical velocity components are present that are larger than the fall velocity of the particles a sufficiently large proportion of the time. Mr. Bagnold definitely concluded that the turbulence in the air had a negligible effect in transporting the sand of the size he was considering. In fact, from his description of the process of saltation, this type of movement can occur without the presence of turbulence if a sufficiently high drag and forward velocity could be developed without inducing turbulence. Naturally, this is not possible with air or ordinary liquids. In regard to this particular point, Mr. Shields[2] showed experimentally that bed-load movement takes place at a Reynolds number when the viscous boundary layer still covers the particles. All this leads to the conclusion that bed-load movement, in the sense considered by Mr. Shields, and movement by saltation can be brought about by forces that do not necessarily arise due to the presence of turbulence. Of course, if turbulence is present, particle movement by these methods may be profoundly affected. In the writer's opinion, this is the intrinsic difference between bed-load and saltation-load movement and transportation by suspension; the latter type of movement is due entirely to turbulence.

In his classic experiments and observations, Mr. Bagnold defines a so-called "threshold" wind velocity and also relates the total sand movement to the mean flow conditions. This definitely contradicts the author's statements relating to critical conditions and the possibility of defining bed-load transportation as a function of mean flow.

In the "Introduction" the author lists four axiomatic statements on which he bases all of his analyses. According to him these statements are the results of previous studies; however, since the results referred to are published in a paper not readily available to American engineers in general, it would have been better to give a little more background material and present the basis for these fundamental statements. For instance, the concept of an "average step" certainly needs further elaboration.

In the "Derivations" the factor p is defined as giving "* * * the number of steps that start from any given place during the time it takes to remove one particle." Exactly what is meant by "place" and the "time to remove one particle"? Is "place" the area covered by one particle? In order to make p_s in Eq. 2 dimensionless the author introduces, very arbitrarily, a time t, which he chooses to call the time required for a particle to settle a distance equal to its own diameter. This particular t certainly could have been defined in many other ways. Therefore, it appears that at this point in particular the author departed from a theoretical analysis of the physical processes and stepped over into the realm of abstract dimensional analysis.

In the "Derivations" the author refers to bed-load movement as being due solely to the "lift" on a particle. The lifting forces that act on a particle resting on the bed are certain definite hydrodynamic forces caused by the flow over the particle. However, bed-load movement is not due entirely to such lifting forces; impact and pushing forces are certainly also present. Of course, all

such other forces are also probably proportional to $D^2 v^2 \rho_f$ as given by the author for the lift forces, although at low velocities the liquid viscosity may be of importance. Eq. 10 is not correct for the velocity at the distance from the wall of the particle diameter. J. Nikuradse's velocity equation for sand-roughened pipes gives for the value of v at the edge of the protuberances the value $8.5 \sqrt{\frac{\tau}{\rho_f}}$.

The data presented by the author seem to be functionally related in the manner indicated, particularly for the smaller sediment loads. However, for the larger sediment loads it is extremely doubtful if the measurements included solely bed load in the sense that the writer interprets bed load. In regard to the sand mixtures, such mixtures cannot be described completely by any mean effective diameter unless the size-frequency curve is of the same type and shape for all such mixtures.

The writer agrees with the author that there is no single, universal law which will describe bed-load transportation. The reason for this is that different sets of forces come into play for different stages of movement. For small ratios of sediment size to laminar boundary layer thickness, the forces causing movement are certainly different from conditions when the laminar layer is no longer present. As shown by Paul Nemenyi,[21] if any analysis or correlation of experimental data is to be really useful for extrapolation, the entire "picture" must be kept in mind. The complete picture of bed-load transportation is probably divided into a number of phases, each having a slightly different set of predominating forces. The various phases are probably separated by critical regions. The transition from pure rolling and sliding to combined rolling, sliding, and "jumping" or saltation is such a critical region. The coming into play of the vertical velocity component of the turbulence is another such critical region. All these various phases of sediment transportation will become immensely clarified if some parameter can be found against which data can be plotted for all the phases of sediment transportation, from the simple rolling and sliding of the particles, when the grains are still covered by a laminar boundary layer, to when the major part of the material is being moved in true suspension. Mr. Shields made a start in this direction by use of the parameter $v_f\, D/\nu$, in which v_f is the friction velocity, $\sqrt{\frac{\tau}{\rho_f}}$; D is the particle diameter; and ν is the kinematic viscosity. Perhaps the parameter $\frac{v_s}{v_f}$ (in which v_s is the settling velocity) may serve this all-important function. This particular parameter is a function of the Reynolds and Froude numbers of an individual particle.

This paper should certainly serve the very important function of stimulating thought in regard to the problems of sediment transportation, and particularly so-called bed-load movement, along directions different from those followed by engineers in the past years. The paper should also show definitely that the

[21] "The Different Approaches to the Study of Propulsion of Granular Materials and the Value of Their Coordination," by Paul Nemenyi, *Transactions*, Am. Geophysical Union, 1940, Pt. II, p. 633.

innumerable, purely empirical bed-load equations now scattered throughout the literature are probably of limited application and certainly should not be used blindly for conditions widely different from those for which the data used in developing the equation were obtained. Eq. 18 certainly seems quite generalized; nevertheless, the limitations of its applicability should be carefully noted. The important question about this equation is in regard to what exactly was taken or considered as "bed load" in the experiments on which it is based. This point should be more definitely clarified both for Mr. Gilbert's data[8] and that obtained by the author in Zürich.

O. G. HAYWOOD, JR.,[22] JUN. AM. SOC. C. E. (by letter).[22a]—An interesting analytical derivation of a bed-load formula is presented in this paper, which has the decided advantage that all of the arbitrary constants introduced have a definite physical meaning. However, proceeding backward from his final empirical expression to the evaluation of some of these constants leads to results sufficiently doubtful to reflect upon its validity.

From Eqs. 15*a* and 19, $A = 0.465$. The order of magnitude of constants A_1 and A_2 may be estimated by assuming spherical grains, thus giving $A_1 = \frac{\pi}{4}$ and $A_2 = \frac{\pi}{6}$. The general magnitude of λ_0, if it is a constant as assumed, may be estimated from observations of bed-load movement. For a grain of 1 mm diameter, a travel distance in a single step of 10 cm is not an unreasonable estimate, or $\lambda_0 = 100$. Then $A_3 = \dfrac{0.465 \times 100 \times \frac{\pi}{6}}{\frac{\pi}{4}} = 31$; and, from Eq. 6,

$$t = \frac{31}{0.75}\sqrt{\frac{0.1 \times 1}{980 \times 1.65}} = 0.32 \text{ sec},$$

which appears to be an absurdly high value for the time to remove one particle of 1 mm diameter from the bed. From the definitions of t, p, and p_s, t is evidently that almost instantaneous transition period when a particle passes from a state of rest to a state of motion, and which the writer would consider more properly of the order of a few thousandths of a second.

A similar analysis of the other equation constant, $B = \dfrac{A_2}{135\, A_4} = 0.391$, leads to equally disturbing results. Again assuming spherical grains in order to approximate the values of the constants, $A_2 = \frac{\pi}{6}$ and A_4 then computes to be 0.01. Thus, the coefficient of lift is 0.01 and constant for all conditions satisfying the author's basic equations. It must be noted, however, that Eq. 9 is identical to the generally accepted equation for the drag on a particle. Moreover, numerous experiments have shown that the coefficient for drag

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[8] "The Transportation of Debris by Running Water," by Grove Karl Gilbert, *Professional Paper No. 86*, U. S. Geological Survey, Washington, D. C., 1914.

[22] Capt., U. S. Army, Engr. Battalion, Fort Buchanan, Puerto Rico.

[22a] Received by the Secretary July 7, 1941.

varies decidedly with the turbulence around the particle (Reynolds' number), and that the coefficient has a minimum value of the order of 0.15. It is evident that the lift must exceed the weight of the particle before it will be lifted from the bed. It appears decidedly unlikely that particles could withstand a dragging force of at least fifteen times their weight before moving. In other words, if the coefficient of lift is that indicated by the author's results, the particles composing the bed would all be shoved along the bed long before the lift forces could become of sufficient magnitude to be effective, and the derivation based on this lift equation fails.

Returning to a more general view of the paper, the writer does not concur in the statement in item 2(*c*) of the "Introduction" that "The average step of a certain particle seems always to be the same even if the hydraulic conditions or the composition of the bed changes." This statement is important, as it forms the "backbone" of the analytical procedure, and its validity appears to be confirmed, as the value of λ_0 is found empirically to be constant for most of the data. From the writer's observations of bed-load movement, it appears that the steps increase in length as the velocity of flow increases. Although most of these observations were on light-weight materials, the phenomenon of equal steps was never noted by the writer and was never mentioned by any of his co-workers at the U. S. Waterways Experiment Station, Vicksburg, Miss., who had done extensive experimentation with sand movement. The fact that λ_0 actually does appear to be constant indicates to the writer not that the assumption of equal steps is correct but instead that λ_0 is an arbitrary constant that does not have the physical meaning attributed to it.

Finally, the evident discrepancy of certain of the data presented in Fig. 3 is explained as a result of differential sorting of the grain mixture forming the bed. The author states that this hypothesis is not subject to direct proof. The U. S. Waterways Experiment Station, during the conduct of the investigations analyzed by Mr. Einstein, made periodic analyses of the material moving into the sand trap. It would appear that these data should furnish sufficient direct proof as to whether or not appreciable sorting is occurring. Data for

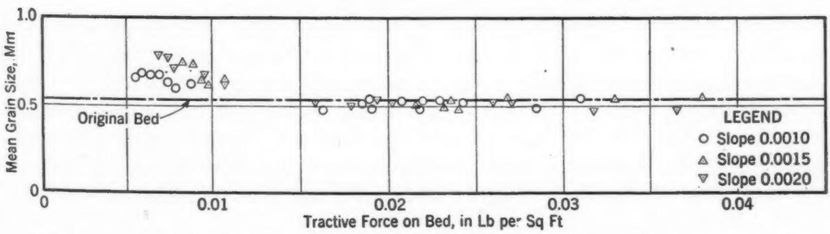

FIG. 4.—MEAN GRAIN SIZE OF MATERIAL IN MOTION; EXPERIMENTATION STATION, SAND 3

one sand are reproduced in Fig. 4. The curve is typical, showing that the larger grains move first, but that, as movement increases in intensity, the material in motion becomes representative of the material in the bed.

However, even without these data, the explanation appears faulty from the plotted data presented in the paper. It appears that, if the hypothesis is

correct, the grouping of points along curve (S) in Fig. 3 is caused by the formation of a coarse protective layer of grains, and the plotted points would fall along curves (1) and (2) if the grain size of this protective layer were used in the formulas instead of the mean grain size of the original bed material. The same analysis may be applied in the reverse direction. Knowing the location of the plotted data and the bed-load equation, the grain size of the protective layer may be determined.

From Eqs. 16*a* and 16*b*, $\phi \sim \frac{1}{F} \times \frac{1}{D^{1.5}}$; and $\psi \sim D$. Taking sand 6, for example, there is a grouping of points at about $\phi = 0.0025$ and $\psi = 6.0$. By cut and try, it may be found that the points fall on curve (1) if $D = 0.125$ cm, giving $\phi = 0.00025$ and $\psi = 24$. Thus, if the author's explanation is correct the protective layer is composed of grains of an average diameter of 0.125 cm.

However, the reference from which Mr. Einstein obtained these data gives a grain-size distribution curve of this sand. There are no particles as large as 0.125 cm in the sand mixture, and less than 2% by weight of the particles composing this sand mixture are even half as large. Obviously, sorting alone is not the answer.

The writer has also noted, with interest, the contents of Appendix II regarding elimination of side-wall influence, as he has made a somewhat extensive investigation of this subject.[23] Although the author's method has been used by many investigators, published data actually verifying the method are rather rare. Most data on open channel flow in laboratory flumes have an experimental error of such magnitude as to obscure small variations in roughness. Natural channels, on the other hand, do not tend to have a marked change of roughness at some particular depth. The ideal channel to investigate would be one of sufficient size so that errors in measurement have negligible weight, and which has channel boundaries that have a definite change of roughness at some stage, with uniform roughness below this stage, and uniform but different roughness above. Some drainage ditches approximate this condition, and an analysis has been made of data published in the Department of Agriculture's bulletin, "Flow of Water in Drainage Channels."[24]

After rejecting channels because of insufficient data, insufficient change of roughness with depth, erodible banks, or undue effect of foliage, the following eight channels were considered to be reliable: Twenty Mile Creek, in Mississippi; Bogue Phalia, in Mississippi; Forked Deer River, in Tennessee; Sugar Creek, in Tennessee; Ditch No. 1 near Chaffee, Mo.; special dredged channel, Lake Fork, Ill.; Kaskaskia River, in Illinois; and Two Mile Slough, in Illinois.

From Eqs. 25 to 28, it may be easily derived that the general equation for the Manning's *n* of the entire channel is:

$$n^{1.5} = \frac{\sum_{m=1}^{N} n^{1.5}{}_m \, P_m}{P} \qquad (34)$$

[23] "Flume Experiments on the Transportation by Water of Sands and Light-Weight Materials," by O. G. Haywood; submitted in 1940 to the Massachusetts Institute of Technology, at Cambridge, Mass., in partial fulfilment of the requirements for the degree of Doctor of Science.

[24] "Flow of Water in Drainage Channels," by C. E. Ramser, *Technical Bulletin No. 129*, U. S. Dept. of Agriculture, November, 1929.

Most of the dredged drainage channels have slight flow for the major part of the year, and a greatly increased flow for short periods of time. Thus, the lower part of the channels tends to be free of vegetation, whereas the banks are more or less thickly vegetated. If it is assumed that the perimeter is composed of sections of two uniform roughnesses (that is, the bottom and the banks) the general equation may be written

$$n^{1.5}\,P = n^{1.5}_1\,P_1 + n^{1.5}_2\,P_2 \qquad (35)$$

in which P_1 is the wetted perimeter of the lower section of the channel, P_2 the remainder of the wetted perimeter, and n_1 and n_2 their respective roughness coefficients.

The value of P, of course, is dependent on the depth of flow. For flow in the lower part of the channels, the value of P_1 varies with the depth of flow, and P_2 is zero. For greater depths of flow P_1 is constant and P_2 varies with stage. Thus, Eq. 35 may be written in two parts:

$$n^{1.5}\,P = n^{1.5}_1\,P_1 \qquad (36a)$$

if the flow is in the lower part of the channel and

$$n^{1.5}\,P = \text{constant} + n^{1.5}_2\,P_2 \qquad (36b)$$

if the vegetated banks form part of the wetted perimeter.

When the value of $n^{1.5}\,P$ is plotted against wetted perimeter, the points therefore will lie on two straight lines. One line passes through the origin, with its slope determined by the value of "n" for the lower part of the channel. The slope of the other line is determined by the roughness coefficient of the banks, and intersects the lower line at a value of perimeter corresponding to that stage at which flow against the vegetated banks first occurs.

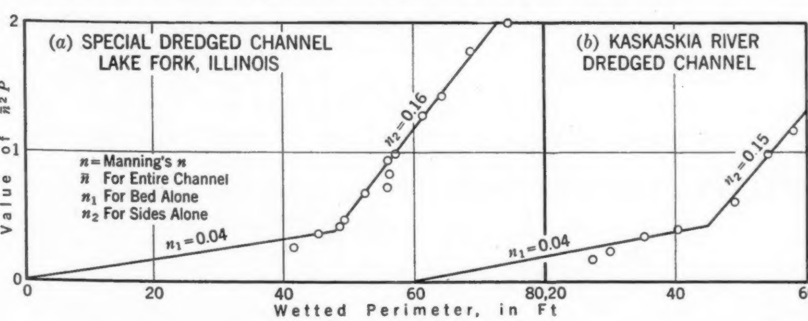

FIG. 5

Two of the graphs made by the writer are reproduced in Fig. 5. It will be noted that the data check the predicted straight lines very closely. All of the aforementioned drainage ditches were plotted in similar graphs, and none were out of accord with the reasoning. Although these graphs verified the method for large channels, its application to laboratory flumes was still considered questionable. Certain of the author's assumptions appeared valid for large

values of the Reynolds number, but more questionable for lower values such as those obtained in flumes. Some experimental data taken by the author in a flume with smooth walls and sand-roughened bottom are shown in Fig. 6.

As a further check on the method, the writer attempted an entirely different approach based on the Prandtl-Kármán theory of flow,[25] following to

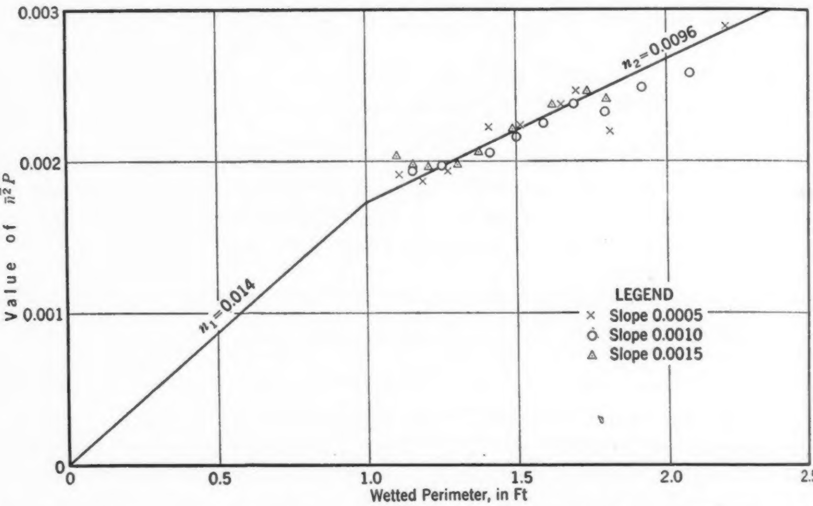

FIG. 6.—RECTANGULAR FLUME WITH SMOOTH WALLS AND SAND-ROUGHENED BOTTOM (WIDTH = 1 FT)

some extent the reasoning of G. H. Keulegan[26] and H. Schlicting.[27] The basis of this method is the division of the flume channel into three sections as shown in Fig. 7, and computing the flow in each section from the Prandtl-Kármán equation, using

$$\frac{V}{\sqrt{\frac{\tau}{\rho}}} = 5.5 + 5.75 \log \left(\frac{y}{\nu} \sqrt{\frac{\tau}{\rho}} \right) \qquad (37a)$$

for flow near the smooth side-walls, and

$$\frac{V}{\sqrt{\frac{\tau}{\rho}}} = 8.5 + 5.75 \log \left(\frac{y}{k_s} \right) \qquad (37b)$$

for flow near the rough bottom. In these dimensionless equations, V = velocity at any point at distance y from the boundary, τ = shearing stress on boundary, ρ = fluid density, ν = fluid kinematic viscosity, and k_s = "equivalent sand roughness" of boundary.

[25] "Applied Hydro- and Aeromechanics," by L. Prandtl and O. G. Tietjens, McGraw-Hill Book Co., Inc., New York, N. Y., 1934.

[26] "Laws of Turbulent Flow in Open Channels," by G. H. Keulegan, *Research Paper RP1151*, *Journal of Research*, National Bureau of Standards, December, 1938.

[27] "Experimentelle Untersuchungen zum Rauhigkeitsproblem," by H. Schlicting, *Ingenieur-Archiv*, Vol. VII, No. 1, February, 1936 (abridged translation in *Proceedings*, Am. Soc. C. E., November, 1937, Translations, p. 16.)

The position of the dividing lines between the sections was determined by the relationship between shearing stress on the walls and volume of water effective against them. For any given condition of flow, the total shearing stress and the total rate of flow are known, and the distribution should be such as to make the computed velocity along the dividing line the same, regardless of whether computed from the wall stress or from the bottom stress. Discussion, at some length, of the assumptions and limitations of the method and full derivation of the necessary equations are contained in the writer's thesis.[23]

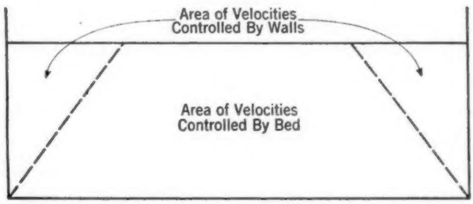

FIG. 7.—SCHEMATIC OF FLUME CROSS SECTION

An alternate method of determining the position of the dividing lines between the sections is to locate them so as to obtain the maximum flow for the given total boundary stress. Except for this change in the method of determining the position of the dividing lines, this "maximum discharge" method is identical with the one just discussed.

In Table 1, the first four columns cover the assumed conditions of flow—that is, the slope, depth, discharge, and mean velocity—the latter two mutually dependent quantities being dependent on the roughness of the bed.[23] The effective discharge is that part of the discharge that is effective against the bed; that is, the total discharge minus that part dissipating its energy against the walls. Computations for the effective discharge under the various conditions of flow were made by two methods, as shown in the table, the "writer's method" being that based on the Prandtl-Kármán method, and the "author's method" being that described in the paper.

The tractive force on the bed is the total boundary shearing stress minus the stress acting against the walls. In addition to computations based on the "writer's method," the "author's method," and the "maximum discharge method" (described briefly herein), computations are also shown for the two limiting values, the first based on the walls offering no resistance and the second on the walls offering resistance equal to that of the bed. The first is the familiar du Boys tractive-force equation,

$$\tau = \gamma D S \qquad (38a)$$

in which τ = drag, or boundary shearing stress, γ = unit weight of fluid, D = depth of water, and S = slope of energy gradient or, for uniform flow, slope of water surface. The second,

$$\tau = \gamma R S \qquad (38b)$$

is the same equation with the hydraulic radius substituted for the depth.

The substantial agreement of the writer's and the author's methods, especially for the values of bed tractive force, will be noted. Although it is fully realized that this agreement does not definitely prove that either method gives

correct results, the two methods are based on quite different assumptions and analytical procedures, and their substantial agreement is encouraging. As extreme accuracy in analyzing bed-load data is not necessary because of the large experimental error always present, it is concluded that either of the two

TABLE 1.—COMPARISON OF SIDE-WALL CORRECTION METHODS

Slope	Depth, in ft	Discharge, in ft per sec	Mean velocity, in ft per sec	Effective Discharge, in Cu Ft per Sec: Writer's method	Effective Discharge, in Cu Ft per Sec: Author's method	Tractive Force on Bed, in Lb per Sq Ft: du Boys $\gamma D S$	Tractive Force on Bed: $\gamma R S$	Tractive Force on Bed: Writer's method	Tractive Force on Bed: Author's method	Tractive Force on Bed: Maximum discharge method
(a) GLASS-WALLED FLUME, WIDTH 1.0 FT, WALLS SMOOTH										
0.0001	0.4	0.167	0.42	0.125	0.119	0.0025	0.0014	0.0017	0.0018	0.0017
0.0005	0.1	0.060	0.60	0.054	0.051	0.0031	0.0026	0.0026	0.0027	0.0027
0.0005	0.1	0.031	0.31	0.030	0.029	0.0031	0.0026	0.0029	0.0029	0.0029
0.0005	0.4	0.491	1.23	0.316	0.275	0.0125	0.0069	0.0073	0.0070	0.0078
0.0005	0.4	0.416	1.04	0.311	0.275	0.0125	0.0069	0.0083	0.0083	0.0084
0.0005	0.4	0.416[a]	1.04[a]	0.310[a]	0.275[a]	0.0125[a]	0.0069[a]	0.0081[a]	0.0083[a]	....
0.0005	0.4	0.296	0.74	0.253	0.236	0.0125	0.0069	0.0097	0.0100	0.0098
0.0005	0.8	1.246	1.55	0.422	0.478	0.0250	0.0096	0.0102	0.0096	0.0114
0.0005	0.8	0.849	1.06	0.597	0.553	0.0250	0.0096	0.0159	0.0163	0.0155
0.0015	0.4	0.772	1.93	0.577	0.494	0.0375	0.0208	0.0249	0.0240	0.0243
0.0015	0.4	0.556	1.39	0.475	0.434	0.0375	0.0208	0.0290	0.0292	0.0280
0.0015	0.8	2.024	2.53	0.969	0.935	0.0750	0.0288	0.0370	0.0346	0.0355
0.0015	0.8	1.576	1.97	1.110	0.996	0.0750	0.0288	0.0476	0.0474	0.0421
(b) CONCRETE FLUME, WIDTH ABOUT 2.3 FT, WALLS SMOOTH										
0.0020	0.4	2.413	2.62	2.13	1.96	0.0500	0.0371	0.0407	0.0404	....
0.0020	0.4	1.51	1.64	1.34	1.36	0.0500	0.0371	0.0425	0.0434	....

[a] Water temperature 16° C (60° F); all other computations are for a water temperature of 24° C (75° F).

methods, and probably any other method based on rational consideration of velocity and shear distribution throughout the cross section, is sufficiently accurate. However, it should be noted that, if the average tractive force on the bed is computed by Eq. 38(*a*), serious error is introduced, particularly in deep, narrow channels. It is not intended herein to question the validity of the du Boys expression for large, natural channels where the depth is small in comparison with the width of the channel.

SAMUEL SHULITS,[28] ASSOC. M. AM. SOC. C. E. (by letter).[28a]—This scholarly probe into the realm of a universal law for the transportation of bed load is inspiring and merits the careful consideration of researchers in solids transportation. Since the ultimate end of all such analytical inquiries is a rational and practical implement for engineers, it may be interesting to conjecture what the reaction of a river engineer might be to the proposed method.

[28] Chf., Engr. Dept. Research Centers, U. S. Waterways Experiment Station, Vicksburg, Miss.

[28a] Received by the Secretary July 14, 1941.

The largest part of available experimental data on bed-load movement is that measured by G. K. Gilbert, A. Schoklitsch, E. Meyer-Peter, and the U. S. Waterways Experiment Station—and perhaps F. Schaffernak. Others have made a few scattered flume tests and then produced a bed-load formula, based on their own tests and those of the aforementioned authorities. All the formulas are different, with varying claims for their applicability, although some can be shown to resemble each other very closely. The author presents a law of transportation, the empirical coefficients for which are determined from the Zürich[7] (Meyer-Peter) and Gilbert[8] data and then finds that the U. S. Waterways Experiment Station measurements do not fit very well. Professor Meyer-Peter and the U. S. Waterways Experiment Station have offered their own formulas for their measurements, whereas Mr. Gilbert arrived at no generally usable equation. Professor Schoklitsch calculated another formula with his own and the Gilbert data. Of what avail is all this research if different investigators can create different results from the same basic data? Can the river engineer take one formula and use it with assurance or with some indication of its inherent error? The maze becomes worse when one realizes that not all the proposed formulas have been mentioned in this paragraph.

The author does not favor the concept of a critical value pertaining to the flow at which transportation begins. This factor is of physical significance, since there are flow conditions at which no bed-load movement occurs—a demonstrable fact with which river engineers like to deal. Little is gained by discarding the idea of a critical value, as the complicated formulas of this paper reveal. Why discard it when it is usually of little influence in practice? To illustrate this view, consider merely as an example the Schoklitsch bed-load formula,[29] derived from simple rational premises:

$$G = C\,(Q - Q_0) \qquad (39a)$$

In Eq. 39*a* G is the bed load, Q the instantaneous discharge, and Q_0 the critical discharge at which movement begins—all in cubic feet per second; whereas C is the dimensionless bed-load coefficient. Eq. 39*a* is also dimensionally pure. In actual calculations, Q_0 is negligible in comparison with Q, and its exclusion or inclusion is simply a matter of judgment, still necessary regardless of the bed-load formula preferred. The formula then becomes

$$G = C\,Q \qquad (39b)$$

There is no valid logical objection to the concept of such a critical parameter. Its magnitude is not yet definitive, to be sure, but that is the problem of research and does not vitiate the significance of this physical concept. Similarly, the author refers to grain diameter and settling velocity, which are two parameters still far from being "nailed down" in a utilitarian sense.

The difference between the Schoklitsch formula, for example, and the Einstein law is, briefly: The former predicts the mathematical relationship

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[7] "Neuere Versuchsresultate über den Geschiebetrieb," by E. Meyer-Peter, H. Favre, and A. Einstein, *Schweizerische Bauzeitung*, Vol. 103, No. 13, March, 1934.

[8] "The Transportation of Debris by Running Water," by Grove Karl Gilbert, *Professional Paper No. 86*, U. S. Geological Survey, Washington, D. C., 1914.

[29] "The Schoklitsch Bed-Load Formula," by S. Shulits, *Engineering*, June 21 and 28, 1935, p. 644.

between certain parameters, and then strives by means of experimental data to prove the basic reasoning and to determine certain coefficients; whereas the author devises two dimensionless functions and the determines empirically (from experimental data) their mathematical relationship (f) and certain coefficients (A and B). The first method is usually accepted as the best research procedure. As far as usefulness is concerned, it cannot be claimed that the second method is superior, although it may not be inferior. The river engineer may like to use Eq. 18, which states that ϕ is a constant times ψ; but he will then have to use constants and a function (f) which, strictly, fit only the Zürich and Gilbert data. Thus, there is no assurance of the universality of the constants or function, even if dimensionless characteristics are desirable.

Since Eq. 18 can be written:

$$\phi = \frac{\psi}{0.465\ e^{0.391}} \qquad (40)$$

the possibility of a straight-line plot is indicated, rather than the semilogarithmic presentation in Fig. 3. Has the author investigated this possibility?

JOHN S. MCNOWN,[30] JUN. AM. SOC. C. E. (by letter).[30a]—The equation presented by the author, for bed-load movement, marks a forward step in the solution of this complex problem. A phenomenon that is subject to so many variables does not lend itself readily to a general analysis. For this reason the tendency of the experimental points to approach a single curve for movement of a uniform bed load, as shown in Fig. 3(a), is remarkable. The principal variables included in the experimental data used in this paper are amount of bed load, depth (or hydraulic radius), and particle density. Of necessity other variables such as water discharge, water velocity, and channel slope are included because of their dependence on the principal variables.

Some factors of probable importance are apparently omitted in the analysis. That the effect of a nonuniform bed is not included is obvious from a comparison of Fig. 3(a) and Fig. 3(b). Whether or not the shape of particles used in the two sets of experiments has an effect is not clear as no mention is made of this factor by the author. The plotted data show no trend that could be accounted for in this way. The application of this equation to the data obtained by Ho Pang-Yung [31] using irregularly shaped particles would be of assistance in settling this point. Also, the writer is of the opinion that the solution of the bed-load movement will be incomplete until the degree of turbulence is included as a variable. This cannot be done, of course, until an instrument for measuring turbulence conveniently is devised.

The agreement obtained is all the more remarkable as some of the underlying assumptions are admittedly uncertain. The existence of a constant ratio between the length of step and the size of particle seems improbable to the writer. Under this condition all particles in a uniform bed would have to take

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[30] Instr. of Math., Mechanics and Hydraulics, Univ. of Minnesota, Minneapolis, Minn.

[30a] Received by the Secretary July 17, 1941.

[31] "Abhängigkeit der Geschiebebewegung von der Kornform und der Temperatur," by Ho Pang-Yung, *Mitteilungen der Preussischen Versuchsanstalt für Wasser-, Erd- und Schiffbau*, Heft 37, Berlin, 1939.

steps of the same size. Also a particle of a certain size would have to make the same size of steps under all conditions of movement. This implies a regularity that the writer has never observed in bed-load movement. If the same result were to be obtained by the assumption of a variable step length the plausibility of the derivation might be increased. This can be done by a slight change in the method of obtaining Eq. 1.

The number of particles per second that cross the line labeled "Section" in Figs. 1 and 8(*a*) must be equal to the number of particles starting a step of

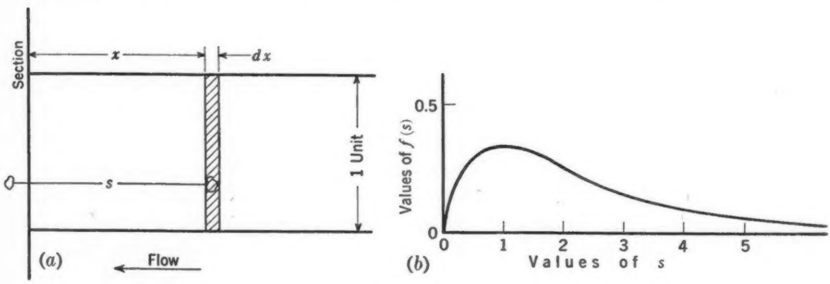

FIG. 8

variable length in one second that is long enough to cross the line. The two sets of particles are not identical but the equality must exist for continuous flow. The number of particles per unit width, beginning a step in one second from the differential area, which is a distance x from the line, is given by

$$N = \frac{dx}{A_1 D^2} p_s \qquad (41)$$

as shown in Fig. 8(*a*). The probability that a step will cross the line is designated as p_x. The functional relationship defining the frequency distribution of the variable-step length s should satisfy the following conditions: (1) The probability of making a step of zero distance is zero, (2) the probability of making a step of infinite distance is also zero, (3) the function must exist and be positive at all points between zero and infinity, and (4) the total probability or area between the distribution curve and the s-axis must be equal to one. One function (see Fig. 8(*b*)) that satisfies these conditions is

$$f(s) = s\, e^{-s} \qquad (42)$$

This curve may be thought of as a special case of the family of skewed frequency curves generally designated as Pearson's Type III. The probability that $s > x$ is given by

$$p_x = \int_x^\infty s\, e^{-s}\, ds = (x + 1)\, e^{-x} \qquad (43)$$

The total number of particles beginning steps that are long enough to reach or cross the line per unit width is then

$$\int \frac{dx}{A_1 D^2} p_s\, p_x = \frac{p_s}{A_1 D^2} \int_0^\infty (x + 1)\, e^{-x}\, dx = \frac{p_s}{A_1 D^2} 2 \qquad (44)$$

From the integral the term 2 is seen to have the dimension of a length and is approximately three fourths of the mean step length for the distribution that was assumed. By considering this value as simply a characteristic length, the length, L, may be substituted for it by a change of units and the author's expression, $\frac{p_s L}{A_1 D^2}$, is obtained for the number of particles crossing the line, per second per unit width. The introduction of a constant would have no effect on the final result so that L could be taken as the average length of step. The author's assumptions on step length would then be applied to the mean value rather than to a single step.

In Eqs. 9 and 10 the effective velocity in determining the lift is assumed to be the wall velocity. The criticism that the author applies to the concept of critical movement would apply here also. Were the lifting force dependent on the wall velocity alone a critical velocity would necessarily exist. The wall velocity is probably not as important in the movement of particles as is the vertical component of the instantaneous velocity. An increase in the velocity criterion used should indicate an increase in the probability that the lift would exceed the weight of the particle rather than an increase in the lift itself. Using a turbulence criterion such as the root mean square of the vertical component of the instantaneous velocity, a gradual transition would result because of the nature of turbulence. Under comparable experimental procedures the wall velocity and the turbulence velocities may have a nearly constant ratio. Should this be the case no serious error would be introduced by the use of the wall velocity. This question, like so many others, must await a more complete experimental study of turbulence.

The critical values that have been used particularly in formulas for bed-load movement based on the du Boys concept are more likely misnamed than incorrectly conceived as the author maintains. Their use can be interpreted as simply the introduction of a constant to fit the general equation to the experimental data. The measurements corresponding to this particular value mark the end of the transition between no bed load and conditions that fit the particular formula. Some such value is necessary just as the quantity A in Eq. 17 is necessary in the author's derivation.

The value τ in Eqs. 10 and 11 designates the boundary shear, not the variable internal shear. If τ were replaced by τ_0 this fact would be shown more clearly.

This paper should certainly serve as a valuable analysis on which further developments in this intricate problem can be based. The methods and equations used by the author are somewhat complex but, in general, basically sound. The author is to be complimented for his excellent paper.

Corrections for *Transactions:* In Eq. 13, page 356, change $(\rho_s - \rho_f)^D$ to $(\rho_s - \rho_f)D$; and in Eq. 18, page 357, change $e^{-0.391}\,\psi$ to $e^{-0.391\psi}$.

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DISCUSSIONS

ON THE METHOD OF COMPLEMENTARY ENERGY

Discussion

BY R. V. SOUTHWELL, ESQ.

R. V. SOUTHWELL,[37] ESQ. (by letter).[37a]—By this communication the writer wishes to record his appreciation of Dean Westergaard's very clear presentation of the "Method of Complementary Energy." The use of a quasi-energy function, defined as in Eq. 9, was propounded in one of the contributions to a symposium of engineering mechanics held at Ann Arbor, Mich., in the summer of 1935. The writer does not remember, however, that the notion was then attributed by any speaker to Engesser, and he was not aware that it had been propounded already. It must be of interest to all teachers who are concerned to understand the fundamentals of stress-strain theory.

Whether it will have equal appeal to the engineer whose first concern is to calculate correctly, only time can show. Use has so familiarized the concepts of kinetic and potential energy that as an investigator one is kept straight, so to speak, in one's working by a kind of physical intuition, and as a teacher one is likely to regard those concepts as natural and easy. It is salutary, perhaps, to find how groping are one's first attempts to use the complementary notion: One may thereby be made more sympathetic to the difficulties of students! Clearly, time must be found for a serious attempt to master the new technique, and to assess its sphere of usefulness.

Meanwhile, the writer can offer only one comment in regard to the comparison of methods given under the headings "Buckling of Column with Hinged Ends" and "Application to Vibration of Beams." Dean Westergaard has given two examples in which the error resulting from a use of Rayleigh's principle exceeds 20%. The writer suggests that these examples are not really fair to the principle, for the reason that modes such as Eqs. 24 and 61 are excluded (tacitly) in the argument by which it is derived. That argument is based on the concept of conservation of energy, and a formula such as Eq. 27

NOTE.—This paper by H. M. Westergaard, M. Am. Soc. C. E., was published in February, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1941, by Messrs. I. K. Silverman, and George R. Rich.

[37] Prof. of Eng. Science, Oxford Univ., Oxford, England.

[37a] Received by the Secretary June 9, 1941.

presumes that no energy can enter or leave the system. A mode in which both slope and curvature are non-zero at the ends of a strut, however, must entail there either input or output of energy. Neglecting this "leakage," it is not surprising that one should obtain seriously inaccurate results. Similar remarks apply to the second example.

The writer's reason for emphasizing this point is that an analogous condition for close accuracy should presumably be recognized when the complementary method is used. It would be worth while to examine whether this is so, and whether (for example) it excludes the use of a mode as shown in Fig. 3(*b*). In fact, when two methods have such close similarity, it is to be expected that every precaution that must be taken in using the first has its analogue in a precaution that must be taken in using the second. Provided that such precautions have been stated and are generally recognized, the method of complementary energy seems likely to play an important part in the study of plastic distortions, where the notion of elastic strain-energy has such limited utility.

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DISCUSSIONS

CRACK PREVENTION PROGRAM, HIWASSEE DAM

Discussion

BY MESSRS. DOUGLAS MCHENRY, REGINALD H. THOMSON, AND W. R. WAUGH

DOUGLAS MCHENRY,[4] ESQ. (by letter).[4a]—The author has presented a straightforward objective account of the procedures which were successfully used to control cracking during the construction of a large concrete dam. It might be well to emphasize the thought that Hiwassee, like every other major dam, represents a particular case in respect to both design and construction, and that the control methods which were satisfactory in this case may be insufficient or uneconomical elsewhere. On this account the paper would not have lost in value or interest had it included more discussion of the reasoning which led to the adoption of certain procedures and the rejection of others, together with a final evaluation of the effectiveness of the various items listed in the cost schedule.

On future projects where cracking control is required, the Hiwassee experience may well serve as a guide, but scarcely as a model to be followed exactly. The list of factors to be considered in conducting a control program may reach rather formidable dimensions. It will include such items as properties, topography, and structure of the foundation, properties of the materials of construction, details of design, river diversion scheme, climate, and (not the least important) the personnel of the design and construction groups. All of these items can, and should, be taken into account. Many of the details can be accorded only qualitative consideration, and must be handled in accordance with common sense and experience. Many cannot be covered by advance specifications because the conditions requiring treatment are often unpredictable.

A successful crack control program must combine (very much after the fashion of a successful program of foundation treatment) a rigid general specification, based upon preliminary study, with flexible detail specifications which may be adapted to various unpredictable conditions.

NOTE.—This paper by O. Laurgaard, M. Am. Soc. C. E., was published in March, 1941, *Proceedings*.

[4] Engr., U. S. Bureau of Reclamation, Denver, Colo. (formerly Engr., TVA).

[4a] Received by the Secretary July 23, 1941.

The writer was in close contact with the Hiwassee program from the time of its inception until the completion of the dam, and gained a definite impression that the relative values of the different phases of the program were about proportional to their relative costs, with the exception of the item "low casting temperatures." It appears, now that the job is completed, that the expenditure for cooling the mixing water would have been better justified had it been increased sufficiently to permit the use of slush ice in the mixers (as at Friant Dam, in California); or, if it had been distributed among the other phases of the program. The advantage gained by pre-cooling is dissipated to some extent during the period of exposure between lifts; so pre-cooling will be most effective with a rapid placing schedule.

The evidence favoring some type of low-heat cement for massive concrete construction is now established rather definitely. Former fears regarding construction difficulties due to its slower rate of hardening have been largely dispelled, and experience has shown that it need cost little or no more than modified cement. Superficial temperature studies may be deceptive in regard to the effect of type of cement. Under certain construction conditions it is possible that low-heat cement will produce a greater temperature rise than modified cement. The advantage lies chiefly in the slower early rate of heat generation (especially advantageous with artificial cooling), the more favorable shape of the temperature gradients, and the greater extensibility of the concrete.

The decision to use thin lifts on the foundation and on old concrete surfaces was based upon theoretical studies of temperature distribution, and the resulting stresses. Such studies were necessarily incomplete because of the present lack of a satisfactory means of relating strains and stresses during the early life of concrete. It is gratifying that field performance indicated that this phase of the control program was fully justified.

The cost item of $11,064 for cooling the concrete in place artificially represents, in the writer's opinion, one of the most worth-while items of expenditure in the program. Among other things, it has indicated the complete feasibility of applying embedded-pipe cooling to local regions where a degree of volume constancy is required, either to prevent cracking or for other reasons. It is of interest to compare the cost of the adopted control methods with the cost of an alternative scheme, probably equally effective, involving artificial cooling of the entire dam. With complete cooling the items of thin casting lifts, low casting temperatures, cooling the aggregate, and local artificial cooling might have been eliminated, but all others would have been largely, if not wholly, retained. The minimum cost of a complete embedded-pipe cooling system may be taken as $0.144 per cu yd of concrete (the cost for Grand Coulee Dam), or $115,000 for the entire structure. The total cost under this scheme would have been $231,000, compared with the actual cost of $210,621. The difference is scarcely great enough to be significant as an indication of the relative economies of the two schemes because of insufficient data on their relative merits. In this particular case, grouting the contraction joints before putting the structure into service was not considered necessary. In cases where early grouting is essential, complete artificial cooling is definitely required.

The author's balancing of saving in cement against the cost of the control program to indicate a net saving is most interesting. This balance apparently carries an inference that the dam as actually constructed, with a low interior cement content and relatively free from cracks, is equal in quality to the same structure with higher strength interior concrete and a normal amount of cracking. The writer is inclined to concur in this view, but recognizes it as an excellent subject for speculation.

The inclusion of the $29,000 item for the scientific program is likewise of interest; this inclusion acknowledges a debt to the similar research investigations of the past that are to a large extent responsible for the success and economy of the Hiwassee control program.

Reginald H. Thomson,[5] Hon. M. Am. Soc. C. E. (by letter).[5a]—This is a paper of unusual excellence because it gives a clear and concise report on the numerous different operations that had as their common objective the prevention of cracks and the construction of the most durable wearing surfaces in the Hiwassee Dam. Furthermore, a financial statement on the program is submitted, and the reader is afforded an opportunity of appraising this phase of concrete dam construction on a dollars-and-cents basis. Although he differs with the author on some matters, the writer would nonetheless express his appreciation of this distinctly superior paper. The fact that the total cost of the dam is nowhere given renders percentage studies based upon total cost impossible.

During recent years the building of concrete dams has become a matter of increasing care and concern to those charged with the responsibility of their construction and maintenance. For this there is good reason, since some of the older dams have failed to possess either the watertightness or "permanence" so confidently and enthusiastically attributed to them when they were first built. Dams are expensive structures at best, and heavy maintenance and repair bills following heavy initial investments are always unsatisfactory. Particularly are they so when unexpected.

The engineers furnishing the ten or eleven formulas, under which this construction was to be completed, showed by their provisions that they fully realized that water was one of the greatest of solvents, that in passing through cracks either in concrete or granite it dissolved and carried away some of the substance, and in so doing not only enlarged the crack but weakened the structure. The rules for the distribution of the cement throughout the structure showed an intelligent comprehension of the relation of heat in the newly made blocks of concrete to their expansion and contraction in the process of curing.

There was also a definite comprehension of the fact that either rock or masonry exposed to weather is subject to elements provided by nature for the destruction of masonry by the aid of water in connection with heat and cold. It was also finally detected and falteringly admitted that water is a destructive

[5] Cons. Engr., Seattle, Wash.

[5a] Received by the Secretary July 25, 1941.

agent in the concrete mix itself, and that its proportionate quantity must be regulated carefully.

Engineers finally came to a realization that excessive mixing water was the cause of many troubles. Dams and other structures built with sloppy mixes were leaking, weathering, and eroding, whereas older dams of the "dry-tamped" period were not. With flawless logic it was reasoned that a marked return toward former practice would largely restore the former qualities. It also was observed that concrete mixtures with more cement in them resisted weathering and the penetration of water better than the leaner mixes. Methods of mixing so as to combine the aggregate into a somewhat unified body were studied also.

Out of the accumulated experiences has come a very marked improvement in modern dam construction that permits a central mass of concrete of reasonably low cement content, a water content as low as possible, and an aggregate gradation of maximum density. Having established these values for the particular project, thenceforth it is demanded to maintain them and secure uniformity in batching, mixing, placement, and curing. The final result comes from the minute attention given to the adopted formula—or to its misuse.

The tables in this paper, beginning with Table 1, constitute valuable data for the student of concrete behavior. Here are given the weights of seven different batch mixes, and the weights of cement, sand, and large aggregates of four different sizes, together with the pounds of water used. This is followed by records of compressive strength achieved by the various field mixes running up to one year. This provides a field for specific investigation for inquisitors through many years to come.

Table 2, comparing concrete to certain composition in the Norris Dam with concrete of different composition in the Hiwassee Dam, shows the persistence of study and forward look on the part of the Hiwassee constructer. Table 5 gives the temperatures prevailing when the various masses were deposited. Certainly, the care and watchfulness taken in securing and recording the facts given in these tables mark a class of construction personnel far ahead of the ordinary, and, through such careful work and intelligent study of results, concrete so made should enjoy the word "permanent." How much of the special work done under this \$210,621 contract is at this date specifically unusual it is difficult to affirm definitely.

The use of low lifts (approximately 5 ft) with long hours given for each lift to enable the concrete to adjust its chemical combination and return to quite nearly normal temperature, using a minimum of cooling pipes throughout, as against the common practice of 10-ft lifts with an abundance of cooling pipes, is a process that can be commented upon only with data furnished from structures of similar size, and this comparison the writer is not able to make.

It is true that the three casting lifts are charged with only \$38,621 of the total increased cost, and that at 6¢ per sq ft for the cost of cleaning up, and with no knowledge as to the price paid per hour, the 6¢ cost carries little information for a separate job. On such works as the writer has been engaged

upon, time of completion of the structure has been of great value. Here, with 5-ft casting lifts, five days (or one day per ft) is given for curing of concrete. This was done to avoid cumulative heat and resultant expansion. Since all this work was done on a cost-plus arrangement, and since the writer is ignorant as to the terms of the agreement, he cannot give any opinion as to the probable final cost compared with the cost that would have followed with 10-ft lifts and standard cooling pipes.

The report on diagonal keyways is very interesting and indicates that the usual belief that stresses occur normally at right angles to the face of a structure is reasonably correct.

The paper refers to tests made during the first few months of concreting to determine the effect of grinding in the mixers, which tests were used to correct the combined aggregate grading of the material going into the mixers, and to obtain a desirable grading in the fresh mixed concrete. This process is one which the writer does not quite comprehend. Was it to determine the number of revolutions to be made in the mixing drum, or was the rock aggregate so soft that the reduction made in the size in the mixers had to be accounted for? This is a refinement to which the writer is not accustomed.

Engineers who follow contractors' methods of "getting things done," rather than laboratory methods, will no doubt question the value or the worth of the $210,621 cost added to the possible lesser cost that might have been incurred with less attention to details.

One can only "guess at" or roughly estimate the probable cost of the dam, except by listing the quantities of materials used in its construction, but cannot place the probable cost at less than, say, $12,000,000. Now, an insurance cost of $210,621 placed upon a $12,000,000 structure, to insure longer life, is a very small sum. Personally, the writer feels that the need of refacing the face and spillway of this dam has been postponed by this care for such a number of years that it will make this sum a ridiculously small one to have been incurred in order to produce the added life.

It is understood that there is a committee of engineers in the employ of the federal government which is now examining every known dam in the United States, and preparing a report in which all salient points relating to construction will be set out. When that has been published, many theories will no doubt be dissolved, and a better light given by which to design; but the tables given in this paper will continue to be of value.

W. R. Waugh,[6] Assoc. M. Am. Soc. C. E. (by letter).[6a]—In recent years more and more study has been given to the problem of temperature cracking in mass concrete. An analysis of the causes surrounding the occurrence of a crack usually has suggested a possibility of eliminating the recurrence of a crack under similar conditions by controlling the circumstances that were known to have caused the original crack. A diligent record, made during construction, of the location and circumstances of occurrence of cracks, is

[6] Associate Materials Engr., TVA, Hiwassee Dam, N. C.

[6a] Received by the Secretary July 28, 1941.

invaluable as an aid in studying the causes, and suggesting means, of controlling cracking. At Norris Dam such a record was made as a part of a program of special-instruments investigation which included the measurement of concrete temperatures at selected points in the dam. Much of the experience gained from a study of these data, along with knowledge gained from the experience of other agencies, was translated into the special program adopted for Hiwassee Dam. The success of the program is attested by the small number of cracks that developed.

As indicated by the author, some cracks did occur and careful records were made of locations and circumstances of occurrence. These indicate the need for continued study of causes and improvements in the technique of prevention. Refinements in the construction program offer a means of eliminating circumstances under which cracking is likely to occur. The elimination of large differentials in height between adjacent blocks is one means of avoiding circumstances that are conducive to cracking. High contraction-joint faces exposed for long periods nearly always result in cracking; however, the construction program can usually be arranged economically to eliminate most of these high exposed faces. In some instances it is necessary to leave certain blocks low for stream diversion during construction. When such a situation arises, special measures can be taken to control the conditions of temperature in the adjacent blocks with the high exposed faces so that cracking will not occur. The mass was cooled artificially in a circumstance of this nature at Hiwassee Dam, with success.

In the closing paragraphs of the paper under the heading "Cost of Program: Reduced Cement Content"), the author has presented a summary in which he purports to show that an actual saving in total cost of the structure was achieved as a result of the program. In several respects the writer cannot agree with the author's method of computing what he calls "Additional Cost," and there also arises a question as to the validity of the credit claimed for a saving in cement.

Considering first the summary of additional cost and using the author's nomenclature and explanation under the heading "Additional Cost," the estimate in Table 9, Col. 3, is submitted as the writer's estimate of additional cost. Comparing these values with the author's summary (Col. 2, Table 9), it will be noted that three items (Nos. 1, 5, and 8, Table 9) have been omitted and one item (No. 7) changed. First, the cost of the scientific program (No. 1) has been omitted because this program was conducted to gain additional knowledge concerning stress, strain, contraction joint openings, foundation uplift pressures, and interior temperatures during construction and with the structure in service. It is true that some of the thermometer installations aided in the control of artificial cooling in one block, and this was a benefit to the program; but largely the scientific program did not benefit the crack prevention program directly and, therefore, should not be charged against it. Second, some small benefit, no doubt, was gained from the cooling of the aggregate (No. 5, Table 9) after the rinsing process was completed due to evaporation of free water. The rinsing process, however, had an entirely

different function, and the benefit which did accrue was small and purely incidental; and, therefore, this charge should not be made. Third, the charge for "curing and winter protection" (No. 8, Table 9) should be omitted because

TABLE 9.—COMPARATIVE COST ESTIMATES, CRACK PREVENTION PROGRAM, IN DOLLARS

No.	Description (1)	Author (2)	Writer (3)
1	Scientific program	29,000	....
2	Use of low-heat cement	58,115	58,115
3	Thin casting lifts	38,621	38,621
4	Low casting temperatures	41,820	41,820
5	Cooling the aggregate	3,059	....
6	Artifically cooling the concrete in place	11,064	11,064
7	Use of steel reinforcement	14,867	3,814
8	Curing and winter protection	14,075	....
9	Total additional cost of crack prevention	210,621	153,434
10	Reduction in cost for saving in cement	245,028	123,000
11	Surplus or saving	34,407	....
12	Increase in cost of project	....	30,434

these are normal functions of the concreting program that would have been conducted regardless of whether or not any special measures for controlling temperature cracking had been adopted. Fourth, as to the "use of steel reinforcement" (No. 7), only that which was used in the special mats intended for the purpose of preventing the extension of two cracks which had already occurred should be charged to the program, because the steel in the roof of the galleries would normally have been used in any event.

In proof that an actual reduction in cost to the structure resulted from the program, the author has taken a credit for cement saved. The cement content of 0.80 bbl per cu yd for interior (mass) concrete was a natural outcome of having selected a maximum allowable water-cement ratio which resulted in this cement requirement since for the desired consistency the water requirement was a rather definite quantity. In assuming what he calls "the predicted one barrel of cement" per cubic yard as a basis for the saving computed, the author has used a value that is subject to question since this prediction was made during the very early investigations and before a maximum allowable water-cement ratio was selected. The saving he has computed, therefore, is subject to question. The value of using the lowest possible cement content commensurate with desired properties of the hardened concrete both from the standpoint of pure economy and the reduction in tendency to crack is beyond question, but in this case the manner of computing the saving is questioned. At Norris Dam a cement content of 0.90 bbl per cu yd for the interior concrete was adopted. Using this value in computing the credit for cement saved would seem more logical. Thus the credit would be about half of that used by the author, or roughly $123,000 (No. 10, Table 9). Applying this credit against the suggested "Additional Cost" (No. 9), an estimated increase in cost to the structure of $30,434 would result.

It does not seem logical that a program so valuable to the finished structure could be conducted without cost. This would seem contrary to the old adage

that, "You can't eat your cake and have it too." Even if the program had cost the project all of the $153,434 proposed as the "Additional Cost" (which is probably not true) and if no credit was taken for cement saved (which would indeed be an extreme viewpoint), the value to the structure in reduced maintenance and increased life should be much more than the additional amount expended on the program. In the preliminary planning the improved quality of the structure was the consideration that prompted the adoption of the program. It was anticipated that the program would cost something, but it was fully appreciated that the improvement in quality of the structure would be worth far more than the additional cost incurred.

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DISCUSSIONS

DESIGN OF ACCELERATION AND DECELERATION LANES

Discussion

By Messrs. W. L. Waters, R. A. Moyer and Donald T. Davidson, and Stephen E. Butterfield

W. L. Waters,[14] M. Am. Soc. C. E. (by letter).[14a]—Although this paper is very interesting, it is somewhat academic and it is based on assumptions, so that the tables and formulas given may create an unwarranted feeling of confidence in a designer who uses them.

The paper may be criticized for neglecting the practical features of the problem as distinct from the academic ones. In any highway where high speed (say, 40 miles per hr) is permitted on the outside lane, and where the traffic is fairly dense (say, 1,000 cars per hour in that lane), the only safe procedure is for the incoming motorist to come to a full stop and await an opening in the traffic. From the converging acceleration lane good vision of the oncoming traffic is impossible, and any attempt to break into a line of oncoming high-speed traffic at this point is dangerous.

The author's method of designing the deceleration lane is very safe; but the entry of cars into high-speed traffic is always a danger point. For the latter, the most practical and safe plan seems to be to limit the speed in the outside lane, place conspicuous warning signs one-quarter mile ahead of any entrance, and make all incoming traffic come to a full stop.

R. A. Moyer,[15] Assoc. M. Am. Soc. C. E., and Donald T. Davidson,[16] Jun. Am. Soc. C. E. (by letter).[16a]—In the paper on the design of acceleration and deceleration lanes, Mr. Mitchell has focused attention on certain relatively new design details which highway engineers are slowly recognizing as worthy of consideration in the design of the modern highway. The planning of details

Note.—This paper by Adolphus Mitchell, Assoc. M. Am. Soc. C. E., was published in March, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1941, by Messrs. H. F. Holley, D. W. Loutzenheiser, Hawley S. Simpson, and Milton Harris; and June, 1941, by T. F. Hickerson, M. Am. Soc. C. E.

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[14a] Received by the Secretary June 20, 1941.

[15] Research Associate Prof., Highway Eng., Iowa State College, Ames, Iowa.

[16] Research Graduate Asst.-Highway Engr., Eng. Experiment Station, Iowa State College, Ames, Iowa.

[16a] Received by the Secretary July 14, 1941.

in the method of handling traffic at intersections and entrance and exit lanes will always be a challenge to the highway designer's ingenuity. There are also certain fundamental principles in design involving vehicle behavior with its limitations, and driver behavior with the driver's limitations, which are gradually being recognized by engineers and for which standards of performance are being established. As the results of field tests and related research yield new and more exact information on these subjects, it is to be expected that closer agreement in these standards will result.

In general, the writers are in agreement with the standards that Mr. Mitchell has recommended, and consider them quite reasonable on the basis of the results of tests conducted by the writers in 1941. In this discussion certain refinements in the design of the curves are offered, with comments in regard to certain other elements in the design proposed by the author which appear to be worthy of consideration.

The author computed the values for weaving distance in both the deceleration and acceleration lanes on the basis of a reverse curve composed of two symmetrical circular curves. He states (see heading "Deceleration Lanes"): "The assumption of an **S**-path of travel is a theoretical expedient entirely, and there is no doubt that drivers would find a spiral path more natural than a reverse circular path." He further states: "The formula derived on the basis of the **S**-path contains all the variables involved in the natural path." The writers have taken exception to the latter statement because an examination of the author's formulas, and the very fact that no provision is made for the natural spiral path in the formulas, indicates that the important variable C, the rate of change of acceleration, is not considered. The value of C reflects the driver's ability or skill in steering a car along a definite path from a tangent to a curve or from one curve to any other curve with a reasonable degree of accuracy in a given distance or length of time. The use of the reverse circular curve or of the straight-line transition from one lane to the other, as shown in Fig. 1, adds little to the economy of construction or to the appearance of the design. This type of construction is almost certain to provide too abrupt a change in direction and too short a length in the weaving distance because it does not provide a natural path that the driver can follow easily.

To clarify this important phase of design of exit and entrance lanes, values of weaving distance were computed on the basis of a natural path composed of four equal spiral curves transitional throughout using the same speeds and values of W as in Fig. 2. In addition to the mathematical analysis, a series of carefully controlled road tests were run to determine minimum lengths of weaving distances using a test car whose characteristics were well known from previous tests on highway curves.

In the mathematical analysis for the determination of the weaving distance, the standard spiral formulas, as recommended by the Iowa Engineering Experiment Station [4] and by the U. S. Public Roads Administration,[17] were used.

[4] "Skidding Characteristics of Automobile Tires on Roadway Surfaces, and Their Relation to Highway Safety," by Ralph A. Moyer, *Bulletin No. 120*, Iowa State College Eng. Experiment Station, Ames, Iowa. 1934.

[17] "Transition Curves for Highways," by Joseph Barnett, Bureau of Public Roads, U. S. Dept. of Agriculture, U. S. Government Printing Office, Washington, C. D., 1938, pp. 7–26.

The proposed curve layout is shown in Fig. 15, and from this it is evident that the weaving distance L may be computed as follows:

$$L = 2\left[T_s + T_s \cos \Delta\right] = 2\left[T_s\,(1 + \cos \Delta)\right] \ldots\ldots\ldots\ldots (16)$$

Since the tangent distance T_s and the length of each spiral L_s are approximately equal,

$$L = 2\left[L_s\,(1 + \cos \Delta)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

The standard formula for length of spiral L_s is

$$L_s = \frac{3.16\ V^3}{C\ r_c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

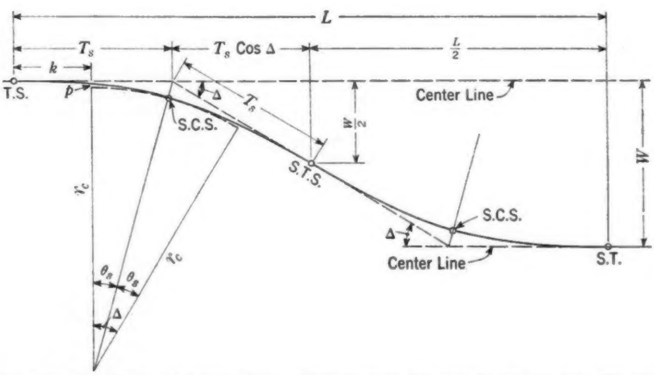

FIG. 15.—REVERSE CURVE COMPOSED OF FOUR EQUAL SPIRALS TRANSITIONAL THROUGHOUT

in which V is the speed in miles per hour, C is the rate of acceleration in feet per second[3], and r_c is the radius of the circular curve in feet. Substituting this value for L_s in Eq. 17,

$$L = 2\left[\frac{3.16\ V^3}{C\ r_c}\,(1 + \cos \Delta)\right] \ldots\ldots\ldots\ldots\ldots (19)$$

The standard curve formula for the determination of the radius r_c is

$$r_c = \frac{0.067\ V^2}{f + e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which f is the coefficient of friction and e is the superelevation in feet per foot.

Substituting the value for r_c in Eq. 19, and using a value of e equal to zero, will reduce the value of L to

$$L = 2\left[\frac{47.1 f\ V}{C}\,(1 + \cos \Delta)\right] \ldots\ldots\ldots\ldots\ldots (21)$$

For a spiral curve that is transitional throughout, the intersection angle Δ is equal to the sum of the two spiral angles θ_s. Since the standard formula for the spiral angle θ_s is

$$\theta_s = \frac{L_s\ D_c}{200} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which D_c is the degree of the circular curve, then

$$\Delta = \frac{L_s\, D_c}{100} \qquad (23)$$

The spacing W between the center line of the highway and of the exit or entrance lane can be computed for any value of L by using the formula

$$W = 2\, L_s \sin \Delta \qquad (24)$$

TABLE 2.—"WEAVING DISTANCE" VALUES FOR REVERSE CURVES COMPOSED OF FOUR EQUAL SPIRALS, TRANSITIONAL THROUGHOUT

Speed, in miles per hr	f	R_c, in ft	D_c, in degrees	$C = 2$			$C = 3$			$C = 4$		
				L_s, in ft	W, in ft	L, in ft	L_s, in ft	W, in ft	L, in ft	L_s, in ft	W, in ft	L, in ft
20	0.20	134.0	42.8	94.5	122.6	333.0	63.0	57.2	238.0	47.0	32.4	182.0
20	0.16	167.5	34.2	75.5	65.6	287.0	50.3	29.8	197.0	37.7	16.8	149.0
20	0.12	223.5	25.6	56.5	28.2	222.0	37.8	12.8	150.0	28.0	7.1	113.0
20	0.10	268.0	21.4	47.2	16.6	187.5	31.4	7.4	125.0	23.6	4.2	94.0
30	0.20	301.5	19.0	141.5	128.0	535.0	94.5	58.4	369.0	70.8	33.0	280.0
30	0.16	376.0	15.2	113.5	67.4	444.0	75.6	30.2	300.0	56.8	17.0	226.0
30	0.12	503.0	11.4	85.0	28.6	338.0	56.5	12.6	226.0	42.4	7.1	169.0
30	0.10	604.0	9.5	70.6	16.6	282.0	47.2	7.4	188.5	35.3	4.2	141.0
40	0.20	536.0	10.7	188.5	130.0	730.0	126.0	58.7	496.0	94.2	33.0	374.0
40	0.16	670.0	8.6	151.0	68.0	596.0	100.7	30.4	400.0	75.5	17.1	302.0
40	0.12	895.0	6.4	113.0	28.2	450.0	75.4	12.6	301.0	56.5	7.1	226.0
40	0.10	1,070.0	5.4	94.5	16.8	377.0	63.0	7.4	252.0	47.3	4.3	189.0
50	0.20	837.5	6.8	236.0	131.0	926.0	157.0	58.4	624.0	118.0	32.8	470.0
50	0.16	1,047.0	5.5	189.0	68.4	750.0	126.0	30.2	503.0	94.3	17.2	376.0
50	0.12	1,395.0	4.1	141.5	28.6	565.0	94.4	12.8	377.0	70.8	7.2	283.0
50	0.10	1,675.0	3.4	118.0	16.4	471.0	78.5	7.4	314.0	59.0	4.4	236.0
60	0.20	1,210.0	4.7	282.0	129.0	1,110.0	188.0	58.0	748.0	141.0	32.4	562.0
60	0.16	1,510.0	3.8	226.0	67.0	899.0	151.0	30.0	603.0	113.0	17.0	451.0
60	0.12	2,010.0	2.9	170.0	29.0	679.0	113.0	13.2	453.0	84.8	7.3	338.0
60	0.10	2,415.0	2.4	141.2	16.6	565.0	94.4	7.6	377.0	70.6	4.2	282.0

A summary of the computed values for weaving distances L and lane spacings W for speeds from 20 to 60 miles per hr for values of friction f and rates of change of acceleration C in the usual operating range is given in Table 2. The values of coefficient of friction used in these computations ranged from 0.10 to 0.20. A value of $f = 0.20$ is generally accepted as a reasonable design value for speeds of 20 miles per hr or less, and a value of 0.16 is the standard value for speeds from 30 to 60 miles per hr. For speeds greater than 60 miles per hr a value of $f = 0.10$ is preferred. The values of C can be treated in like manner—that is, use a value of $C = 4$ ft per sec³ for speeds of 20 miles per hr or less, $C = 3$ ft per sec³ from 30 to 60 miles per hr, and a value of $C = 2$ ft per sec³ for speeds greater than 60 miles per hr.

As a result of the mathematical analysis, it was apparent that for any given values of f and C only one value for length of weaving distance L and one value for spacing W was possible if four equal spiral curves, transitional throughout, were used. To determine the length L for any given value of W, changes in the design values of f or C are required. To obtain smaller values of W than those determined using the recommended design values for f and C, a reduction

in the value of f is the safest procedure. To obtain larger values of W than those determined using the recommended design values for f and C, a reduction in C is the safest procedure. It should be noted in Table 2 that, with $W = 16.8$ ft for a speed of 20 miles per hr, $f = 0.16$, and $C = 4$, the length L is 149.0 ft; whereas with very nearly the same distance $W = 16.6$ ft at the same speed, $f = 0.10$, and $C = 2$, the length L is 187.5 ft. This increase of 38.5 ft in the length of L is an increase of only 25% and indicates that the changes in weaving distance within the ordinary range in design values of f and C will not vary by more than from 25% to 30%.

On the basis of the foregoing analysis Fig. 16 for weaving distance was prepared using a constant value of $W = 12$ ft and values of a equal to 0, 3, 6, and

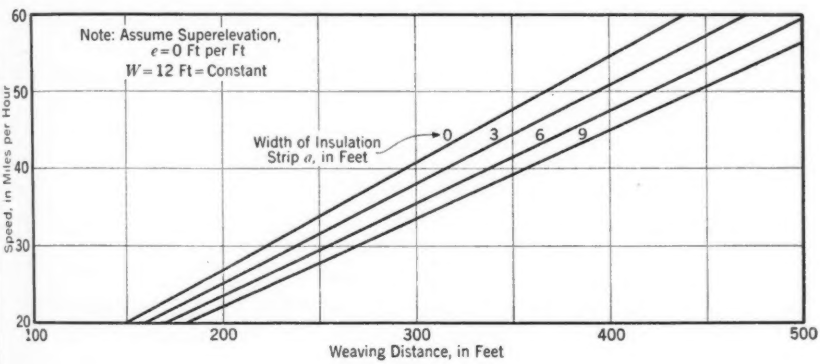

FIG. 16

9 ft for speeds ranging from 20 to 60 miles per hr. When comparing the values of L in Fig. 16 with similar values in Fig. 2, it will be noted that the latter are approximately 40% less than the former. This difference in the lengths of L is the logical difference between circular reverse curves of the same radius and spiral curves with radii varying from an infinite radius to that used by the author for the same conditions of speed, lane spacing, and values of f.

As a check on the mathematical analysis, road tests were run with a test car of known steering characteristics on curves. This car has been used on many tests on curves and, on the basis of the results of tests by the General Motors Corporation on many different makes and year models of cars,[18] may be considered as an average type of car in respect to steering and handling on curves. The car was equipped with a ball bank indicator that was used to aid the driver in holding to the maximum specified side friction and the maximum curvature at each speed. By holding the side friction to the maximum value of $f = 0.16$ recommended by the author, the minimum lengths in the weaving distance obtained in these tests checked rather closely with the values given in Fig. 2. Since the test values were the minimum lengths that could be obtained without exceeding the value of $f = 0.16$, it follows according to the mathematical

[18] "Marking Highway Curves with Safe Speed Indications," by R. A. Moyer and D. S. Berry, *Proceedings*, Highway Research Board, National Research Council, Vol. 20, 1940, pp. 399–428.

analysis for the determination of L using spiral curves that large values of C were developed in these tests. With relatively small intersection angles Δ, accuracy in the steering of the car was not a critical item, which indicates why values of C equal to 6 and 8 ft per sec^3 were possible. For design purposes values of C equal to 3 or not greater than 4 ft per sec^3 are to be preferred and this greatly simplifies the steering of the car from one lane to the other.

The design proposed by the author may be criticized on the basis that the lengths of the deceleration and acceleration lanes are unnecessarily long. The introduction of spirals as the natural path that the cars will follow will further increase these lengths and, where local conditions require it, some modification in the design standards would have to be made. Under these conditions a reduction in the cruising speed or the design speed at the entrance to the exit lanes would be an important consideration. Also, motor deceleration might be used only for speeds greater than 40 or 50 miles per hr for a fixed distance such as one half the weaving distance to be followed by higher rates of stopping by the use of brakes than are recommended by the author. A stopping rate of 8 ft per sec^2, which corresponds to a value of f equal to 0.25, has been established on the basis of extensive tests at Iowa State College, Ames, Iowa, as a desirable maximum comfortable rate of stopping and is the maximum rate recommended for design purposes in locations where frequent stopping is necessary. Such a procedure would reduce the total length of the deceleration lane by one third or one half and would still retain many of the desirable features referred to by the author as justifying the cost of construction of deceleration lanes.

In examining the design proposed by the author for the acceleration lane and the benefits to be derived from such a design, the writers are not convinced that the traffic will be able to use it as the author intends it should be used. The deceleration lane will work out as intended by the author because normally the driver is free to leave the highway at the designated exit point if he desires to do so. However, entering the highway presents an entirely different situation and where traffic is heavy enough to warrant the use of an acceleration lane, the opportunities to enter the highway freely without interference are likely to be available only for a limited percentage of the traffic entering the highway. This is a phase of traffic behavior that should be investigated to determine the extent to which traffic entering from an acceleration lane can do so freely with varying volumes of traffic on the highway.

Although an acceleration lane designed solely as such has the advantage in reducing delay both for the turning traffic and the through traffic on the highway, the writers contend that an important function of the acceleration lane should be to provide ample space in which the turning traffic can merge safely into the through traffic lane without requiring the turning traffic to stop or the through traffic to reduce speed seriously. To satisfy this requirement a greater length should be provided in which the traffic can merge by reducing the length of the insulation strip and by replacing the cinders or gravel braking section (which traffic would not use) by a paved section of the same length which the traffic would use. Full acceleration to the cruising speed is not necessary when entering the highway and a reduction in the length of the insulation strip to one

half the length shown in this section of the acceleration lane should be quite satisfactory, especially under conditions where a reduction in the total length of the acceleration lane recommended by the author is necessary. Observations of traffic for many years have revealed to the writers that traffic which is not legally required to stop by signs or similar devices will stop only as a last resort and for this reason it is questionable if there is any justification for a cinder or gravel braking section. An improvement here would be to cover this section with a distinctive type of low-cost paved surface that could be used either for braking or for merging the turning traffic with through traffic. Such a treatment would provide greater flexibility in the use of the acceleration lane and would reduce delays and traffic interference by an appreciable amount as compared with that possible if the design proposed by the author is used.

There is little doubt but that the proposals made by Mr. Mitchell are in the right direction. The trend in the control of traffic today is to provide a smooth continuous flow of traffic along highways and through intersections in such a way that delays, large speed differentials, and unnecessary stops are eliminated, and so that traffic can leave and enter through highways freely. The savings in the cost of vehicle operation and in time occasioned by the elimination of delays and traffic stops mount rapidly as the volume of traffic increases. Road tests conducted by the writers have revealed that when cars are stopped every other block the gasoline mileage is cut in half, the average speed is reduced by one third, and the tire wear is more than six times as great as when no stops are required. Furthermore, the design of highways to provide a smooth continuous flow of traffic will not only reduce operating costs and save time but, as the author has clearly stated, it should reduce highway accidents.

The author is to be congratulated for having made a careful analysis of the many details in vehicle and driver behavior which are certain to play a more prominent part in the design of highways in the future. It is hoped that his paper will stimulate research in field observations of traffic behavior and in the development of standards of design that will be adopted by highway departments.

STEPHEN E. BUTTERFIELD,[19] Esq. (by letter).[19a]—State traffic engineers, engineers dealing with the control of rural and high-speed traffic, and students of rural accident statistics are well aware of the large number of accidents that result directly from the conflict arising between slow-moving, decelerating, or accelerating vehicles and through-moving vehicles, traveling at a constant, high rate of speed. Of equal economic significance is the accompanying congestion, resulting in time losses and lower over-all speeds, which occurs at points of egress from or access to high-speed highways carrying substantial volumes of traffic. It is apparent that many highway designers have not been equally cognizant of the importance of providing a facility that permits a vehicle to diverge safely from, or merge with, the fast through-moving stream of traffic. In many of these cases where highway designers have attempted to provide accelerating and decelerating facilities,

[19] Traffic Engr., State Highway Dept., Hartford, Conn.

[19a] Received by the Secretary July 15, 1941.

it is apparent that insufficient thought has been given to their functional design or to the study of their component parts. The exit and entry ramps to the Merritt Parkway, in Connecticut, and to the Hutchinson River Parkway, in Westchester County, New York, are examples of these earlier and inadequate facilities.

In view of these facts and in view of the vast program of parkway, freeway, and dual highway construction upon which the United States is engaged, Mr. Mitchell's treatise is extremely timely. The author has clearly outlined the essential elements that should be considered in the design of accelerating and decelerating lanes. Although this subject has been studied previously in some detail,[20] Mr. Mitchell has now successfully presented a concise treatment of the component elements in the design of accelerating and decelerating lanes. These elements are treated in a logical sequence and in a strictly analytical manner. Also, the author has carefully assembled and interpreted, in the light of their value to the design of accelerating and decelerating lanes, the available research material applicable to this subject.

From a review of this paper, it should now be clear to most highway engineers that there is no longer need for making major assumptions or hypotheses in the designing of accelerating and decelerating lanes. Further research on some of the minor controversial matters relating to this subject doubtless will throw much-needed light on certain phases of the problem, such as to what extent motor deceleration should be considered and at what point motor deceleration gives way to brake deceleration.

Mr. Mitchell has presented a sound mathematical approach to the matter of weaving distance, a problem which formerly has been subject to the greatest diversity of opinion. An insulation strip, in the form of a true physical barrier, is of the greatest importance to insure the correct use of the decelerating lane and to insure that the decelerating process occurs entirely off from the through-moving lanes. This insulation strip should form a sufficient barrier so that no motorist will be tempted to ride over it in order to enter the exit ramp. Except for a short distance at its approach end, this insulation strip should not be a low mountable curb. The color scheme of this barrier area, and of the decelerating lane itself, should offer such contrast to the highway proper that the point of divergence will be clear to the approaching motorist.

In his presentation, the author has allowed the motor deceleration distance to overlap the weaving distance, thus permitting a part of the through-pavement area to be used for the act of deceleration. This means, in the example presented, that with a cruising speed of 50 miles per hr, a vehicle under motor deceleration will be traveling at approximately 41 miles per hr at the end of the weaving distance. This is the point at which a vehicle is finally entirely off from the through-moving lanes. Should not the deceleration distance be treated separately from, and introduced at the end of, the weaving distance? This would insure freedom from any possible conflict between the decelerating vehicle and a following through-moving vehicle. This would be consistent with the design of the accelerating lane in which no attempt at merging with

[20] "Accelerating and Decelerating Lanes," by Wortham W. Dibble, Harvard Bureau for Street Traffic Research, June, 1938; also "Accelerating and Decelerating Roadway Areas," by Stephen E. Butterfield, Yale Bureau for Street Traffic Research, June, 1939.

the through stream of traffic is made until the full cruising speed has been attained.

In the use of certain values as the cruising speed for the through-moving traffic, one should not lose sight of the fact that the existing cruising speed or the design cruising speed may not be the one that will be adhered to in the not too distant future. Lest the highway under construction become antiquated soon after its completion, it is wise to consider the cruising speed as that average speed to be expected on the projected highway at a time ten or fifteen years in the future.

Some highway designers are still reluctant to admit the necessity of providing accelerating lanes, even though they do admit the very real need of decelerating lanes. It is generally conceded that if either the decelerating lane or the accelerating lane must be sacrificed, it should be the accelerating lane in favor of the decelerating lane. The crux of the argument revolves around whether full security is to be provided to the following through-moving motorist at the point of exit, and yet no security provided to him against a suddenly entering vehicle at a point of entry. Since vehicles are likely to be grouped, even with a light volume of traffic on the through lanes, an entering vehicle may encounter merging conditions not drastically different from those encountered under high volume conditions.

The inability of the motorist of average intelligence and long driving experience to judge accurately his distance from, and the speed of, an approaching vehicle is well known. Because of his inability to judge the speed of an approaching vehicle and because of his normal impatience to delay, is it not far better to remove the possibility of a human error by providing a facility which largely removes the human element? With the provision of grade separations, freedom from access to abutting properties, decelerating lanes, and the other natural attributes of a freeway, a sense of security is afforded the motorist that does not actually exist unless merging lanes are provided which permit vehicles to enter the highway at the cruising speed. It follows naturally that the higher the speed of the through route the greater is the need of accelerating lanes and the greater is the inability of the motorist who is stopped at the entry to the expressway to judge the speed of the approaching vehicles.

So-called accelerating lanes in the form of extra pavement widths have been provided at the entry point to certain parkways located in Eastern United States. The failure of vehicles to use these extra pavement widths as accelerating lanes should not serve as a measure of their value; rather it should serve as an indictment of their inadequate design. When carefully designed, and in accordance with the general principles outlined by Mr. Mitchell, accelerating lanes will be found to be a highly functional and an urgently needed parkway facility.

This paper should be welcomed by all engineers and highway designers confronted with the problem of parkway and expressway design. Mr. Mitchell has so segregated the design elements that it should be a relatively simple matter to adapt the general method to the treatment of any individual problem or to substitute for the submitted data later data derived from further research on the individual elements.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

MISSOURI RIVER SLOPE AND SEDIMENT

Discussion

By Samuel Shulits, Assoc. M. Am. Soc. C. E.

Samuel Shulits,[9] Assoc. M. Am. Soc. C. E. (by letter).[9a]—Captain Whipple's paper merits attention as an analytical treatment of a problem in river regulation. Too frequently river engineers opine qualitatively on the behavior of rivers—one reason why the science of river engineering has moved forward so slowly.

The assumption of an average slope as the basis of the improvement of the Missouri River seems untenable. The word "average" implies a time interval that must affect the magnitude of the "average." It is not easy to accept the tenet of an average slope when the author states that the slope had been increasing for several decades prior to the improvement program and that there have not been enough data to predict the future trend. An average slope does not "jibe" with the recognized fact that the eventual Missouri River slope depends primarily on its sediment-carrying characteristics, which can scarcely be related to an average gradient. If the long view is taken, an average has no significance; and if a short range is taken, the application of the average to a specific problem seems precarious. The writer believes that there is no basis for the rule of thumb that a river has an average slope to which it must return regardless of the regulation measures adopted.

Although detailed studies have not been made for the paper, the effects of Fort Peck Reservoir and of irrigation projects are deemed to be of negligible future influence between Sioux City, Iowa, and Rulo, Nebr. The present state of knowledge is inadequate to permit such conclusions. Is it not possible that a very slow progressive degradation may occur below Fort Peck Reservoir due to a desilting of silt-laden flood flows in the reservoir? The Bureau of Reclamation is measuring, very carefully, the morphologic behavior of the Colorado River below Hoover Dam almost all the way to the Mexican border.

Of particular interest is the application to the Missouri River of some of the findings since 1930 in the field of bed-load formulas. Since these formulas

Note.—This paper by William Whipple, Jr., Assoc. M. Am. Soc. C. E., was published in March, 1941, *Proceedings*.

[9] Chf., Engr. Dept. Research Centers, U. S. Waterways Experiment Station, Vicksburg, Miss.

[9a] Received by the Secretary July 14, 1941.

are the keystone of the conclusions on the equilibrium slope of the Missouri River, it is incumbent to subject them to a rigorous critique.

The Schoklitsch bed-load formula (Eq. 2) is imputed to be based principally on flume experiments and it is asserted that its empirical form renders its application to natural streams questionable unless verification can be made on a large scale. Then the belief is expressed that the Straub formula (Eq. 3) is more applicable to the conditions in the Missouri River. There is little evidence, if any at all, for these contentions because: First, as far as the writer is aware, the Schoklitsch formula is the only one for which there is any large-scale verification and it has been confirmed for three rivers;[10] second, the Schoklitsch formula has a good rational basis, whereas the Straub formula is the result of substituting the Manning velocity formula into the du Boys bed-load formula, and the latter is predicated on an assumed manner of sand motion known to be untrue;[10] and third, both formulas found their claims and coefficients on flume investigations—the numerous tests of G. K. Gilbert (mainly) in the case of Professor Schoklitsch and the comparative few of F. Schaffernak in the case of Professor Straub. These three facts render untenable the author's assumption that the Straub formula is more representative of Missouri River conditions; besides there is no physical evidence to substantiate the claim for the Straub formula.

The surmise is incorrect that the Schoklitsch formula contains some inherent assumption of the relation of width to slope or discharge. The formula is based almost entirely on the Gilbert measurements, using the bed load and discharge per unit width of flume, as is the case with all such formulas. The difficulty here, not remedied by any bed-load formula known to the writer, is the fact observed in rivers[11] that movement occurs only over a part of the bed. There is no movement near the sides or banks. The width of the moving strip seems to vary directly with the discharge, but this is not incorporated into the Schoklitsch formula.[12]

The point is made that the Schoklitsch formula does not have more factors to vary for given conditions and yet it is assumed that G, Q, and u in the Straub formula must be the same for the improved or unimproved channel. This leaves the roughness coefficient, $\frac{1.486}{n}$, and the width as the only variables to reflect altered channel conditions. Since the improved and unimproved channels are stated to have equal roughness coefficients, the width remains the only variable factor, and it occurs in the Schoklitsch formula. It may be inferred, therefore, that the Schoklitsch formula is also applicable to the Missouri River problem under discussion. Actually, the magnitude of n, when the bed load is in motion, can be 20% to 35% over its value at the critical discharge of incipient movement,[13] a possibility neglected in this paper.

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[10] "The Schoklitsch Bed-Load Formula," by S. Shulits, *Engineering*, June 21 and 28, 1935, pp. 644–646, and 687.

[11] "Beobachtungen ueber Geschiebefuehrung," by S. Kurzmann, Munich, 1919; also "Untersuchung ueber den Schwebestoff und Geschiebefuehrung des Inn naechst Kirchbichl (Tirol)," by L. Muehlhofer, *Die Wasserwirtschaft*, 1933, p. 1.

[12] "Der Geschiebetrieb und die Geschiebefracht," by A. Schoklitsch, *Wasserkraft und Wasserwirtschaft*, No. 4, 1934, p. 37.

[13] "Beitraege zur Frage der Geschwindigkeitsformel und der Rauhigkeitszahlen fuer Stroeme, Kanaele und geschlossene Leitungen," by A. Strickler, *Mitteilungen des Amtes fuer Wasserwirtschaft*, No. 16, Bern, Switzerland, 1923.

The computation of the equilibrium slope is based on the valid assumption that the critical discharge is negligible. This changes the Straub formula to:

$$G = \frac{u}{c^{1.2}} S^{1.4} Q^{1.2} \quad (4)$$

If the Schoklitsch formula is treated in the same way,

$$G = \frac{86.7}{D^{0.5}} S^{1.5} Q \quad (5)$$

Current knowledge of solids transportation is still too meager to permit a positive decision for 1.4 instead of 1.5 for the exponent of S of for 1.2 instead of 1 for the exponent of Q. Consequently little can be said for or against the one equation that cannot be applied to the other—and any choice must be arbitrary to a considerable degree. The situation is similar to the choice exercised in the use of exponential velocity formulas, always a matter of personal familiarity with the formula and its coefficients. With regard to Eqs. 4 and 5, the selection simmers down to whether one prefers to choose a value of u (which is determined by the grain diameter) and estimate c (a roughness coefficient which, for some reason, is made to be $\frac{1.486}{n}$), or to select the effective grain diameter (D) in the Schoklitsch formula.

The resemblance between the two formulas becomes even greater upon closer analysis of the coefficient u. It can be shown from experimental data that:

$$u = \frac{K}{D^{0.75}} \quad (6)$$

in which K is a constant, equal to 111,000 if D is expressed in millimeters. Eq. 4 then becomes:

$$G = \frac{K}{c^{1.2}\, D^{0.75}} S^{1.4} Q^{1.2} \quad (7)$$

so that there is $D^{0.75}$ to compare with $D^{0.5}$; and the case is just as strong for the one as for the other. Since considerable latitude is possible in practice in the choice of c or n for rivers like the Missouri, close agreement should be possible between Eqs. 5 and 7 by using in the latter a value of the roughness coefficient well within the limits of accuracy of the coefficient. Thus it is not just to condemn the Schoklitsch formula for the Missouri River and to uphold the Straub formula as "probably more applicable."

The writer would be interested to know what value of u was used and how it was obtained in the section, "Capacity"; what c or n was employed; and then how the equilibrium slope of 0.76 ft per mile was computed.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ANALYSIS OF BUILDING FRAMES WITH SEMI-RIGID CONNECTIONS

Discussion

BY MESSRS. S. D. LASH, DEAN F. PETERSON, JR., AND R. W. STEWART

S. D. LASH,[8] ESQ. (by letter).[8a]—It is gratifying to find that attention is being directed toward problems connected with the design of beams in steel building frames. The fact that considerable economies are possible without change in construction procedure makes it the more surprising that this subject has been comparatively neglected by practising engineers.

The authors do not claim that the methods presented by them are suitable for the practical design of beams in steel building frames. However, since the methods of analysis are presented as a basis for simpler design procedures, it appears legitimate to consider the paper from the point of view of design rather than analysis.

The calculation of end moments in beams with semi-rigid connections differs from most other design calculations, inasmuch as it is necessary to rely upon the results of laboratory tests for the properties of the connections. Many investigators have assumed that the relation between applied moment and angular deformation for any semi-rigid connection is approximately linear, since this assumption is the obvious way of introducing modifications of regular design procedures. The laboratory investigations referred to by the authors have shown, however, that such an assumption is incorrect for most types of semi-rigid connections. The relation between moment and angular deformation is not linear, and the behavior of a connection on first loading differs considerably from its behavior on subsequent reloadings. In an experiment on a steel frame, if the loads are applied, removed, and reapplied, measurements of strains being taken on the reloading, these strains will depend upon the reloading curves for the connections, but the total strains will be those corresponding to the initial loading curves.

NOTE.—This paper by Bruce Johnston, Assoc. M. Am. Soc. C. E., and Edward H. Mount, Esq., was published in March, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1941, by Maurice P. van Buren, Assoc. M. Am. Soc. C. E.; and June, 1941, by Wayne W. Smith, Leonard P. Zick, Jr., Juniors, Am. Soc. C. E., and Conrad C. Wan, Esq.

[8] Acting Secretary, National Building Code, National Research Council of Canada, Ottawa, Canada.

[8a] Received by the Secretary June 9, 1941.

Since moment-angle curves for semi-rigid connections are not linear, it would appear to be necessary to introduce some form of "limit design" when determining allowable restraining moments. Thus, for example, if a factor of safety, or load factor, of n is desired, the allowable restraining moment at working load should be $\frac{1}{n}$ times the computed restraining moment at n times the working load.

It should also be pointed out that the "connection constant γ," referred to by the authors, depends not only upon the type of connection, but also, in the case of flange-angle connections, at least, upon the depth of the beam. The constant as determined experimentally cannot be used for beams of a depth other than that used in the original tests unless suitable corrections are made.

In order to make the foregoing points more specific, Fig. 16 has been prepared using information previously published by the Steel Structures Research

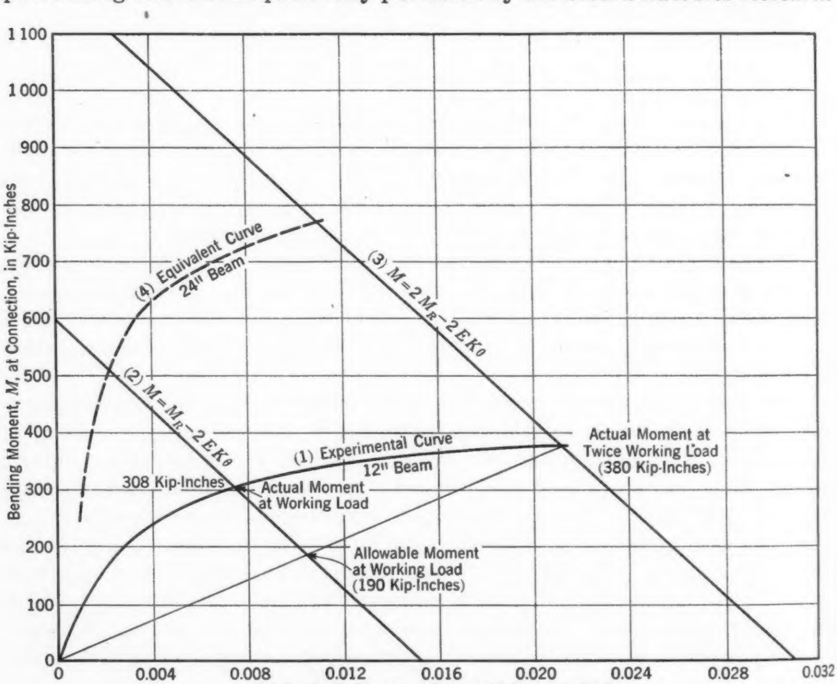

Fig. 16

Committee in Great Britain.[9] Curve (1) represents the moment-angle relationship for 4-in. by 4-in. by ½-in. flange-angle connections on a beam of 12-in. depth. Actually, this curve defines a lower limit obtained by combining the curves resulting from tests on seven riveted specimens submitted by different fabricators.

[9] Final Report, Steel Structures Research Committee, Dept. of Scientific and Industrial Research of Great Britain, H. M. Stationery Office, London, p. 282.

For simplicity it will be assumed that a beam is attached to completely rigid columns. In such a case for symmetrical loading, the end moment may be conveniently determined by solving the equations graphically:

$$M - f(\phi) \qquad (23)$$

representing the curve of the connection; and

$$M = M_R - 2\,E\,K\,\phi \qquad (24)$$

representing the slope-deflection equation.

In Fig. 16 two lines, (2) and (3), are drawn corresponding to slope-deflection equations for two different magnitudes of load applied to a 12-in. beam 20 ft long. Line (2) corresponds approximately to a working load and line (3) to a load twice as great. In the first instance the end moment is 308 kip-in. and in the second, 380 kip-in. Thus, for a load factor of two, the allowable moment at working load should be 190 as indicated by line (5) instead of 308 kip-in. This diagram will also serve to indicate that the working range of the curve for a connection is actually quite extensive. For case shown it will extend at working loads from angular deformations of about 0.002 radians up to 0.12 radians and, if overload is being considered, up to twice these values. For this reason it is desirable to take experimental readings over a somewhat greater range than has usually been done in the past.

Fig. 16 also shows, approximately, the effect of depth of beam upon the moment-angle relationship obtained. The moment-angle curve (1) shown is based on tests using 12-in. beams. If 24-in. beams had been used in these tests, it is probable that the curve would have been similar to that shown by curve (4). If it is desired to present the properties of connections without relation to depth of beam, a convenient way of doing so is to transform the ordinates and abscissas from M and ϕ to $\frac{M}{D}$ and $\phi\,D$, respectively. The ratio $\frac{M}{D}$ may be thought of as the pull on the connection and $\phi\,D$ as its linear deformation. The application of this method to web connections has not been investigated.

In frames having semi-rigid beam connections, the most reasonable approach to the problem of making allowance for width of members appears to be to assume that the columns are continuous members to which the beams are attached. After all, this is what actually occurs. It then appears logical to assume the effective length of a column as the length measured between the neutral axes of the beams, and the effective length of a beam as the length measured between the faces of the columns.

In the case of flange-angle connections particularly, this method leads to an overestimate of column moments, since the loads are transmitted to the columns by the connection angles as concentrated loads and not as moments at the neutral axes of the beams. No appreciable error will be introduced by assuming the maximum moment in the column to be the moment at the estimated level of the point of contraflexure of the flange angle connected to it. Although this correction may be worth making from the point of view of the design of the column, it is doubtful if it is worth making when considering the

distribution of moments throughout a framework, since an appreciable rotation of the end of the beam may result from local shear deformation of the column in the vicinity of the connection. Results published elsewhere[10] have shown that this deformation may increase the total rotation of the column at the level of the beam by 30% or 40%.

The National Research Council of Canada has recently published Part 3 (Engineering Requirements) of the National Building Code (a model code for the use of Canadian municipalities). In this Code an attempt has been made to give some weight to the considerations mentioned by the authors at the beginning of their paper. In particular, it is required that bending moments in columns other than those supporting a symmetrical arrangement of beams of approximately equal span shall be investigated, and the stresses resulting from them shall be provided for. With this mandatory requirement is a permissive requirement stating that "where a beam is restrained at either end due allowance may be made for such restraint." The Code does not restrict the designer to any particular method for estimating moments in columns or restraining moments in beams, but acceptable methods of doing these things are given as Appendixes. A description of the method given for computing allowable end moments was published in 1941.[11]

DEAN F. PETERSON, JR.,[12] JUN. AM. SOC. C. E. (by letter).[12a]—The methods presented by the authors have made an excellent rational start toward a better basic understanding of what actually occurs in a loaded frame in which the members are connected in the customary way. Expressing the laboratory-determined "stiffness" of the connection in terms of the "stiffness" of the connecting member "opens the door" for the application of the "moment-distribution" and "slope-deflection" methods. The writer believes, however, that the most direct and simple method for solving such an elastically complex structure as a building frame with semi-rigid connections is the "flexure factor" method.[13] Using this method it is unnecessary to solve either simultaneous equations or to make the rather involved adjustments of the "moment-distribution" method. Since no mention is made of the application of the "flexure factor" method, either in the text or in the references given, it may add to the completeness of the discussion to call attention to it.

To illustrate the application of this method, the writer has chosen the simple frame of Fig. 17(*a*). The "stiffness" of any member is the moment required at one end, with the other end freely supported, to produce an angle change of unity in the direction of the end tangents. In Fig. 17(*b*) the proportionate stiffnesses of the members are determined from the $\frac{M}{E\,I}$-diagrams.

[10] Final Report, Steel Structures Research Committee, Dept. of Scientific and Industrial Research of Great Britain, H. M. Stationery Office, London, p. 357.

[11] "The Design of Beams in Steel Frame Buildings," N. R. C. 992, National Research Council of Canada, 1941.

[12] Project Engr., Savage River Dam, Upper Potomac River Comm., Western Port, Md.

[12a] Received by the Secretary June 25, 1941.

[13] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521.

Fig. 17(*c*) is the traverse of the elastic curve due to an unbalanced moment of 100 at Point *B* and Fig. 17(*d*) is the resulting moment diagram. For a vertical load of 10 kips at a point 9 ft from Point *B* the "fixed-end" moment at Point *B*, by the "moment-area" method or by Eq. 13, is 13.86 kip-ft and $M_{BC} = M_{BA} = (0.8020)(13.86) = 11.15$ kip-ft. The writer was able to check this value by means of Eqs. 4.

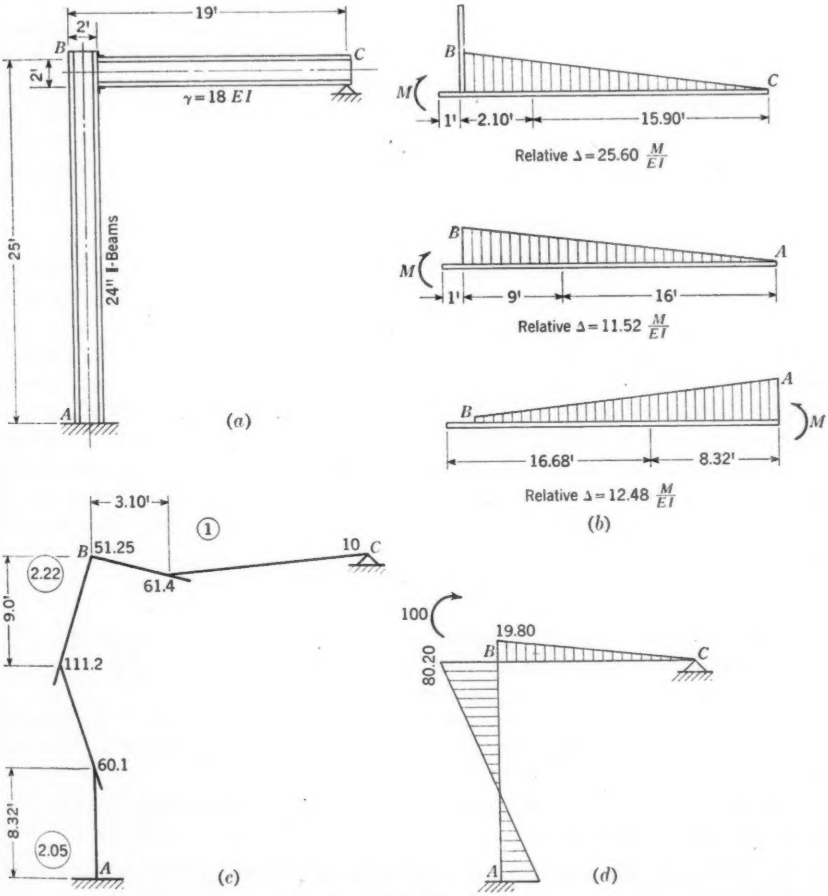

Fig. 17

The writer doubts that the effect of sidesway should be generally ignored. If sidesway were allowed in his example M_{BA} would be decreased from + 11.15 kip-ft to + 6.54 kip-ft, and M_{AB} would change from + 5.55 kip-ft to − 6.54 kip-ft. What would be true, however, of such buildings as powerhouses or low warehouses would probably not be so true of a multi-story office building.

The authors are to be complimented on an excellent research project. Perhaps the time will come when the designer will be able to predict to a

reasonable extent what the value of γ will be for a certain connection, or, better still, be able to design a connection for a predetermined value of α.

R. W. STEWART,[14] M. AM. SOC. C. E. (by letter).[14a]—Considerable effort has been made by the authors to overcome the complexities which result when the effect of semi-rigid connections is incorporated into the computation of the bending moments in a steel frame.

Each of the methods presented has defects which render its use difficult. This may account for the authors' statement that the methods of analysis are too complex for ordinary design use, but can be made expeditious by the use of charts and diagrams in connection with simpler design procedures. It may also account for the fact that the general case, which would occur if an unsymmetrical beam having variable moment of inertia were used between the riveted joints, was not included in the scope of the paper.

A specific list of the defects in the authors' solutions is as follows:

(1) The constants used in end-moment distribution, which are based on the moment required to produce unit rotation at one end of a beam when the other end is fixed, require long series of algebraic terms to express their values if a member is affected by asymmetry or other special conditions like the yielding joints treated by the authors. A glance at the right-hand column and then at the left column (Case I) of Table 1 will disclose how the expressions for moment-distribution constants expand when conditions other than the simplest are introduced. If these constants were further encumbered by unsymmetrical tapering members between the riveted joints, the computation of α and β would become complex, making the use of the formula very tedious and difficult.

(2) The slope deflection equations ($4a$ and $4b$) are based on $K = \frac{I}{l}$-values which are applicable only to members of uniform section between joints. The introduction of unsymmetrical tapering members would add substantially to the difficulty of computing the necessary constants.

(3) The use of the authors' methods independently of a set of charts pertaining thereto would require having at hand, for reference, formulas that are too complicated to remember.

(4) The authors' methods offer no easy facility for detecting errors which may occur in computations.

The following solution of Fig. 11 entirely eliminates defect 3 attributed to the authors' methods, as any one familiar with this method of attack would not use any references to solve any of the authors' probelms, except a steel handbook giving the moments of inertia of rolled beams. It greatly alleviates defect 4 in that an automatic check involving a single setting of a slide rule will verify a large portion of the solution. It alleviates to a considerable degree defects 1 and 2 by using "illustrated" member constants which are simpler than moment-distribution constants and which can be altered without difficulty to include the general case of unsymmetrical members between joints.

[14] Engr. of Bridge and Structural Design, City of Los Angeles, Los Angeles, Calif.

[14a] Received by the Secretary June 30, 1941.

Fig. 18(*a*) represents an $\frac{M}{I}$-diagram due to a moment at one end of a beam shown in Fig. 11, the other end being hinged. It requires no explanation; Fig. 18(*b*) is the appurtenant graph of the tangents to the elastic curves in the beam, known as a traverse of the elastic curves, in which Δ represents the

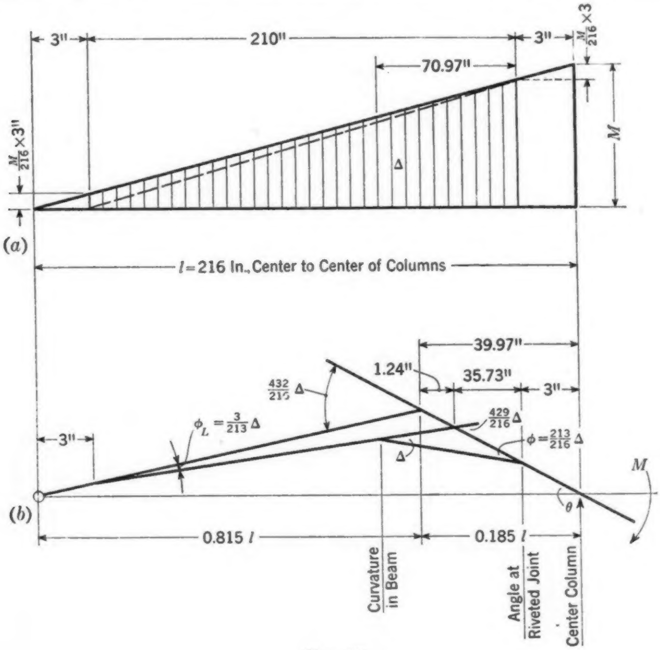

FIG. 18

curvature between joints. To evaluate ϕ, which represents the angle of yield at a joint, Eqs. 2 and 5 yield

$$\alpha = 2\,E\,K\,\frac{\phi}{M} \qquad (25)$$

For the flexure of a beam of constant section, hinged at one end, $\Delta = \frac{M}{2\,E\,K}$. For $\alpha = 1$ (which is the condition for Fig. 11), by solving each of these equations for M, it is found that $\phi = \Delta$ for the case in which Δ is the curvature represented by a triangular $\frac{M}{I}$-diagram. For Fig. 18(*b*), ϕ will equal the area of the $\frac{M}{I}$-diagram under the dotted line in Fig. 18(*a*). With this explanation and the traverse principle[15] that in traverse triangles the lengths of the sides (considered as their horizontal projections since altitudes are negligible) are proportional to the opposite angles, Fig. 18(*b*) can be readily sketched and evaluated. The difference between Figs. 18(*a*) and 18(*b*) is of interest. Fig. 18(*a*) is a moment-area diagram that cannot show joint yield angles or joint rotation angles. Fig. 18(*b*) is a traverse diagram that shows all the elements

[15] *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521.

of the flexure. The moment-distribution constants in Table 1 are necessarily based on flexure due to the existence of moments at both ends of a beam which is why they become more complicated for unsymmetrical conditions than Fig. 18(*b*) which deals with a moment at one end only. The stiffness factor com-

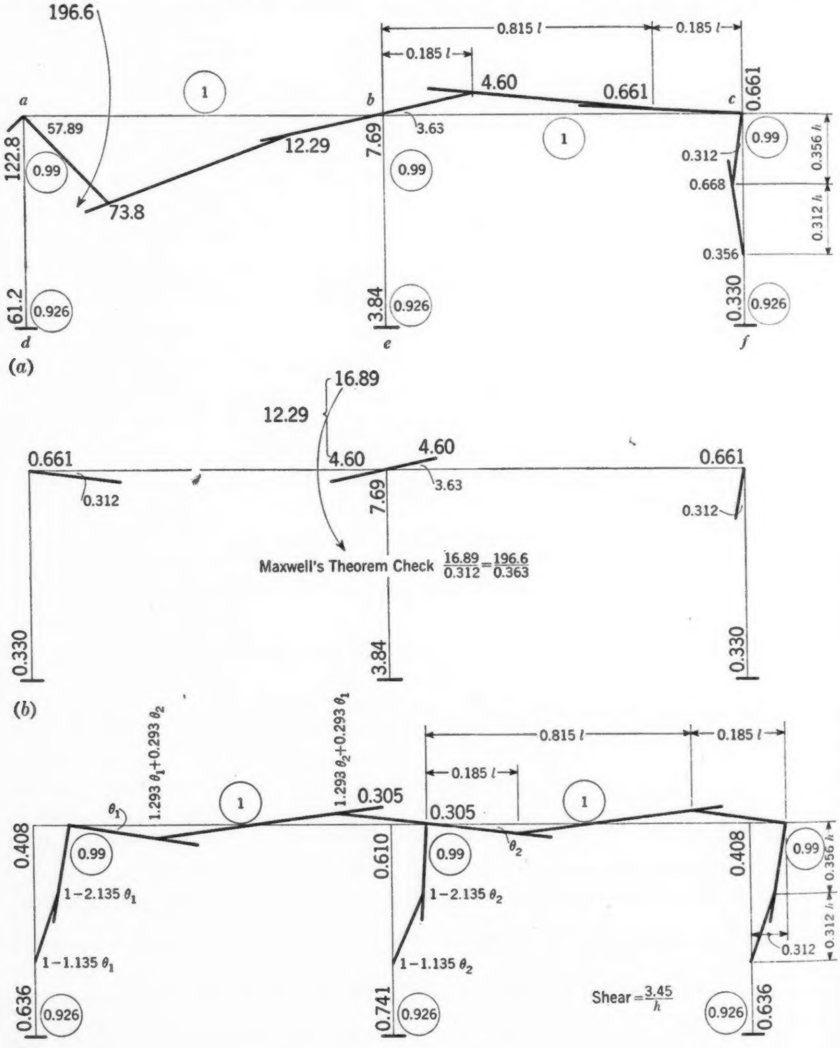

FIG. 19

puted from Fig. 18(*b*) is first computed as the moment necessary to give the value of unity to the angle shown as $\frac{432}{216}\Delta$, which represents the over-all curvature in the beam. Top and bottom stiffnesses for the columns are computed by a procedure that is similar but is simplified because there are no ϕ-angles

in the columns. After all stiffness factors are computed, they are multiplied by a factor which will reduce the deck stiffness to unity in order to simplify subsequent arithmetical work.

The computation for Fig. 19(*a*) is a much faster procedure than the uninitiated would suspect. When the traverse computation arrives at the top of a column both column moments are obtained by one setting of the slide rule as they bear the same ratio to the moments written on column *cf* that the angle at the top of the column bears to the angle at the top of column *cf*.

Fig. 19(*b*) is nearly all copied from Fig. 19(*a*). The computations for Fig. 19(*b*) take less than one minute. As soon as they are complete a single slide-rule setting will disclose whether either Fig. 19(*a*) or 19(*b*) contains an error.

There is more than one way to compute side-sway moments using the traverse method. Fig. 19(*c*) represents an elastic curve traverse converted into an amplified form of slope deflection which can be applied to the two-story structures treated in the authors' paper. The key to this method is to express the values of the angles which govern the stiffnesses in terms of the θ-angles and then proceed as in slope deflection.

Table 7 shows the moments computed from Fig. 2. The check with the authors' moments is exact.

TABLE 7.—MOMENTS COMPUTED FROM FIG. 19

Description	*d*	*a*	*ba*	*be*	*e*	*bc*	*c*	*f*	Shear
$\frac{179.38}{196.6}$ × Fig. 19(*a*)..........	55.8	112.0	11.2	7.0	3.5	4.2	0.6	0.3	$158.2 \div h$
$\frac{179.38}{16.89}$ × Fig. 19(*b*)..........	3.5	7.0	130.5	81.6	40.7	48.9	7.0	3.5	$101.3 \div h$
Moments without side sway...	59.3	119.0	141.7	88.6	44.2	53.1	7.6	3.8	$56.9 \div h$ (check)
Johnston-Mount..............	59.3	119.0	141.7	88.6	44.2	53.1	7.6	3.8	....
Side-Sway Correction:									
$\frac{56.9}{3.45}$ × Fig. 19(*c*)..........	10.5	6.7	5.1	10.2	12.2	5.1	6.7	10.5	$56.9 \div h$
Moments with side sway....	48.8	112.3	146.8	98.8	56.4	48.0	0.9	6.7	Zero
Johnston-Mount...........	48.8	112.3	146.8	98.8	56.4	48.0	0.9	6.7	....

To summarize: The methods demonstrated by the authors to establish a basis for the design of structures with semi-rigid joints required the use of derived constants and derived formulas which are inherently very complex.

The use of the basic constants of flexure[16] in an orderly manner assisted by a pictorial graph of the flexure will eliminate the necessity of making reference either to special slope deflection equations or to formula for constants in an independent solution of problems of this class. ("Independent" solution means one not dependent on a set of charts and diagrams prepared by some one else. In court testimony in a building failure case the solution should be independent.)

It will also eliminate most of the anxiety regarding the possibility of errors.

Corrections for *Transactions:* In March, 1941, *Proceedings:* Add "$+ V'_A + b_{AB}$" to the right side of Eq. 13, page 415; on page 417, line 17, change "α to γ"; on page 418, to the caption of Fig. 8, add "(Load Spacing: 4 Ft 5 In., 5 Ft 2 In., and 4 Ft 5 In., Equals 14 Ft 0 In.)"; see also corrections in May, 1941, *Proceedings*, page 949.

[16] *Transactions*, Am. Soc. C. E., Vol. 102 (1936), pp. 41–44.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE SUSPENSION BRIDGE TOWER CANTILEVER PROBLEM

Discussion

BY FRANCIS P. WITMER, M. AM. SOC. C. E.

FRANCIS P. WITMER,[7] M. AM. SOC. C. E.[7a]—A complete method for analyzing the stresses in a suspension bridge tower under a variety of imposed conditions is presented in this paper. It is particularly interesting that the author has based his development upon one of the almost forgotten members[3] of the series that were classics in their field fifty years ago. It is unfortunate that the more detailed study upon which this paper was based[2] could not have been presented in full. The derivation of Eq. 4 would then have been clear to one who happens not to be acquainted with Professor Robinson's valuable little treatise published in 1882, and who is not in a position readily to consult the complete paper in the Engineering Societies Library in New York City.

For the usual case of a tower without eccentricity at the top and with a variable moment of inertia, the writer has had satisfactory results by using Eq. 19 of Case VIII, first determining an equivalent moment of inertia I by equating Δ in Eq. 20 of Case IX to the deflection for a cantilever with constant moment of inertia, $\Delta = \frac{F\,h^3}{3\,E\,I}$. The value of I, so found, may then be used in the quantity $n = \sqrt{\frac{R}{E\,I}}$ without serious error. The author has applied a similar procedure to several numerical cases with results that seem to justify this conclusion, particularly when it is remembered that the fiber stress in the tower due to bending is generally small in comparison with the direct unit stress. Incidentally, Eq. 19 may be readily derived by the writer's method, which is cited in the paper.[4]

NOTE.—This paper by Blair Birdsall, Assoc. M. Am. Soc. C. E., was published in April, 1941, *Proceedings*.

[7] Director, Civ. Eng., Univ. of Pennsylvania, Philadelphia, Pa.

[7a] Received by the Secretary July 12, 1941.

[3] "Strength of Wrought Iron Bridge Members," by S. W. Robinson, Van Nostrand Science Series, No. 60, D. Van Nostrand, Publisher.

[2] The more detailed study from which this paper was prepared has been placed on file for reference at Engineering Societies Library, 33 West 39th Street, New York, N. Y.

[4] "Basic Formulas for Combined Flexure and Direct Stress," by Francis P. Witmer, *Civil Engineering*, December, 1937, p. 855.

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DISCUSSIONS

CONSUMPTIVE USE OF WATER FOR AGRICULTURE

Discussion

BY MESSRS. RHODES E. RULE, EDGAR E. FOSTER, CHARLES H. LEE, HARRY F. BLANEY, AND J. L. BURKHOLDER

RHODES E. RULE,[44] ASSOC. M. AM. SOC. C. E. (by letter).[44a]—For many years it has been known to irrigation engineers who have investigated the subject that a close correlation exists between maximum temperatures and consumptive use. The authors' paper is refreshingly practical in its application of this relationship. Too often the results of extensive research are submitted in involved and inconclusive treatises beyond the grasp of the average engineer who has neither the time nor equipment to attempt to translate them into terms of useful application.

Determination of valley consumptive use lends itself to several methods of approach. By pursuing these courses separately and then reconciling the results as the final step it is possible to arrive at a conclusion with a probable error not larger than required by the uses to which the data may be put. The authors have added another avenue of attack which is simple and apparently effective, subject to certain limitations.

The following discussion relates to specific details taken in the order of their presentation in the paper and concludes with data for two Southern California basins which, as will be shown, do not accord with the empirical relationship developed by the authors.

Definitions.—Omission of the term "nonrecoverable deep percolation loss" from the definition of valley consumptive use clears up an inconsistency. This term should never have been introduced by the Committee on Duty of Water[3] since, as the authors correctly state, it "is a form of outflow independent of the factors influencing transpiration and evaporation."

NOTE.—This paper by Robert L. Lowry, Jr., M. Am. Soc. C. E., and Arthur F. Johnson, Assoc. M. Am. Soc. C. E., was published in April, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1941, by E. B. Debler, M. Am. Soc. C. E.

[44] Engr., Hydrographic Div., Met. Water Dist. of Southern California, Banning, Calif.

[44a] Received by the Secretary June 6, 1941.

[3] "Consumptive Use of Water in Irrigation," Progress Report of the Duty of Water Committee of the Irrigation Div., *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 1349–1377.

A confusion of terms exists here. "Deep penetration," "deep percolation," or simply "penetration" and "percolation" have been used variously to designate that part of the moisture reaching the earth's surface which passes downward to below the root zone. As such it is not lost to available water supply and is certainly not consumptive use. Under ordinary conditions it ultimately reaches the permanent water table and becomes available for recovery by pumping or for use where it reappears at the surface as effluent seepage. In all but a few exceptional cases this availability for use is within the valley where the deep penetration originally occurred and there is no net loss of supply to the valley. On the other hand, if "nonrecoverable deep percolation" is taken to mean ground water which escapes the valley through deep subterranean channels or simply as underflow at the lower end of the area it should be included in "valley outflow" even though its estimation be a mere guess. Certainly if its quantity is large in a particular case it becomes an important item for investigation and estimate. If carelessly lumped in with consumptive use the latter would be too high and the results misleading.

Lest the foregoing be thought to be mere quibbling over terms of no particular consequence it should be realized that, in spite of its basic simplicity, knowledge of consumptive use has been hampered by lack of agreement and confusion over definitions and, perhaps, a misunderstanding of the physics and geology of subsurface waters. Involving nothing more complicated mathematically than simple arithmetic the inventory of valley-water supplies is one subject concerning which there should be little disagreement as to methods and definitions.

Field-Plot Experiments.—In the discussion of plot experiments the authors have failed to recognize the valuable and determinative work done by H. F. Blaney, M. Am. Soc. C. E., F. J. Veihmeyer, Colin A. Taylor, Assoc. M. Am. Soc. C. E., and others. The thought seems to persist that plot experiments have failed to account for deep percolation and surface losses. This was the conclusion of the Committee on Duty of Water,[3] and was unquestionably true of the earlier investigations; but there are now at hand the results of much definitive work in which these losses have been measured or can be calculated from the experimental data. Some of these data are reported in papers referred to by the authors.[9, 10] The principal deficiency is in variety of crops and climatic conditions. Much of the available information relates to tree crops in California.

By proper use of the field-plot method consumptive use can be measured under actual field conditions with remarkably small error. Essential to success is careful selection of a location having uniform soil and representative crop conditions. Having such a location the soil moisture is determined by sampling before and after measured applications of irrigation or rain water. Sampling is extended to below the root zone. Surface runoff can be trapped and measured or kept to a negligible quantity by suitable control. Evapora-

[9] "Rainfall Penetration and Consumptive Use of Water in Santa Ana River Valley and Coastal Plain," by H. F. Blaney, and Colin A. Taylor and A. A. Young, Assoc. Members, Am. Soc. C. E., *Bulletin No. 33*, California Div. of Water Resources, 1930.

[10] "Regional Planning—Part VI, The Rio Grande Joint Investigation in the Upper Rio Grande Basin in Colorado, New Mexico, and Texas, 1936–1937, Part III, Water Utilization," by H. F. Blaney and others, National Resources Committee, 1938, pp. 295–427.

tion and transpiration are readily computed from the decrease in measured soil moisture between applications of water. Plotted foot by foot, the consumptive use follows smooth curves which permit ready detection and evaluation of deep percolation since any excess of applied water over that accounted for by these curves, plus runoff, may be ascribed with certainty to deep percolation. By careful application of water to suit the crop demands, deep percolation may be practically eliminated. The only qualifying conditions are that the water table must be beyond the reach of plant roots, the soil should be uniform, and the crop representative.

There is no surer method for the determination of consumptive use by individual crops or measurement of evaporation from bare soils than by the use of field plots if suitable locations are available and the work carefully done. It is admittedly expensive but the results justify the cost.

Integration Method.—The authors state that the integration method used by Mr. Blaney depends upon tank experiments. This is only partly true. Long-continued and successful use of the field-plot method provides the background for estimating many of the unit values in the integration method as applied by Mr. Blaney and his associates. Tank experiments become necessary where a high water table exists and the field-plot method cannot be applied.

In the last analysis the integration method relies on the judgment of the person applying it. If this judgment is to be good it must be based on a thorough knowledge of the experimental field including field-plot, tank experiments and all other sources of information.

Quantitative values of annual consumptive use by individual crops are known to cover a wide range. Under fixed climatic conditions consumptive use varies directly with total leaf surface and, with annual crops, the length of the actual growing period. Double cropping results essentially in a doubled consumptive use on an equal area. The data of Fig. 2 must be recognized as being based on a variety of vegetative types and farming practices which, when averaged, produce a high correlation between consumptive use and effective heat as defined by the authors. The same degree of correlation would not exist as between different crops. Grape vines consume far less water per acre than mature citrus trees or alfalfa. This is reasonable from a consideration of the relative leaf surfaces. A valley in which vineyards predominate doubtless would produce a point which would fall well below the curve shown. It follows that successful use of the authors' method in estimating valley consumptive use must be predicated upon a variety of conditions which, in the aggregate, produce "average" results. Specialty crops require special attention and analysis.

Knowledge of consumptive use will be served best by concerting research effort toward accurate determination of use by the more common crops under true field conditions and the relation of the resulting experimental data by some such common denominator as effective heat in order that reasonable estimates may be made for regions where experiments have not been undertaken. It is probable that comparison on the basis of relative pan evaporation would be more satisfactory but temperature records are available nearly everywhere and the effective heat method has a correspondingly wider field of utility.

All this discussion sums up to the writer's opinion that the integration method, based on reliable experimental data and checked by an inflow-outflow computation, provides the best method for estimating existing consumptive use.

The effective heat method appears to offer real promise in two ways:

(1) As a basis for comparison in estimating unit values of consumptive use for regions where experimental data are lacking (these would then be most useful of application in the integration method); and

(2) In estimating average consumptive use for proposed projects where temperature records alone are available and where a preliminary value will suffice.

Effective Heat.—Selection of 32° F as a base for computation of effective heat is subject to question and further analysis. A rigorous study of the subject belongs in the province of pure science and involves many variables. It is doubtful if a better solution could be obtained than by further investigation along empirical lines.

A glance at Fig. 1 suffices to show that, for the particular data from which this block diagram was prepared, mean maximum temperatures above, say, 52° F would produce a better monthly correlation with consumptive use and, roughly, each 10° above 52° would result in monthly consumptive use of 0.1 ft in depth.

The authors state that comparative studies were made on other bases before 32° was selected. Nevertheless, lacking definite evidence to the contrary, the writer inclines to the belief that a somewhat higher base would be more satisfactory.

Length of Growing Season.—The authors' method of calculating the length of growing season is tedious and the results do not appear to warrant the work involved. Further, since it is arbitrary, it fails to take account of farming practice which varies from place to place. Where full information regarding such practice is available or can be readily obtained in the field it offers a much better basis for computation.

Failure of the method is most clearly shown in Southern California for which no data have been given in the paper. For all practical purposes the use of twice-repeated five-day moving averages results in a 365 day growing season for the coastal plain and interior valleys of Southern California. It is obvious that some other criterion must be adopted for this location.

Careful attention to cultural practices will provide the basis for estimating an average growing season for any given area.

In many parts of California annual row crops are planted during March and April and require from 90 to 120 days to mature. After the spring crop is harvested some of the acreage is replanted. On the average, taking double-cropping into consideration, the "growing season" for row crops probably does not exceed 150 days. Following harvest some farmers practice weed control by cultivation; others permit weeds to flourish and later plow them under for green manure. Clearly variations in consumptive use by such crops are dictated more by individual cultural practice than by any arbitrarily defined growing season.

Similarly, tree crops will have widely varying uses depending upon practice with respect to cover crops now being used to an increasing extent by California farmers. Deciduous trees without a cover crop will have a growing season of perhaps 250 days, whereas a cover crop would result in appreciable consumption during every month of the year.

In other words the length of the season during which plants will grow is not controlling unless there are growing plants to utilize the available heat during this season.

The authors have based their studies on the assumption of an adequate water supply. Perhaps this should be further qualified by the assumption of an "adequate plant supply." As pointed out this is not always true, particularly where the frost-free period is long. It is concluded that cultural practice is more important than an inflexible definition of growing season based on minimum temperatures.

Annual Variations in Consumptive Use.—Annual variations in calculated consumptive use by the inflow-outflow method are usually such as to discourage reliance on one-year computations. To the causes mentioned by the authors should be added the impossibility of evaluating, accurately, water in transit to the water table. Where the preceding year has been exceptionally wet and the depth to the water table is large (in Southern California it may be several hundred feet) the quantity of water in transit may be sufficient to result in gross errors in calculated use during a single year. Experiments by the writer demonstrate that a number of years may elapse before influent seepage at the valley margin reaches the main ground-water body by lateral percolation. Vertical seepage to the water table is not usually so delayed but may require a year or more in older alluvium.

As distinguished from these apparent but not true variations there also must be considered the normal variation resulting from rainfall differences. The authors have corrected for dry years by adding a quantity considered adequate to insure "normal consumptive use." Presumably the object was to arrive at a set of figures of consumptive use based on a full evaporation opportunity. The difficulty here lies in the fact that heavy rainfall encourages, and scanty rainfall discourages, all types of vegetative growth, including a variety of weeds and grasses which are heavy consumers of water. This variation in use is independent of effective heat. The net effect is the same as if the equivalent area were increased during wet years and decreased during dry years. "Normal consumptive use" is not a constant but an average. If an addition is made for dry years a subtraction would be equally in order for wet years.

Data for San Jacinto Valley, California.—Studies[45] by Mr. Blaney, A. A. Young, Assoc. M. Am. Soc. C. E., and Paul A. Ewing, reported in 1941, provide the basis for the following analysis of an interior valley in Southern California.

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[45] "Utilization of the Waters of Beaumont Plains and San Jacinto Basin, California," Progress Report, U. S. Dept. of Agriculture, SCS, 1941.

The San Jacinto Basin comprises 162,100 acres, of which (1939) 31,500 are irrigated, 95,500 dry farmed, 28,200 in native vegetation, and 6,900 miscellaneous (water surfaces, towns, etc.).

Irrigated crops of the basin consist of some 10,000 acres of irrigated trees, both citrus and deciduous; 6,000 acres of alfalfa; 12,000 acres of row crops; and 2,500 acres of miscellaneous crops including grain; 1,000 acres were classed as prepared land—idle.

Consumptive use for an 18-yr period, 1922 to 1939, was computed at 198,500 acre-ft by the inflow-outflow method, and for 1938–1939 was estimated as 204,200 acre-ft by the integration method. Consumptive use on irrigated lands approximates 2.3 ft.

The writer has estimated the equivalent area at 90,000 acres with a consumptive use of 2.25 ft.

Mean maximum temperatures are not published for any station in San Jacinto Basin but records are available for two nearby stations having similar climatic conditions and less than a degree difference in mean temperature.

Applying the authors' moving average scheme for determination of the growing season it was found that for all practical purposes the entire year would be so classified. Since it was apparent that this was not in accord with the facts a different method was adopted.

Based on cultural practice as observed in the field the "growing season" ranges from a minimum of about 150 days for row crops to a maximum of 300 days for trees. By an approximate system of weighting according to acreage a mean growing season of 214 days was estimated. In general this covers the period from April through October and coincides with the irrigation season for this location.

Computed average effective heat during the 18-yr period, for the average year and for a 214-day season (April 1 to October 31), was as follows:

Length of season (days)	Day-degrees
365	17,700
214	11,900

Plotted in Fig. 2, on the basis of 11,900 day-degrees and a consumptive use of 2.25 ft, it is seen that the point falls below the defined curve. In terms of consumptive use it falls 0.4 ft short of the amount indicated by the curve, even after adjusting the growing season as explained herein.

Three apparent reasons come to mind to explain the discrepancy. First is the difficulty and expense of water development from which has grown a frugal system of irrigation practices. Much of the supply is pumped by individual farmers or sold on a unit basis, both conducive of careful husbanding of irrigation water. Second, the average crop grown may consume less water than the average in the valleys studied by the authors. Third, throughout most of the basin the water table lies well below the root zone, thus limiting consumptive use to water applied to the surface, either artificially or naturally.

Data for San Gabriel Valley, California.—Mr. Conkling made extensive investigations in the San Gabriel Valley,[46] in the intermediate climatic zone of the South Coastal region of Southern California. Of 133,000 acres in the basin, 85,000 were irrigated in 1926; 33,000 were fallow, and 15,000 classed as waste. The equivalent area, as estimated by the writer, was 103,000 acres. Consumptive use from inflow-outflow computations from 1923–1924 to 1926–1927 averaged 188,000 acre-ft. By integration the use was estimated to be 195,000 acre-ft or 1.9 ft in depth on the equivalent area.

Temperature records are published for the City of Pasadena, Calif., at the westerly edge of the basin. The mean effective heat for three seasons, 1924–1926, was 10,600 day-degrees for a growing season from April 1 to October 31.

Plotted in Fig. 2 the point for San Gabriel Valley falls below the curve, consumptive use being 0.6 ft less than would be inferred from the effective heat. As in the San Jacinto Valley this is in spite of a materially shorter growing season than would have resulted from using the authors' method of determination.

Conclusions.—The situation in Southern California is such that modifications of the proposed method must be made before it will yield satisfactory results. As has been shown the growing season must be ascertained by reference to agricultural practice and field observation of the growth habits of the local plant life.

Also, the water supply situation must be considered. Shortage of supply is the rule if viewed in relation to regions such as have been studied by the authors. This does not necessarily imply an actual shortage of water required to produce commercial crops. More nearly the truth is that scarcity and high cost of water have promoted a careful conservation of all available water so that a portion of what would be normal consumptive use in other parts of the country has been salvaged by cultural methods aimed toward the highest possible duty of water.

It is possible, also, that crop types prevailing in Southern California are such as to have a lower average use than in the regions represented by the curve in the paper.

Finally, and perhaps most important of all, is the situation regarding evaporation opportunity by reason of the ground-water level. Apparent from a reading of the area descriptions in the Appendix is the fact that the study areas chosen by the authors are nearly all regions of high ground water. This is emphasized by the repeated references to artificial drains. Under such conditions the evaporation opportunity is high regardless of whether land is in crop or not. This being the case there exists a near maximum utilization of available heat in evapo-transpiration processes. There is little opportunity to conserve moisture during the period when lands are not in crop. On the contrary, an attempt at such conservation might well result in concentrating alkali near the ground surface to the detriment of the lands.

Contrasted with these conditions an entirely different situation exists where depth to ground water is considerably in excess of the depth of the root zone.

[46] "San Gabriel Investigation—Analysis and Conclusions," by Harold Conkling, Report, Div. of Water Rights, Dept. of Public Works, State of California, 1929.

Here transpiration may be practically stopped during the period between crops, and evaporation is limited to losses following precipitation. Available water supply thus may be utilized more efficiently. Identical crops may be grown with a materially lower consumptive use without affecting yields or depriving plants of water which they are capable of using beneficially.

In this connection it should not go without mention that because of high ground water under cropped lands an enormous quantity of water is unnecessarily consumed each year while on the same stream systems there exist severe downstream shortages. From the comparisons given herein this excess consumption may be in the order of 0.5 ft in depth each year. This constitutes a potential water supply of significant quantity in many regions. Available supply is never exhausted until the water table is kept at sufficient depth below the surface to preclude such losses. In many cases drainage by pumping or substitution of a pumped supply for a portion of the gravity supply would result in benefits such as alkali and weed control which may compensate in large measure for increased costs.

From these general considerations, substantiated in a measure by the data cited herein, it is suggested that in the application of the authors' method to a particular location careful consideration must be given and proper correction made for differences in consumptive use which may arise from: (*a*) Irrigation practice, (*b*) crop types, and (*c*) depth to ground water.

In closing the writer wishes to express again his appreciation of the value and practicability of the authors' work. If most of this discussion has been critical let it be added that it is intended to be constructive. Mere confirmatory discussion is usually little more than sterile.

EDGAR E. FOSTER,[47] ASSOC. M. AM. SOC. C. E.[47a]—The consumptive use of water for agricultural purposes provides one of the crucial problems of irrigation engineering, and the authors are to be commended for searching for some logical basis for solution. It is evident that they have made a careful study of the subject.

Since water used in irrigation is consumed chiefly in evaporation and transpiration, it may be reasonably expected that there should be some relationship between heat and the consumptive use of water; yet the graph, Fig. 2(*a*), shows such a remarkably close correlation (in contrast with the authors' statement that a rather wide variation exists in plotting individual years as shown by Fig. 2(*b*)) that it appeared to the writer that the data should be tested for correlation by the methods of statistics.

The coefficients of correlation between the water used, in acre-feet, and the effective heat, in day-degrees, were calculated for ten stations with records ranging from thirteen to sixteen years, the data for which are given in the Appendix. These coefficients were computed by the formula,[48]

$$r = \frac{X\,Y}{\sqrt{\Sigma X^2 \, \Sigma Y^2}} \quad \text{(1)}$$

[47] Head, Flood Control Section, U. S. Engr. Office, Omaha, Nebr.

[47a] Received by the Secretary June 20, 1941.

[48] "Handbook of Mathematical Statistics," by H. L. Reitz, Houghton Mifflin Co., 1924.

in which: r = the correlation coefficient; X = the deviation from the mean water used for the individual stations; and Y = the deviation from the mean effective heat for the individual stations.

TABLE 7.—CORRELATION COEFFICIENT FOR WAGON WHEEL GAP

Year	U_r, in acre-ft	H, in day-degrees	X	Y	$X\,Y$	X^2	Y^2
1912	1.05	3,990	−0.25	10	− 2.5	0.0625	100
1913	1.30	3,920	0	− 60	0	0	3,600
1914	1.27	4,200	−0.03	220	− 6.6	0.0009	48,400
1915	1.35	3,600	0.05	−380	− 19.0	0.0025	144,400
1916	1.48	3,710	0.18	−270	− 48.6	0.0324	72,900
1917	0.91	3,930	−0.39	− 50	19.5	0.1521	2,500
1918	1.65	4,410	0.35	430	150.5	0.1225	184,900
1919	1.43	4,060	0.13	80	10.4	0.0169	6,400
1920	1.14	3,910	−0.16	− 70	11.2	0.0256	4,900
1921	1.13	4,060	−0.17	80	− 13.6	0.0289	6,400
1922	1.54	4,520	0.24	540	129.6	0.0576	291,600
1923	1.43	3,960	0.13	− 20	− 2.6	0.0169	400
1924	1.02	3,860	−0.28	−120	33.6	0.0784	14,400
1925	1.43	3,540	0.13	−440	− 57.2	0.0169	193,600
Means	1.30	3,980	....	....	....	....	....
Totals	....	....	....	....	+204.7	0.6141	974,500

The method of computation is given in Table 7, which contains the entire work of the calculation of the correlation coefficient for the valley of Wagon Wheel Gap. Substituting the appropriate values from this table in Eq. 1, $r = +\,0.265$, with a probable error, e, of $e = \dfrac{0.6745\,[1 - (0.265)^2]}{\sqrt{14}} = 0.168$.

The characters, U_r and H, have the same significance as in Table 3, Valley (10). It will be noted that the corrected values of the consumptive water are used as given by the authors. In general, the corrected data would give better correlation than the uncorrected values, but because the object is to determine how much water could be used, rather than how much had been used, the corrected values are justifiable.

TABLE 8.—SUMMARY OF THE CORRELATION COEFFICIENT r AND THE PROBABLE ERROR e

Name of stream	MEANS		CORRELATION COEFFICIENTS	
	U_r	H	r	e
Wagon Wheel Gap...	1.30	3,980	+0.265	±0.168
Mesilla............	2.83	12,370	+0.085	0.186
Black River........	1.85	6,950	+0.266	0.174
Mad River..........	2.15	8,760	−0.212	0.179
Skunk River........	2.25	9,340	−0.073	0.168
Sangamon River....	2.43	10,270	−0.254	0.158
Tallapoosa River....	2.75	11,900	−0.089	0.186
White River........	2.58	11,260	+0.010	0.180
Trinity River.......	2.82	14,460	−0.150	0.176
Cypress Creek......	2.99	14,500	+0.282	±0.172
Total............	....	....	+0.120	....
Mean............	....	....	+0.012	....

The computed correlation coefficients of the ten basins, together with their probable errors, are given in Table 8.

As is shown in the theory of statistics, the value of the correlation coefficients varies from − 1.0 for an inverse mathematical relationship, through values approximating 0.0 for purely chance, to 1.0 for a direct mathematical relationship. From Table 8 it will be seen that the correlation coefficients between consumptive water used and the effective heat as defined and calculated by

the authors vary from − 0.254 to + 0.282 with a mean of 0.012. There are five positive values and five negative values. The probable errors of the coefficients, being dependent upon the number of observations as well as upon the magnitude of the deviations, are large (in some cases several times as large as the coefficients themselves), but in no case will the combined coefficient and probable error be great enough to show significant correlation. The low values of the coefficients, the diversity of the signs, and the relatively large probable errors show conclusively that there is no correlation between consumptive water and the effective heat for the individual years.

Nine of the ten valleys in Table 8 are not in regions where irrigation is practised. From this it may be inferred reasonably that the correlation is not caused by inaccuracies in estimating the area equivalent in rate of use to the cropped area, or changes in crop distribution or crop damage, but may be attributed to the lack of correlation in the various hydrological data.

The correlation coefficient was not computed for the averages of the effective heat and water in Table 1 because the data are not entirely comparable on account of the variable number of years of record. A high degree of correlation is plainly evident between the average water used and the effective heat as shown by Fig. 2(*a*), and a computed coefficient would probably have little added significance.

There now arises the question of what is the reason for the high correlation between the means as shown by Fig. 2(*a*), whereas there is no correlation between the water used and the effective heat for the individual years. A part of the improved correlation is probably obtained by averaging the inaccuracies noted by the authors. These sources of error are: Inaccurate records of discharge; inaccurate estimates of ground water; inadequate precipitation and temperature records; variability in the climatic elements; inaccurate estimates of the water used; and changes in crop distribution and reduction in the water demand because of crop damage. Since all these factors operate each year to a varying degree, it is likely that the variations caused by these factors offset each other to a considerable extent. However, there will very likely still be deviations that will be reduced further by averaging a series of data of successive years.

Nevertheless, in addition to the valid averaging of deviations, a kind of spurious correlation is obtained in the averages of water used and effective heat, which is the result of dividing the individual yearly values by the same factor—in this case, the number of years of record. This false or exaggerated type of correlation is pointed out by various writers of mathematical statistics.[49,50] Improvement in the correlation between two variables is caused by operating on the concurrent data in a series of observed values with a common divisor. It is not possible to estimate how much apparent improvement in the correlation may be due to this cause but it may be considerable. The uncertainty is enough to vitiate the results observed in Fig. 2.

In view of the lack of correlation for individual years, it is evident that the method proposed could not be used to estimate the year-to-year require-

[49] "Mathematical Analysis of Statistics," by C. H. Forsyth, John Wiley & Sons, Inc., 1924.

[50] "Frequency Curves and Correlation," by W. P. Elderton, Cambridge University Press, 1938.

ment of consumptive water with any degree of certainty since, for any given value of annual heat, it would be a matter of chance whether or not the computed depth of water would be sufficient for that year. The use of Fig. 2(*a*) would provide a somewhat more reliable value for the average annual consumptive water but, because of the spurious correlation obtained by the use of averages as data, the reliability of the value would not be great.

It is hoped that the authors will continue their search for means of estimating the quantity of consumptive water used in agriculture. A method utilizing more than one climatic element would probably be more successful. However, a high degree of correlation cannot be expected, although a coefficient should be higher than 0.50.

Charles H. Lee,[51] M. Am. Soc. C. E.[51a]—This important subject has been presented in an analytical manner from limited, although widely distributed, data, and the authors have developed a simple method of computing consumptive use of water that is very promising. From this as a base, they point out that it is an easy step to determine water requirements for crops and from thence to farm delivery and diversion requirements, all of which are essential data for the design of an irrigation project. They are to be commended for the broad view they have taken of the problem and their recognition that many of the earlier studies were of limited scope or objective, had differing concepts of the meaning of consumptive use, and hence have little value in a generalized study. An example of this fact was the frequent use of stream flow depletion as a measure of valley consumptive use. In 1930 the writer pointed this out in commenting upon the Report of the Duty of Water Committee of the Irrigation Division on consumptive use of water in irrigation.[52] At that time he suggested the adoption of the climatic instead of the crop year and the topographic or geologic unit of area instead of the project area. It is noted that the authors have proceeded in accord with these ideas.

The authors state that their basic data of consumptive use have been obtained from stabilized irrigation areas in which general crops are grown and which have adequate water supply and full project development. An added condition has been absence of nonrecoverable loss by deep percolation. These conditions should be considered and, if necessary, adjusted in applying their results to other areas.

The authors have developed a master curve for consumptive use (Fig. 2), well supported by plotted points, which they state (see heading "Valley Areas Studied") "represent practically the entire range of growing conditions and types of agriculture found in the United States." This statement is believed to be rather broad and to overlook entirely the Southwest, the Pacific Coast and other areas where growing conditions and types of irrigated agriculture differ widely from those in the cold mountain valleys that make up most of the irrigated areas selected for study. From the standpoint of production and economic importance, the omitted areas are of greater importance than are

[51] Cons. Engr., San Francisco, Calif.

[51a] Received by the Secretary June 23, 1941.

[52] Discussion by Charles H. Lee of "Consumptive Use of Water in Irrigation," Progress Report of the Duty of Water Committee of the Irrigation Div., *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1385.

those for which representative valleys have been included. The master curve, therefore, should be accepted conditionally and should be considered as applicable only to the Rocky Mountain and humid sections of the United States until proved to have wider scope.

TABLE 9.—PRECIPITATION AND CROP DATA FOR THE VALLEY AREAS STUDIED

No.	Valley (see Table 1)	Precipitation (In.)		Length of growing season (days)	Vegetation		
		Average annual	Growing season		Cropped	Pastured	Wooded
	(a) Irrigated Valleys						
1	New Fork	10.2	4.3	120	Meadow, hay, and pasture		
2	Michigan and Illinois	10	5	....	Grass, hay, and willows		
3	Southwest Area, San Luis	7 to 8	5	....	Hay, grain, and potatoes		
4	West Tule Lake	8.5	0+	140	Alfalfa, cereals, and potatoes		
5	Garland Division of Shoshone Project	....	....	....	Alfalfa, grain, sugar beets, and potatoes		
6	North Platte	17	13	165	Alfalfa, sugar beets, and cereals		
7	Mason Creek and Boise	10	Little	....	Alfalfa, clover, hay, and grain		
8	Uncompahgre	9	4.5	145	Alfalfa, cereals, sugar beets, and onions		
9	Mesilla	8	5.3	240	Cotton, alfalfa, and native vegetation		
	(b) Non-Irrigated Watersheds						
10	Wagon Wheel Gap "*A*"	21	....	....	....	....	100[d]
11	Black River	31	31 −	164	62	30	28
12	Mad River	38	21	180	33	38	29
13	Skunk River	32	21	205	68	26	6
14	Sangamon River	36	36 −	220	78	18	4
15	North Fork of White River	42	26	245	25	28	47
16	Green River	45	23 +	245	40	40	20
17	Tallapoosa River	52	....[a]	....	42	15	41
18	East Fork of Trinity River	....	21.6	215	75	16	9
19	Cypress Creek	43.9	31	280	50	20	30
20	San Jacinto River	....	....[a]	....	33[b]	38[c]	29[e]

[a] Adequate for crop requirements. [b] Cultivated. [c] Prairie. [d] Forested. [e] Pine forest.

In order to define more clearly the limitations of the master curve, Table 9 has been compiled from data in the Appendix, and gives both average annual precipitation and that during the growing season, the length of growing season, and vegetation. It appears that, in most of the selected valleys, 50% or more of the precipitation falls within the growing season. In contrast to this, in the Southwest, and particularly on the Pacific Coast, practically no precipitation falls within the growing season.

It is further apparent that the principal irrigated crops are limited to alfalfa, native hay, grain, potatoes, and cotton, omitting entirely all the extensive orchard, vineyard, field, and vegetable crops raised in the Southwest and on the Pacific Coast, as well as rice, lettuce, cantaloupes, celery, artichokes, beans, seeds, and many other specialty crops of which large acreages are irrigated. With respect to the length of the growing season, the only irrigated area considered by the authors exceeding 165 days is Mesilla Valley, whereas

in most of the irrigated sections of the Southwest and Pacific Coast the growing season exceeds 225 days and in some favored localities 300 days. In addition to differences in crops in the two regions, there are also vast differences in the methods of handling water and in the economics practised in its distribution and application. Some of these differences may well have sufficient influence upon consumptive use to overshadow that of effective heat.

The writer regards the element of precipitation as one most likely to cause confusion and error in the use of the master curve. The mean consumptive-use values that support the curve have apparently been built up with the inclusion of total annual precipitation (see heading "Method of Study: Measurement of Use of Water"). This varies for the selected valleys from 7 in. to 10 in. (with one instance of 17 in.) and 50% or more falls within the growing season (see Table 9). Considering irrigated areas throughout the West, precipitation varies from 3 in. to more than 20 in. and on the Pacific slope occurs mostly during the winter season. Precipitation is ordinarily disposed of by: (1) Interception; (2) immediate evaporation from the soil; (3) runoff; (4) temporary absorption as soil moisture, returning to the atmosphere later as soil evaporation and transpiration; and (5) deep percolation passing beyond the reach of plant roots and ultimately becoming ground water. The relative distribution of total precipitation among these fractional elements varies greatly, depending upon the season of the year, the frequency of storms, the intensity of precipitation, the type of vegetation, the slope of the ground surface, the character and condition of the soil, etc. With precipitation corresponding to that in the valleys selected by the authors, the deduction of 7 in. to 10 in. in compiling water requirements from the master curve would probably not involve appreciable error; but an attempt to apply this assumption in valleys where the precipitation exceeds 10 in., and occurs during the winter, is likely to cause a very serious error. For this reason, the writer suggests that the master curve be supported by consumptive-use values that include only soil moisture at the beginning of the growing season (or precipitation during the growing season, whichever is more appropriate), rather than total precipitation, thus omitting the features that are foreign to consumptive use. Established methods are now available for measuring available soil moisture within the root zone at the beginning of the irrigation season so that this quantity is no longer indeterminate.[53] Its use in areas with little or no precipitation during the growing season, and the use of precipitation during the growing season in other areas, for the purpose of determining consumptive use, is far more logical than the use of total precipitation. Conversely, when computing water requirement from the master curve, either soil moisture at the beginning of the growing season or precipitation during it should be deducted.

The writer is in agreement with the authors that with present improvements in method and equipment many of the uncertainties of tank data have been eliminated. Careful consideration must still be given to the matter of exposure, however, to insure dependable results from tank observations.

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[53] "Rainfall Penetration and Consumptive Use of Water in Santa Ana River Valley and Coastal Plain," by H. F. Blaney, Colin A. Taylor, and A. A. Young, *Bulletin No. 33*, California Div. of Water Resources, 1930, Pt. II.

The authors refer to the close correlation between transpiration and meteorological phenomena, listing especially evaporation from water surface, air temperatures, solar radiation, and wet-bulb depression readings. The evaporation from water surface probably represents the integrated effect of all the other factors mentioned. As an illustration of this fact, the writer prepared a graph on which were plotted the transpiration ratio and evaporation from water surface for alfalfa during the growing season as measured in tanks by Messrs. Briggs and Shantz at Akron, Colo., from 1911 to 1917,[54] and by A. C. Dillman at Newell, S. Dak., from 1912 to 1918, and at Mandan, N. Dak., from 1919 to 1922.[55] The observations were made in sunken land pans, 8 ft

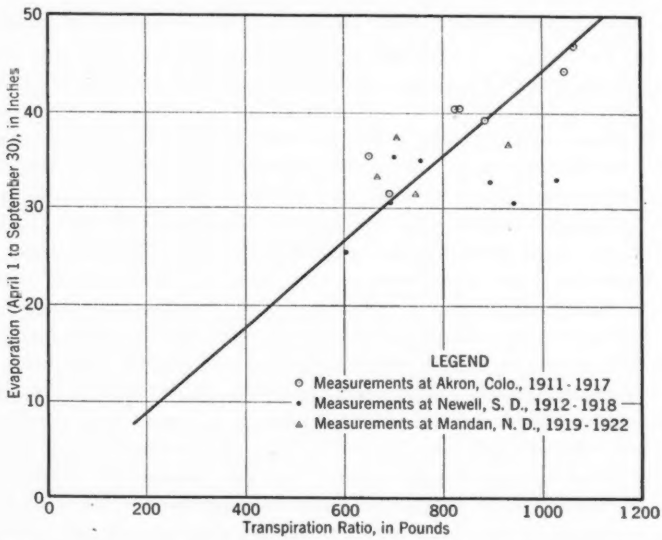

FIG. 3.—RELATION OF EVAPORATION FROM RESERVOIR SURFACE AND TRANSPIRATION RATIO FOR ALFALFA

in diameter, reduced to reservoir surface by the use of a coefficient of 0.94. Although it shows a considerable spread among plotted points, Fig. 3 indicates a definite trend in the form of a straight-line relation.

The authors state (see heading "Influence of Meteorological Factors") that "Although solar radiation gives one of the best correlations with transpiration and evaporation, growing-season temperatures more nearly parallel the cycle of plant growth." The latter statement is not confirmed by the work of R. F. Goudey, M. Am. Soc. C. E., at Los Angeles, Calif. He found that the growth of algae in reservoirs follows the ultraviolet component of solar radiation more closely than it does temperature.[56] The writer believes that there is much yet to be discovered through correlation of evaporation and transpiration with the components of solar energy, and especially with the ultraviolet radiation.

[54] "The Water Requirement of Plants at Akron, Colorado," by H. L. Shantz and L. N. Piemeisel, *Journal of Agricultural Research*, Vol. 34, 1927, Table 27, p. 1144.

[55] "The Water Requirements of Certain Crop Plants and Weeds in Northern Great Plains," by A. C. Dillman, *loc. cit.*, Vol. 42, 1931, pp. 187–238.

[56] "Sun Ray Counts Save Sulphate," by R. F. Goudey, *Engineering News-Record*, April 11, 1940, p. 95.

An especially interesting feature of the authors' paper is the correlation of consumptive-use and watershed losses. Water loss from large land areas in the United States has been the subject of engineering study by two specialized groups—irrigation engineers working in the plains and valleys of the semi-arid and arid West, and water supply engineers working in the humid East and the mountain watersheds of the West. The former have adopted the term "consumptive use of water" to represent the losses in irrigated agriculture by transpiration and soil evaporation. To represent the loss from watersheds by transpiration, soil evaporation, and interception, the latter have used various terms, such as evaporation, evaporation from land areas, evapo-transpiration, total loss, water losses, fly-off, and total evaporation, of which the latter has received much recent use.[57] (The term "total evaporation" has been adopted by the writer as the most expressive short term in current use.)

Although the factors of interception and soil evaporation are of greater importance in total evaporation than in consumptive use, yet otherwise the two quantities (total evaporation and consumptive use) have similarity in that losses under both normally occur with conditions of ample water supply. This fact gives validity to the comparison of water losses from adequately watered irrigated valleys and humid watersheds as attempted by the authors.

The selection of humid watersheds made by the authors is not duplicated in the literature and suggests that they have not followed the line of recent technical papers, beginning with W. G. Hoyt, M. Am. Soc. C. E., in 1936,[58] Joseph Kittredge, Jr., in 1938,[59] and Gordon R. Williams, Assoc. M. Am. Soc. C. E., and others in 1940.[60] These studies have had as their aim the determination of total evaporation from watershed areas by the method of deducting runoff from precipitation. The results of these studies, however, are generally in line with those of the authors. Of especial interest is the correlation made by Mr. Williams of mean annual water loss with mean annual temperature for twenty-eight watersheds in which mean annual precipitation is in excess of 20 in. This relation can be expressed by a straight line, although the departure of individual points from the line is greater than found by the authors for consumptive use and effective heat (see Fig. 2). By eliminating consideration of temperature during the non-growing season, the authors have evidently devised a method of correlation that adheres more closely to the temperature conditions directly related to plant activity and transpiration.

The writer believes that the authors' method will bear further study and has a field of application that will ultimately be useful in water supply studies as well as in irrigation planning.

HARRY F. BLANEY,[61] M. AM. SOC. C. E.[61a]—The treatise on consumptive use of water for irrigated agriculture is excellent, and the authors have de-

[57] "Hydrology of the Physics of the Earth," National Research Council (see chapter on "Transpiration and Total Evaporation," by Charles H. Lee)—publication pending.

[58] "Studies of Relations of Rainfall and Run-off in the United States," by W. G. Hoyt and others, *Water Supply Paper No. 772*, U. S. Geological Survey, 1936.

[59] "The Magnitude and Regional Distribution of Water Losses Influenced by Vegetation," by Joseph Kittredge, Jr., *Journal of Forestry*, Vol. 36, 1938, No. 8, p. 775.

[60] "Natural Water Losses in Selected Drainage Basins," by G. R. Williams and others, *Water Supply Paper No. 846*, U. S. Geological Survey, 1940.

[61] Irrig. Engr., Div. of Irrig., SCS, U. S. Dept. of Agriculture, Los Angeles, Calif.

[61a] Received by the Secretary June 24, 1941.

veloped a practical method of estimating evapo-transpiration based on the relation of measured consumptive use to maximum temperatures above 32° F during the growing season for average water supply and crop conditions. The effect of sunshine and heat in stimulating transpiration was studied as early as 1691, according to a review of literature by the U. S. Department of Agriculture in 1905.[62] For many years irrigation and hydraulic engineers have considered the use of temperature data in estimating consumptive use of water. However, the results usually have not been satisfactory for individual years as the annual variations in rainfall, water supply, crop distribution, and methods of irrigation were not taken into account. For an average consumptive use over a series of years the method is also subject to criticism, when projects under consideration do not have similar crop distribution or the quantities of water available for irrigation per acre are materially different. The authors have made allowance for these varying conditions, but caution should be exercised by those not so familiar with the problems involved. For example, two projects having about the same effective heat unit in day-degrees might have considerable difference in average unit consumptive use if one project had 50% alfalfa and 15% cotton and the other had 20% alfalfa and 60% cotton, since alfalfa may consume 50% more water than cotton.

Differences in distribution of crops is illustrated by a crop survey of the districts in Pecos County, Texas, made in 1940 by the Department of Agriculture in connection with the Pecos River Joint Investigation. The distribution of the irrigated acreage in the Fort Stockton area was: Cotton, 18%; alfalfa, 58%; and other crops, 24%; whereas the distribution in the Imperial-Buena Vista area some 50 miles away was: Cotton, 53%; alfalfa, 26%; and other crops, 21%. The writer computed the consumptive use by both the integration and inflow-outflow methods for these areas, finding that the consumptive use in the Fort Stockton area was considerably more than in the Imperial-Buena Vista area, although there was very little difference in the effective heat units. The greater use in the Fort Stockton area was attributed to a larger percentage of alfalfa and a more abundant water supply.

The authors mention the inflow-outflow and integration methods used in estimating consumptive use of water in the Upper Rio Grande Basin[10] in 1936. In connection with this investigation and the Pecos River study in 1940, the writer reached the conclusion that further research into the relation of available heat to consumptive use of water must be conducted to make the effective heat method comparable in accuracy and reliability to the inflow-outflow and integration methods. Tables 10 and 11 show how closely the results of the inflow-outflow and integration methods in Mesilla Valley will check under favorable conditions. However, conditions are not always favorable for use of these methods.

Before the integration method can be successfully used it is necessary to know unit consumptive use and the areas of various classes of agricultural

[62] "A First Report on the Relations Between Climates and Crops," by Cleveland Abbe, *Weather Bureau Bulletin No. 36*, U. S. Dept. of Agriculture, 1905.

[10] "Regional Planning—Part VI, The Rio Grande Joint Investigation in the Upper Rio Grande Basin in Colorado, New Mexico, and Texas, 1936–1937, Part III, Water Utilization," by H. F. Blaney and others, National Resources Committee, 1938, pp. 295–427.

TABLE 10.—CONSUMPTIVE USE OF WATER IN THE MESILLA VALLEY AREA (NEW MEXICO AND TEXAS), 1936, BASED ON INFLOW-OUTFLOW METHOD[a]

Period	Acre-Ft					$\frac{K}{A}$ in acre-ft per acre
	I	*P*	*R*	$G_s - G_e$	*K*	
January	3,800	7,180	8,600	+3,310	5,690	0.05
February	13,540	1,660	10,700	+1,660	6,160	0.06
March	50,840	1,200	29,400	−3,310	19,330	0.18
April	93,550	830	50,800	−9,940	33,640	0.30
May	89,858	4,050	62,500	−3,310	28,098	0.25
June	105,204	2,300	62,800	−1,660	43,044	0.39
July	123,299	9,850	76,500	−1,660	54,989	0.50
August	123,190	14,080	79,700	−3,310	54,260	0.49
September	63,832	22,540	50,500	+4,970	40,842	0.37
October	12,170	2,940	17,800	+4,970	2,280	0.02
November	9,020	10,120	12,500	+6,620	13,260	0.12
December	7,000	5,430	12,000	+1,660	2,090	0.02
Total	695,303	82,180	473,800	0	303,683	2.75

[a] As determined by the equation $K = (I + P) + (G_s - G_e) - R$; from inflow ($I$), precipitation ($P$), difference in ground-water storage ($G_s - G_e$), outflow (R), and valley consumptive use (K). Total area (A) = 110,418 acres; irrigated area = 82,923 acres; $\frac{K}{A}$ = consumptive use per acre.

TABLE 11.—CONSUMPTIVE USE OF WATER IN MESILLA VALLEY AREA (NEW MEXICO AND TEXAS), AS ESTIMATED BY THE INTEGRATION METHOD, 1936

Classification	Area, in acres	Consumptive Use	
		Acre-ft per acre	Acre-ft
(a) IRRIGATED LAND			
Alfalfa and clover	17,077	4.0	68,308
Cotton	54,513	2.5	136,282
Native hay and irrigated pasture	216	2.3	497
Miscellaneous crops	11,117	2.0	22,234
Total or weighted average	82,923	2.74	227,321
(b) NATIVE VEGETATION			
Grass	2,733	2.3	6,286
Brush	6,933	2.5	17,332
Trees—bosque	3,532	5.0	17,660
Total or weighted average	13,198	3.13	41,278
(c) MISCELLANEOUS			
Temporarily out of cropping	5,569	1.5	8,354
Towns	1,523	2.0	3,046
Water surfaces: Pooled, river, and canals	4,081	4.5	18,364
Bare land, roads, etc.	3,124	0.7	2,187
Total $(a + b + c)$	110,417	2.72	300,550

crops, native vegetation, bare land, and water surfaces. The reliability of the method depends primarily upon the selection of proper unit consumptive-use values.

In the consumptive-use studies conducted by the Division of Irrigation in cooperation with the National Resources Planning Board on the Pecos River in 1940, the writer developed the irrigation efficiency method of determining unit consumptive use for various agricultural crops. Briefly, the irrigation efficiency method consists of estimating evaporation and transpiration losses during the growing season and winter period based on data available as to the number of irrigations and depth of water applied, farm duty of water, and efficiency of irrigation water application for various crops and soils; and precipitation and evaporation from free water surface. Precipitation may be a large factor in winter consumptive use.

A survey of irrigation conditions was made throughout the Pecos River Basin. Water superintendents, county agricultural agents, engineers, and farmers were interviewed to ascertain the quantity of irrigation water supplied to various crops, both for the growing season and the winter period. Precipitation records were collected and estimates made of the efficiency of irrigation for various crops in the different areas. The latter data were supplemented by water-application efficiency studies made on typical farms by measuring the quantity of water applied and taking soil samples before and after irrigation to determine the proportion of soil moisture available for crop transpiration use within the root zone. All this information was used to determine the unit consumptive use of water for various crops in each area. A summary of the results is shown in Table 12.

For many years the Division of Irrigation, U. S. Department of Agriculture, has been correlating observed evaporation and evapo-transpiration data as a means of estimating consumptive use of water. The writer made some studies of this character on field crops at the Denver (Colo.) Irrigation Field Laboratory of the Division in 1919. The adaptability of the evaporation pan as an index for estimating evapo-transpiration losses from moist areas was demonstrated by studies in 1931.[63] For tules growing in a large tank within the confines of a swamp area at Victorville, Calif., the percentage of consumptive use with reference to evaporation from a nearby exposed Weather Bureau pan was 95%. Evaporation from pans was used as an index of consumptive use of water on the Rio Grande in 1936,[10] and on the Pecos River in 1940,[64] when evaporation stations were established in valleys extending several hundred miles along those rivers.

In the Pecos River investigation an empirical formula was developed by the writer and Karl V. Morin[65] for use in computing evaporation from a lake surface and unit consumptive use of water by vegetation having access to ground water, where temperature and humidity records are available. Both evapora-

[63] "Water Losses Under National Conditions from Wet Areas in Southern California," by Harry F. Blaney, Colin A. Taylor, Harry G. Nickle, and Arthur A. Young, *Bulletin No. 44*, California Div. of Water Resources, 1933.

[64] "Consumptive Water Use and Requirements," by Harry F. Blaney, Paul A. Ewing, Karl V. Morin, and Wayne D. Criddle, Pecos River Joint Investigation (Pt. IV-B) Section 2 (mimeographed report of Division of Irrigation, SCS to National Resources Planning Board, 1941).

[65] *Ibid.*, p. 104.

tion and transpiration respond freely to temperature and humidity. Records of temperature, humidity, and evaporation in New Mexico and Texas were used. The first step in the process was to introduce a factor of evaporative force which is the average temperature times the percentage of (yearly) day-

TABLE 12.—NORMAL UNIT CONSUMPTIVE-USE VALUES FOR PRINCIPAL IRRIGATED CROPS, PECOS RIVER VALLEY AREAS (NEW MEXICO AND TEXAS) IN ACRE-FEET PER ACRE (ESTIMATED BY IRRIGATION EFFICIENCY METHOD)

Location	Estimated Unit Consumptive Use				
	Alfalfa	Cotton	Annuals	Other crops	Weighted average for area
(a) NEW MEXICO					
Las Vegas	2.4	...	1.8	1.7	1.9
Fort Sumner	3.1	...	2.3	2.0	2.5
Roswell	3.3	2.6	...	2.1	2.6
Carlsbad	3.4	2.8	...	2.2	2.8
(b) TEXAS					
Barstow	3.5	2.2	...	1.9	2.6
Grandfalls	3.6	2.4	...	2.2	2.7
Imperial	3.3	2.6	...	2.2	2.8
Fort Stockton	4.2	2.6	...	2.2	3.0

time hours for each month, or other time unit considered. A chart using evaporative force as ordinates and use of water in inches as abscissas was developed, on which it was found that plotted observed evaporation pan records, reduced to lake surface by the factor 0.70, appeared thereon as a fairly straight line.

From this the following general formula was developed for estimating annual unit values of evaporation from free water surface and evapo-transpiration by native vegetation for use in the integration method:

$$E = F\,K\,(114 - H) \qquad (2)$$

in which E = annual evaporation in inches; F = average temperature multiplied by percentage of yearly daytime hours = evaporative force (varying with latitude); K = constant; and H = annual average relative humidity (a percentage).

It is desirable to modify Eq. 2 when computing consumptive use of water by vegetation by substituting U for E, as follows: The annual consumptive use, U, in inches, equals

$$U = F\,K\,(114 - H) = u_s + u_w \qquad (3)$$

in which: u_s = growing-season consumptive use; and u_w = winter consumptive use. Then

$$U = f_s\,k_s\,(114 - h_s) + f_w\,k_w\,(114 - h_w) \qquad (4)$$

in which the subscript *s* denotes the growing (summer) season and *w* the winter period. The symbol *f* varies with temperature and latitude whereas *k* will vary with type of vegetation.

The advantage of the evaporative force method over the effective heat method is that it includes the factors of humidity and latitude as well as temperature and length of growing season.

The authors state (see heading "Method of Study: Equivalent Area") that "the cropped land in most agricultural valleys amounts to 85% or more of the gross area." This may be true for some federal projects but the percentage of cropped land is probably considerably less in most agricultural valleys unless areas within the general irrigated boundaries are eliminated.

In the Upper Rio Grand Basin[10] in 1936, the proportion of cropped area to gross area was 56% in the Southwest Area, San Luis Valley; 46% in the Isleta-Belen Area; and 75% in the Mesilla Valley. The author's data (heading "Description of Areas Studied: Mesilla Valley, New Mexico-Texas" (Appendix)) show only 71% in Mesilla Valley, which is more than the average valley percentage.

The Division of Irrigation has computed the annual consumptive use of water by the inflow-outflow method in Mesilla Valley for the period 1921 to 1939, inclusive, and the results check closely with the data given in Table 4, varying from 1.86 acre-ft per acre in 1921 to 3.53 in 1926, with an average of 2.78 acre-ft per acre for the 19-yr period.

Conditions are not always favorable or the expense may be too great to use some of the foregoing methods.

The authors have developed a simplified and unique method for estimating consumptive use under average conditions when a limited time is available. The paper is a valuable contribution to the literature on the subject.

J. L. Burkholder,[66] M. Am. Soc. C. E.[66a]—The method originated by the authors provides for estimating the consumptive use of water in agriculture by correlating mean effective heat, and such consumptive use on extensive areas of cropped land having a range of climatic conditions. The adaptation of mean effective heat as the independent variable for estimating consumptive use is based on the fact that this variable is readily determinable for any locality and on the theory that it is the most important factor influencing the quantity of water consumed by transpiration and evaporation from cropped areas.

A number of other variables affect the consumptive use of water in agriculture. Some of the more important are: Soil types; alkali and drainage conditions; types of crops and their yields; and water application with respect to deficiency or surplus.

On irrigated areas, soil types, and alkali and drainage conditions constitute a complex network of factors that influence the consumptive use. If drainage is quite effective, the net influence of these factors is likely to result in a higher or above normal consumptive use due to increased evapora-

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[66a] Received by the Secretary July 15, 1941.

tion losses from heavy and frequent irrigation. In the reverse case of tight soils and restricted drainage, the "rise of salt" will prove an effective brake to over-irrigation, which would increase evaporation losses. Thus consumptive use might be lowered.

The yield of most common crops under given soil and climatic conditions will vary directly with the quantity of water beneficially applied. For instance, sugar cane will normally increase its yield about 20% for each additional acre-foot of water so applied up to a total application of about 5 acre-ft. Heavy crop yields transpire more water; hence the consumptive use is increased.

On most irrigated projects water is provided to the farms in normal years practically on a demand basis. Only in years of deficient supply is the use curtailed by rationing. The authors have attempted to correct their data for any deficiency in water supply. However, without detailed information on the type of crops produced on the various farms, and on the quantity of water required for each crop, it would be very difficult to establish the amount of the deficiency. Also, under the authors' method of estimating consumptive use, it may be equally important to correct for excessive evaporation due to an overabundance of applied water.

A consideration of the complex nature of the variables affecting the consumptive use of water in agriculture leads to the belief that the annual consumptive use for a given area may be expected to vary considerably from the mean for that area. The correlation between consumptive use and effective heat, therefore, should reflect the true value of the annual consumptive use so that the range of such use can be established.

The consumptive-use data presented in the paper include a total length of record of 42 years for nine irrigated valleys, of from 6,000 acres to 462,000 acres in area, and a total length of record of 141 years for eleven non-irrigated watersheds of from 200 acres to 5,000,000 acres in area. Of the eleven plotted points (Fig. 2) representing consumptive use on non-irrigated areas, nine fall below the line. It is thus apparent that the authors' study is based primarily on records of consumptive use on non-irrigated watersheds. Also, it is apparent that the mean consumptive use on these areas is somewhat lower than mean consumptive use on irrigated areas. This may be accounted for by the fact that crop yields are usually less on non-irrigated areas than on irrigated areas, or the lower consumptive use may be due to deficient or poorly distributed rainfall.

On the irrigated areas the length of the consumptive-use record ranges from as low as one year to as many as 13 years. For non-irrigated watersheds this record has a much narrower range of from 6 years to 14 years. By reducing the annual consumptive-use record of each area to a mean for the entire period of record, the authors seem to have given the same weight to records of one year as to records of 14 years.

The data presented for irrigated valleys do not include areas having high effective heat such as are found in the subtropical climates of southern California and southern Texas. The writer has attempted to extend the range of the authors' data by the inclusion of two such projects. This study has led to the preparation of a new chart (Fig. 4) on which consumptive-use data for

the Barahona Project, Dominican Republic, and the Lower Rio Grande Valley, Texas, have been plotted, together with the data presented by the authors for individual years. It has thus been possible to define roughly the upper and lower limit of consumptive use. The writer is of the opinion that there is an additional advantage in expressing consumptive use as a zone rather than as a mean, since in investigating new projects the upper limit of consumptive use would then provide a safer basis for estimating the required maximum water supply and the capacity of canals and structures. Also, the lower limit may provide a useful gage of minimum water requirements when it becomes necessary to ration the supply.

It will be noted that line 2, Fig. 4, representing the mean of both the authors' and the writer's data, is almost identical in its relative position with

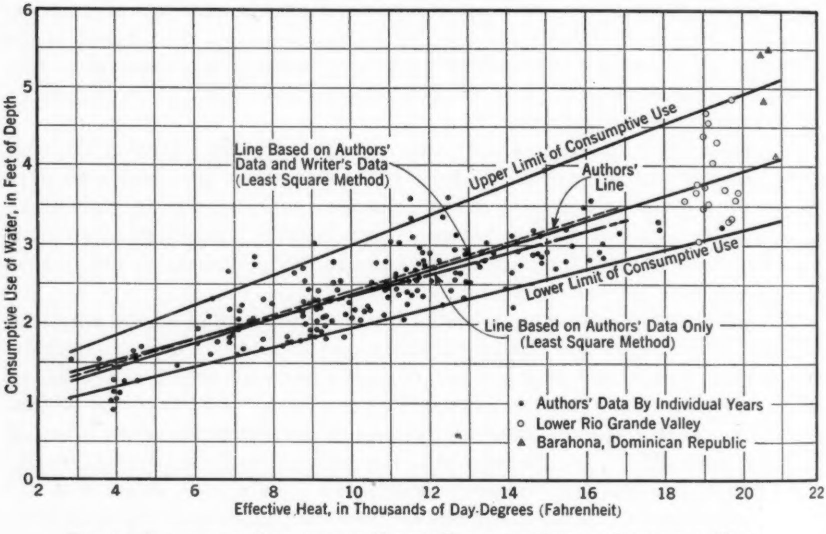

FIG. 4.—RELATION OF CONSUMPTIVE USE TO EFFECTIVE HEAT BY INDIVIDUAL YEARS

line 1 based exclusively on the authors' data. However, if the position of the line representing the mean of the authors' data is fixed by the method of least squares, the authors' line is shifted to position 3.

The data for the Lower Rio Grande Valley include the consumptive use for the years 1922 to 1939, inclusive. The average cropped area for this project is 326,000 acres. Crop distributions are not known for each year, but, in 1929, 53% of the irrigated area was planted to cotton, 19% to orchards, 17% to vegetables, and 11% to grain and sorghum. The growing season includes the entire year. The valley has made rapid strides toward improvement of irrigation facilities and preparation and care of the irrigated lands in recent years, especially since 1928. In this connection it is interesting to note that consumptive use for the years 1922 to 1925, inclusive, is greater than for any of the following years.

Located on latitude 16° North, the Barahona Project is devoted exclusively to the growing of sugar cane, with the exception of a small area of pasture. Irrigation was started on this area in 1920, and the average gross cropped acreage increased from 6,332 in 1921 to 19,354 in 1924. The normal rainfall is about 12 in. and the growing season extends over the entire year. An extensive system of deep open drains was under construction during the years of record. The deltaic soils are relatively tight, and the salt content is high. These conditions forced a reduction in the quantity of water used for irrigation in 1923 and 1924.

The consumptive use for the Barahona area indicates that sugar cane uses more water than the mean requirement for similar climatic conditions as reflected by Fig. 4. The project produced an average of about 30 tons of cut cane per acre in addition to the heavy foliage that was stripped from the cane as it was harvested. It is the sole example of a single-crop area included in the study, and its inclusion results in a steepened line of mean consumptive use.

The authors have made a very able presentation of the subject, and their effort should be commended by those interested in irrigation development. Additional records of consumptive use on irrigated areas are needed to amplify and extend the study presented by them. It is hoped that this paper will stimulate engineers in charge of the many irrigation projects to undertake the required measurements and calculations in order that the range of consumptive use can be more definitely established.

Acknowledgment.—Assistance was rendered by Victor L. Streeter, Assoc. M., Am. Soc. of C. E., in preparing Fig. 4 and the data on the Barahona Project and Lower Rio Grande Valley.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

EXPERIENCES IN OPERATING A CHEMICAL-MECHANICAL SEWAGE TREATMENT PLANT

Discussion

BY MESSRS. R. W. ROWEN, AND GEORGE J. SCHROEPFER

R. W. ROWEN,[13] ESQ. (by letter).[13a]—This was the first large plant to adopt filtration and incineration of undigested sludge. After a careful investigation of the various types of sludge incinerators, it was decided to install three multiple-hearth, mechanically rabbled incinerator units. When they were placed in operation it was found that the filter cake contained less moisture and a higher percentage of combustible matter than anticipated. As a result, no auxiliary fuel was required and, in fact, there was an excess of heat that had to be dissipated. Therefore, hot air from the air preheater units was by-passed directly to the stack, and air at ordinary temperature was used for combustion. The center shaft and rabble arms on the hottest hearths were insulated, and this has effectively overcome the danger of overheating.

Grit and screenings are not incinerated at the Minneapolis-St. Paul plant, although they are now being incinerated at Detroit, Mich., and at several other smaller plants using multiple-hearth incinerators.

The volume of waste, deodorized gases leaving the incinerators at about 1,000° F would produce enough steam for plant heating and all electric power requirements of the plant and still have an excess of steam with only one incinerator unit in operation. The raw sludge filter cake, containing an average of about 65% moisture, 20% combustible material, and 15% ash on the wet basis, has sufficient heat content not only to be self-combustible, but also to furnish all heat and energy required for the entire plant. The economics of heat and power generation are being investigated, and it may prove advantageous to install waste heat boilers and turbo generators.

Another means of utilizing this waste heat is being studied at the Minneapolis-St. Paul plant, which consists of feeding a part of the thickened sludge,

NOTE.—This paper by George J. Schroepfer, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Messrs. F. C. Roberts, Jr., and Rolf Eliassen; April, 1941, by Frederic Bass, M. Am. Soc. C. E.; and June, 1941, by Messrs. Darwin W. Townsend, Lloyd H. Flickinger, Isador W. Mendelsohn, and Ralph E. Fuhrman.

[13] Vice-Pres., Nichols Eng. & Research Corp., New York, N. Y.

[13a] Received by the Secretary August 12, 1941.

containing about 90% water, directly to the incinerator without mechanically dewatering. As the author has pointed out, filtration cost is 72% and incineration is only 28% of the total cost of sludge disposal. By feeding a part of the liquid sludge directly to the incinerator and utilizing the excess heat from the part that is filtered, a further substantial saving in operation costs may be obtained through the elimination of filtration costs on part of the sludge. This is along the line of developments made during 1940 at the Piqua, Ohio, and Ashland, Ohio, sewage treatment plants where all of the undigested, unfiltered sludge is incinerated directly as a liquid in multiple-hearth incinerators, eliminating entirely the cost, mechanical difficulties, and skill formerly required for filtration at these two plants.

The type of sewage treatment and means for sludge disposal at the Minneapolis-St. Paul plant were decided only after long and intensive study of the requirements. A program of investigation, studies, and tests during operation has resulted in economies well beyond those anticipated when the plant was constructed. Frequent meetings of the operating staff for discussion of minor mechanical matters, as well as major developments or theorizing on some idea, have borne fruit.

GEORGE J. SCHROEPFER,[14] ASSOC. M. AM. SOC. C. E. (by letter).[14a]—The writer has enjoyed reading the discussions of his paper and appreciates the time spent by the various writers in the preparation of their comments. The discussions developed no controversial issues, but rather were replete with suggestions that will serve as a source of encouragement in improvement of the operation of the Minneapolis-St. Paul sewage treatment plant.

Professor Eliassen makes an interesting comparison between the costs of sludge disposal by dumping at sea, as practised in New York City, and by filtration and incineration, as used at Minneapolis and St. Paul. It is evident that even the seemingly simple expedient of hauling to sea is a comparatively expensive process, involving costs approximating two thirds to three fourths of those at Minneapolis and St. Paul. As Professor Eliassen has stated, the initial cost of $1,500,000 includes only that of the three sludge vessels. To be comparable it would be desirable to include also the costs of the rather extensive sludge storage building and storage tanks, as well as of dockage facilities and other appurtenances required at the Ward's Island plant in connection with sludge barging.

The performance of the unusually long sedimentation tanks, as discussed by Professor Bass, has been a constant source of gratification to the writer. It appears likely that even greater removals can be effected with graded adjustment of the long effluent weirs—which are now set at a uniform elevation —in relation to the varying detention periods provided at points along the length of the tanks.

Mr. Townsend suggests the desirability of further statements by the writer concerning the question of digestion versus no digestion, prior to vacuum filtration. Throughout the paper the writer attempted to confine himself to

[14] Chf. Engr. and Supt., Minneapolis-St. Paul San. Dist., St. Paul, Minn.

[14a] Received by the Secretary August 18, 1941.

factual information concerning the Minneapolis-St. Paul project. Realizing that every sludge disposal problem is (or at least should be) a special subject of investigation for the peculiar conditions of a particular situation, and remembering the sagacious proverb to the effect that "the most consistent thing about sewage is its inconsistency," the writer would prefer not to generalize concerning the merits or demerits of various combinations of processes of sludge disposal. Referring only to the Minneapolis-St. Paul conditions, however, the writer can state that the decision concerning the filtration and incineration of raw sewage sludge was well justified, as is indicated by the relatively low total costs of sludge disposal at this plant.

Mr. Mendelsohn has requested the writer's views concerning guarantees and maintenance bonds that should be included in specifications. This is an important point. The only equipment on which guarantees and appropriate bonds insuring compliance with these guarantees were required, in connection with Sanitary District contracts, related to the screenings centrifuge, magnetite filters, and incinerators. At the time this equipment was designed in 1935 and 1936, these items of equipment were relatively new, at least in their applications in this particular plant, and in some cases competition was precluded by reason of their special and patentable nature. Operating experience over a period of time has now furnished the designing engineer with what information he requires concerning the performance of such equipment before he makes a selection. Venturing an opinion in a field in which views are certain to differ, the writer would state that qualified engineers, having prepared satisfactory specifications, need not require guarantee bonds on performance of proved equipment in the sanitary engineering field, since the direct and hidden costs resulting therefrom might exceed by several times the risks involved.

Maintenance bonds in the amount of 50% of the contract price for a period of one or two years were required on all equipment. Such bonds were written in connection with the general performance bond without increase in premium. They act as a protection to the qualified manufacturer and contractor, and the city alike, since they serve notice in a forceful way that what is required is well-designed and well-constructed equipment, which will perform successfully and satisfactorily without unusual maintenance costs for a specified period of time.

Mr. Fuhrman presents rather interesting comparative values on the Washington, D. C., and the Minneapolis-St. Paul, plants. As stated, these plants are of comparable size and were constructed and placed in operation about the same time. Enlarging somewhat on the information presented in Mr. Fuhrman's comparison, an interesting point concerning comparisons between various plants can be made:

Description	Washington, D. C.	Minneapolis-St. Paul
Total annual cost of sludge disposal...	$136,815.43	$54,486
Million gallons treated..............	36,705.3	34,200
Tons of dry solids removed...........	37,703.0	12,350
Cost per million gallons treated.......	$3.70	$1.59
Cost per dry tons of solids removed...	$3.63	$4.41

It will be observed that the quantity of sewage treated is approximately the same in the two plants. The quantity of dry solids removed from the sewage in the two plants differs markedly, however, being only about one third in the case of the Washington, D. C., plant. The difference is probably due to difference in strength of the raw sewage and in removals accomplished. It is for these reasons that the commonly used comparison on a million-gallon basis is inapplicable in many cases, even for plants using similar processes. Strength of raw sewage may easily vary from 150 to 300 ppm of suspended solids in two different plants, and removals by sedimentation may differ in the range of 40% to 75%. Using these illustrative examples, it is manifestly improper to compare on a million-gallon basis the costs at a plant having a dilute sewage, on which a low removal is effected, with another plant treating a strong sewage containing quantities of industrial wastes and which removes high percentages of material. For the illustrative range chosen, which is by no means uncommon in practise, one plant might remove and have to dispose of four times the quantity of material that another plant at the other extreme of the range would. It is this fact that in general makes comparisons on a million-gallon basis unsatisfactory and indicates the greater propriety of a comparison on a tonnage basis. Especially is this true of comparisons of sludge disposal costs in which many of the major cost elements vary in direct proportion to the quantity of material handled. The foregoing statements are of a general nature, and are not presented solely because of the specific comparison Mr. Fuhrman has made. It is simply an expression of an opinion that the writer has had for some time.

Returning to Mr. Fuhrman's discussion, it should be stated that in the case of the Minneapolis-St. Paul plant it is possible that at certain times incineration (which is not employed in the Washington, D. C., plant) may be eliminated, at a saving of approximately $1 per ton. This could be accomplished by farmer hauling of sludge cake for fertilizer purposes, at no expense to the District. Rather, it appears quite likely from present indications that a certain income may be realized.

Mr. Rowen points out possible additional savings in connection with incinerator operations that are under consideration. One such possibility that was thoroughly investigated in the period since the paper was presented (January, 1941) and has since become a reality, is the following:

As a result of the elimination of the preheater on one furnace, marked savings in power have been effected. As mentioned under the heading "Sludge Disposal: Incinerators" in the paper, the average power consumption in the two years of operation reported on therein has been 17.3 kw per hr per dry ton. The average power consumption for two months in the furnace from which the preheater has been removed has been approximately 5.5 kw per hr per dry ton. Similar changes will be made in the other two units at an estimated total annual saving in power, replacement, and maintenance costs of $12,000 annually.

Several discussions made mention of flexibility of the plant in regard to the degree of treatment capable of being provided to meet varying river dilution conditions. Although such possibilities might not be desirable in the

very small plants, the writer agrees that flexibility in the degree of treatment is extremely advantageous, especially for a large project in which continuous examination of the requirements of the receiving body is made. After all, the aim is to meet required standards determined upon in relation to stream use, and therefore a certain end product is required, varying with the capability of the stream to assimilate pollution, and with the use to be made of the water.

The writer would like to have answered each point of information raised, but inasmuch as several of them involved matters of a specific rather than a general nature, and since many such questions have already been discussed in other publications, they are not specifically replied to herein.[14b]

At the time the paper was presented, the guarantee tests had not as yet been completed on the magnetite effluent filters, because of certain difficulties, principally those occasioned by displacement of sand in the beds. It is a pleasure to be able to report that as a result of the efforts of the manufacturer the filters have been placed in satisfactory operation and the performance tests relating to removals have been acceptably accomplished. The average removals in the four guarantee tests covering different seasons of the year were as follows:

Suspended solids	ppm
Filter influent	78
Filter effluent	52
Removal by filters	26

The writer wishes to emphasize a matter relative to type of personnel, which is sometimes not sufficiently well recognized, especially on the part of elective public officials. This relates to the desirability of having a sufficient number of qualified professional engineers and chemists on the staff of a sewage treatment plant of any magnitude. Especially is this important when a plant is looked upon as a necessary and considerable investment to be improved and perfected with the rapidly changing status of the art. What might appear on the surface to be a disproportionate expense in "overhead" pays ample dividends in present and future savings in operation and maintenance costs. The Minneapolis-St. Paul Sanitary District is fortunate in having on its staff thirteen engineers and chemists, in addition to a number of others with subprofessional experience, out of a total staff of seventy-nine employees. Lest an impression be created that all such men need be supervisors and "straw bosses," the fact is that many are far from such in the case of the Minneapolis-St. Paul plant, but rather carry their proportionate share of actual work.

The writer appreciates the complimentary remarks concerning the operation of this plant by several of the discussers, and passes on to those who have earned and deserve it whatever credit is due. It is generally realized that to effect important improvement changes, and to increase efficiency and economy, a conscientious, cooperative, and loyal staff is essential.

[14b] The writer will make them available to those desiring more specific information.

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DISCUSSIONS

EVALUATION OF FLOOD LOSSES AND BENEFITS

Discussion

By Messrs. E. L. Chandler, E. F. Chandler, and Charles B. Burdick

E. L. Chandler,[7] M. Am. Soc. C. E.[7a]—Probably there will be no dissent from a statement that the construction of a flood-protection project should not be considered justifiable unless the resulting benefits will exceed the cost. Benefits include the elimination of prospective losses directly attributable to floods, the expansion of existing facilities, and the development of new areas, which will result from improved conditions.

The author refers to "The losses from which the benefits are computed * * *" (see "Introduction"), and predicates his study on the thought that "When the losses are known, the annual benefits can be computed and compared with annual costs." This seems to be a limited and ultraconservative point of view. Any full measure of benefits must include not only computations covering tangible losses but an estimate of future expansion and development of potential resources that will be made available by reason of flood protection. Mr. Foster has not ignored this phase of the subject, but the writer is of the opinion that strict adherence to the methods advocated is likely to result in overemphasis of the specific "losses and damages" due to floods under existing conditions, at the expense of fuller consideration of the many indirect benefits and of benefits to be anticipated from potential developments that will be encouraged by flood protection. Any public development should be built with an eye to the future, and this is especially true of flood-protection works.

The writer realizes that an estimate of future benefits is a matter to be approached with caution. It is entirely possible to rise to flights of fancy that may seem to warrant expenditures for flood-protection works which should not be built. Conservatism must be exercised in making computations contingent on prophecy.

Note.—This paper by Edgar E. Foster, Assoc. M. Am. Soc. C. E., was published in May, 1941, *Proceedings*.

[7] Chf. Engr., Chattanooga Flood Protection Dist., Chattanooga, Tenn.

[7a] Received by the Secretary June 21, 1941.

It happens that there is no question as to the serious need for flood protection for Chattanooga, Tenn. Careful estimate indicates that, with the existing development of the community, measurable damages that would result from the maximum flood to be expected would amount to between two and three times the cost of protective works. On the other hand, there are thousands of acres of land within the city limits that are unusually well adapted for industry and business developments except for the flood menace. These areas, which are well located for service by railroad, highway, navigation, and utilities, now stand idle. No surrounding territory is available that will compare favorably with these areas after they have been guaranteed freedom from flood. No measure of the benefits to be gained from flood protection would be complete without an estimate of development to be confidently expected as the result of protection. Although such estimate is not required to prove the necessity for protection at Chattanooga, it may easily happen that, in other instances, economical justification for worthy projects might fail of recognition unless future benefits were given due weight.

The writer's thought may be summed up by the statement that, although it is poor economics to build works that cannot be justified by the benefits to be received, it is also poor judgment to refuse to make expenditures because of failure to recognize a full measure of benefits that would be gained.

The author states (see heading, "Some Definitions of Wealth and Value") that "value * * * may be defined or measured as the quantity of other goods that may be given or received in exchange * * *," the "other goods" being, in general, money. Again, in referring to the appraisal of property values, he states that "These appraisals must be considered primarily as a sale value * * *" (see heading, "Evaluation by Estimating Depreciated Property Values"). The true value of a piece of property is not necessarily its sale value. Mr. Foster recognizes this elsewhere in his paper, but the writer wishes to place particular emphasis on the fact. It is a common saying that "a thing is worth what you can get for it." Probably such a statement is satisfactory for the purposes of the real-estate dealer or merchant, or the casual buyer or seller, but it should not serve as a criterion for an engineer studying the economics of a project for flood protection; nor will it serve for appraisals of properties to be used as a basis for special benefit assessments in the case of assessment districts. Many factors may tend to render sale price inaccurate as a measure of value for the purpose at hand. Sale prices during boom times or during a period of depression would scarcely be proper. An influence which, perhaps, is more pertinent is the lapse of time since the occurrence of a flood of considerable magnitude. The author correctly states that a marked depreciation of value will follow the occurrence of a serious flood, and that, on the other hand, if a considerable period of time elapses without disastrous flooding, the sale value of properties will reach heights greater than the true values, due to the ignorance of property owners as to potential danger.

Mr. Foster mentions these circumstances under his discussion of evaluation of losses on a "capital basis" and concludes that they "preclude its acceptance for general use" (see heading, "Evaluation by Estimating Depreciated Property Values"). The writer does not agree with this statement. The "capital basis"

method affords a feasible and valuable approach to the problem in many instances, particularly in the case of urban communities where complete protection is to be provided. Mr. Foster states that "The capital-loss method bases the estimated damage on a single flood." This is a limited approach. A study based on the effects of a single flood could scarcely be considered adequate.

In this connection the author states that "An appraisal is made of the affected property after the flood to estimate its value then and the value as before the inundation. The difference between the two values is taken as the flood loss * * *." Why "after the flood"? Probably Mr. Foster has in mind the historic apathy of flood-menaced communities. Generally, this has been such that no flood-protection works have been built except in the wake of serious floods, with the result that appraisals of property values and estimates of benefits have been made while the memory of flood disaster was fresh in mind. It is hoped that Chattanooga will prove to be one exception to this distressing rule. In such event, the situation will be the reverse of that suggested by the author. There has been no serious flood at Chattanooga for a long time and none of consequence since the community reached its present state of development. Flood-protection works have been planned and the Chattanooga Flood Protection District appraisers are faced with the problem of determining values now when the gravity of the flood menace is not generally recognized. Such values must be compared with an estimate of the enhanced values that will result from flood protection. Regardless of which conditions prevail, it is very evident that sound appraisal judgment is necessary.

The author states (see heading, "Benefits from Flood Control") that

> "Levees will eliminate all direct losses caused by inundation up to the stage at which they are overtopped. Beyond that stage the loss is as great as if there were no protection. The annual benefits will equal the annual losses below the elevation of the top of the levee or wall."

This thought should be accepted only with qualification. It appears to be substantially correct in the case of protection for agricultural areas although, even there, the overtopping of a levee, with consequent sudden flooding, is likely to result in losses far greater than those that would result from a flood of corresponding magnitude if there were no protection at all. In the case of protective works for urban communities, levees that would give anything less than complete protection should seldom be considered. The presence of an inadequate levee will furnish a false sense of security that will encourage residents to remain, with their belongings, in low areas in the face of approaching disaster. The sudden overtopping of a levee surrounding a thickly populated district is almost certain to result in loss of life and property far in excess of that which would have occurred if there were no protective works at all.

E. F. Chandler,[8] M. Am. Soc. C. E.[8a]—As viewed by the writer, this paper seems to be the best discussion, well rounded, with brief mention of each of the

[8] Prof., Civ. Eng. and Dean Emeritus, College of Eng., Univ. of North Dakota, Grand Forks, N. Dak.

[8a] Received by the Secretary July 7, 1941.

complete list of possible contributing factors or modifying phases that he has ever seen. It is especially interesting as showing the advance in accepted ideas on this subject during the twenty years, since 1920, and in particular the progress in methods used by the author himself since he first occasionally undertook such investigations.

However voluminous the base data, however elaborate and reasonable the chain of reasoning, there is no absolute certainty concerning the amount and intensity of the rainfall in any assigned future period; there is still less concerning flood flows, and less yet as to the total resulting damages in the future. Rules and formulas assist, but good judgment on the part of the engineer designing flood-protection works is also a necessity.

It can be argued plausibly that the problem of the evaluation of the benefits from flood-protection works, to determine the magnitude and expense of works advisable that will be equal to the cost of the future probable flood losses thereby eliminated, is, for two reasons, the most complicated, difficult, most nearly insoluble, of all engineering investigations. The problem confronts the engineer, however, because he will certainly have occasional floods with their ensuing losses in future years; so he is obliged to try to do the best that he can with it.

The first of these reasons is the uncertainty as to the actual cash loss resulting from any future flood of any assigned height; in business districts, it will vary greatly with business season, with the length of the advance warning, etc., and in agricultural regions even more with the time of year and advancement of the crops. This is not markedly more difficult, however, than the uncertainties and indefinite risks of any business venture.

The second, almost overwhelmingly large, difficulty in making advance estimates of the benefits accruing hereafter from any type or magnitude of flood-protection works lies in the extreme uncertainty as to the probable magnitude of future floods. Rainfall itself is very uncertain in its future times, amounts, and intensities. Even the annual total, in North Dakota, varies in different years from a maximum of about 150% of the normal annual to a minimum of about 50% of the normal; and short storms, of a day, few days, or few weeks, which are usually the cause of the floods, of course, are far more irregular. The portion of the rainfall that becomes floodwater is not logically a fraction of the rainfall, but a remainder after soil absorption, evaporation, small-basin detention en route, etc., have been subtracted. Therefore, doubling the rainfall in any period does not merely double the flood flow but usually multiplies it many times.

For example, in the prairie regions of North Dakota, with a many-year average total annual rainfall of about 18 in., the average total annual runoff in the streams of the region is slightly less than 1 in., although in any single year the total can easily be double the normal, or even much more than that, and in a dry year perhaps be only a quarter, or merely a tenth, of the normal annual total. In some Pennsylvania areas of 40-in. total normal annual rainfall, however, the average annual runoff is about 20 in.; and furthermore, because of the steeper sloping topography, the Pennsylvania flood crests are likely to be more accentuated.

Thus it is self-evident that in any region a slight increase in the total rainfall will presumably cause a disproportionately large increase in the stream flow. It is also self-evident that the peak height of flood flow depends on the intensity and abruptness of the rainfall (especially on small streams) far more than on the total of rainfall, and is very greatly affected by soil conditions, surface vegetation, frozen condition of soil, and a vast number of other factors, all of which the author seems to have included in his lists.

CHARLES B. BURDICK,[9] M. AM. SOC. C. E.[9a]—The Society and the author of this paper are to be congratulated upon this concise statement and scholarly discussion of the methods applicable to the computation of benefits from flood-relief projects. Mr. Foster discusses a subject almost as important as the design and construction of the projects for flood relief. The paper deals with a subject too often neglected, partly on account of the difficulties and uncertainties that are usually involved. There may be cases in which the need for flood-relief projects is sufficiently evident to justify the cost involved in the proper remedy without computation of the benefits. Such cases, however, are very few and must necessarily involve only a few people well conversant with the facts. Even in such cases, if the interested people are called upon to "foot the bills," a very convincing case must be presented if the necessary money is forthcoming, and there must be a marked preponderance of benefits if the project is to be financed.

Every one reasonably familiar with stream flow realizes that a bankfull capacity of creeks and rivers is but a small percentage of the flows that the streams are required to carry as a result of unusual rainfall conditions. It is the rule that at greater or less intervals the adjoining river bottoms must be flooded under exceptional flow conditions. The fact that these occurrences are comparatively rare has led to the occupancy of the bottom lands, in many cases including costly improvements. Thus, there is a flood problem of more or less urgency upon almost every mile of the many thousands of streams that drain the continental United States.

In the days when the doctrine prevailed that works for flood relief must be built at the expense of the benefited property, such works were more or less automatically confined to the most worthy projects. In recent years, a number of exceptionally disastrous floods, which directly affected many people and of which nearly every one became cognizant through the ready means of communication today, have been instrumental in effecting state and federal legislation designed to take the burden largely from the shoulders of the chief beneficiaries of relief. It becomes especially important, therefore, to scrutinize carefully the benefits arising from such works. The public dearly loves to "get something for nothing." Immediately following a great flood, it is good politics to build works for flood relief.

Although there is no doubt that expenditures for flood relief can be justified only by prospective losses without relief, the argument sometimes has been advanced that losses have nothing to do with prospective benefits. This

[9] Cons. Engr. (Alvord, Burdick & Howson), Chicago, Ill.

[9a] Received by the Secretary July 16, 1941.

position has been taken by courts in some cases. The attitude probably arose from legal procedures in ordinary special assessment cases of which many thousands have been tried in which there was no way to determine benefits except through opinion testimony. Immediately following a very disastrous local flood detrimentally affecting property values, it is often practicable to secure well-informed layman testimony that certain benefits to property would arise from adequate flood protection. However, such testimony is unprocurable and would be well-nigh useless in the case of an extensive flood problem involving a large river system. There is nothing more practical that can be done than to determine as accurately as possible the probable losses of the past and to project them into the future as a guide to justify expenditures. The difficulty and the uncertainty of such a task cannot be urged as reasons to omit such an investigation. Unless a preponderance of benefits can be reasonably and clearly shown, measured in dollars, the project should not be built because there are far too many other good uses for money.

Many public expenditures are made in these days that necessarily are only subject to justification on the basis of a "hunch" that they are worth while. These often prove to be valuable improvements, although the reverse is also true. An alleged "social benefit" is a general term frequently heard and usually impossible of measurement. Flood-relief projects can be built only with dollars. Many social benefits can be accomplished only by the expenditure of dollars. Human life, sickness, and death are subject to measurement in dollars at least so far as the general public is concerned. The courts have placed values on such things in many cases, and this type of loss and prospective benefit may very properly be considered where it is an important element.

There are sometimes cases in which it is comparatively simple to show a tangible, strong preponderance of benefits based on indisputable evidence. There are many other borderline cases in which the accuracy of benefit computations may be seriously questioned. The public would do well to confine its flood-relief expenditures to cases in which the preponderance of benefits can be clearly shown.

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DISCUSSIONS

TRAFFIC ENGINEERING AS APPLIED TO RURAL HIGHWAYS

Discussion

BY CHARLES E. CONOVER, M. AM. SOC. C. E.

CHARLES E. CONOVER,[5] M. AM. SOC. C. E.[5a]—It is obviously impossible to bank the pavement adequately on curves of less than 500-ft or 600-ft radius, as the author so clearly indicates. Red flash signals on the right side of the pavement approaching the curve are of real value to the careful driver. The difficulty of providing signals that will curb the reckless driver, unless he sees clearly that his life is greatly in peril, is one that all highway engineers clearly recognize. Signs that have been placed along highways, where fatal accidents have occurred, stating the number killed at the location in question, seem to have a sobering effect on such drivers.

Transition curves, not as yet used to any great extent even on main highways, could be used to advantage on curves of less than 1,500-ft radius. The difficulty of securing the additional property in the shape of long, relatively narrow grooves is clearly recognized by the writer. The cost of such acquisition is always far greater than its real value even in undeveloped areas.

Highway pavement on curves should be wider than on tangents. So far as the writer has observed in traveling through many states east of the Mississippi River, it is not present practice to provide this necessary extra width.

The author refers to the accidents that have occurred on a particular section of two-lane highway with sustained 6% grades, which slow the truck traffic down to a comparatively low speed. He shows that accidents are "piled up" in sections that are on curves, with lack of safe passing distances, due largely to impatient drivers of light cars trying to pass the heavy slow trucks. The remedy is not to widen the pavement, but rather to make it obligatory for trucks to have engines powerful enough and so geared that they can maintain the speed of a passenger car. Rear-end collisions with trucks would lose much of their danger if the bumpers were arranged to be at the same elevation as the bumpers of pleasure cars. In other words, car design and

NOTE.—This paper by Milton Harris, Assoc. M. Am. Soc. C. E., was published in May, 1941, *Proceedings*.

[5] Cons. Engr., Pearl River, N. Y.

[5a] Received by the Secretary July 12, 1941.

truck design are not of less importance in the solution of the accident problem than highway design. Much has been done along this line since 1930, at least within and adjacent to the so-called metropolitan (New York) area.

By Fig. 10, the author portrays vividly the elements of danger always present at crossings, especially in the case of cars making left turns. In urban areas, left turns are often prohibited, and cars are forced, by right turns, to traverse an entire city block in order to accomplish the purpose of a left turn. This affords considerable relief to traffic at very busy intersections. On rural highways, however, the distances usually to be traversed by making right turns around a block, as a substitute for direct left turns, are so great that they compel the making of the left turn as a practical necessity.

At crossings where the traffic on one of the highways is heavy and that on the other is light, a very considerable reduction in the hazards due to such crossings has been effected in many locations adjacent to New York City by installing signal lights so arranged that the red light is set to be normally against the highway with light traffic. A bar is placed transversely in the pavement of the light traffic highway a short distance back of the intersection, electrically connected with the signal light in such a manner that a vehicle passing over it changes the light for that highway for a limited period, the duration of which depends on usual traffic conditions at the intersection.

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DISCUSSIONS

SALTS IN IRRIGATION WATER

Discussion

BY C. S. SCOFIELD, ESQ.

C. S. SCOFIELD,[3] ESQ.[3a]—Mr. Hill, in this paper, undertakes to extend and improve a system of water classification proposed by Chase Palmer[2] before 1911. The writer would be less than frank if he did not state at once that there exists among hydrologists a difference of opinion as to whether Mr. Palmer's classification should be retained in use even if revised. One school holds that it would be better to forget it. The writer does not feel called upon at this time to support either side of that question.

It seems both obvious and uncontrovertable that the methods currently in use for reporting on the quality of water are unsatisfactory. There are almost as many methods of reporting water analyses as there are water analysis laboratories. Under present conditions it requires the services of an expert hydrologist and a computing machine to reduce the findings of two or more laboratories to a common basis so that comparisons between them are possible.

It was not until June 25, 1941, that definite steps were taken to remedy this regrettable condition of affairs. On that date, Committee D-19 of the American Society for Testing Materials adopted the report of its subcommittee on the analyses of industrial waters. The fundamental basis adopted by Mr. Hill is in harmony with the method recommended by that Committee.

However, it is evident that Mr. Hill's objectives lie beyond the relatively simple one of obtaining uniformity of method in reporting water analyses. He has two major objectives: (1) To illustrate by means of diagrams the salient characteristics of two or more waters; and (2) to use these diagrams as an aid in solving certain perplexing hydrological problems along streams, such as finding the sources of salt contamination or finding the places in which salt is deposited.

The writer approves and supports his efforts in the direction of his first objective. These are illustrated in Fig. 1 and Tables 1 and 3. These diagrams show effectively the important differences between the waters of two streams in

NOTE.—This paper by Raymond A. Hill, M. Am. Soc. C. E., was published in June, 1941, *Proceedings*.
[3] Principal Agriculturist, U. S. Bureau of Plant Industry, Washington, D. C.
[3a] Received by the Secretary June 16, 1941.
[2] "The Geochemical Interpretation of Water Analyses," by Chase Palmer, *Bulletin No. 479*, U. S. Geological Survey, 1911.

the Salt River Valley and the progressive changes in concentration and in composition that occur in the Rio Grande between Del Norte and Fort Quitman. No doubt, the essential facts in these two situations would be apparent to an expert hydrologist who examined the tabulated analytical data, but these facts become apparent to a much larger number of persons when they are illustrated as they are in these diagrams.

In respect to the second objective—that of solving problems of the source or the destination of water-borne salinity—the writer finds himself less positive, and in fact somewhat perplexed. The problems involved are of great importance, and any serious attempt to solve them is to be commended; but the writer must admit his own inability to evaluate this method of attack.

It may be appropriate to discuss certain phases of the subject of this paper that Mr. Hill has not emphasized—namely, the nature of the effects of salt constituents on the soil and on plants. One important constituent that is not mentioned in the paper is boron. This element is important because it is one of a small number of elements that are absolutely essential for plant growth. It is important also because in some situations it occurs in irrigation waters and ultimately in the soil solution in such high concentrations as to injure crop plants or even to prevent growth. In other situations (not only in irrigated lands but elsewhere), the quantity of available boron is so small that it is necessary to supply it artificially in order to obtain normal crop growth. Because of these facts, any general discussion of the salts of irrigation water should include boron, and any classification of such waters should take it into account.

The importance of sodium as a constituent of irrigation water is fully recognized in Mr. Hill's classification. The writer will merely point out some of the reasons for its importance. So far as is known, sodium is not one of the elements essential to plant growth. However, it is almost always present in irrigation water and in the soil solution; and whenever salinity concentrations are high, sodium is almost always the dominant basic or cation constituent. Plants absorb very little sodium from the soil solution even when its concentration in that solution is high. It is not known to produce injurious effects within the plant, probably because most plants do not absorb it in large quantities. Its presence in high concentrations in the soil solution is directly injurious to plants chiefly because of osmotic effects of its salts, which retard the absorption of water by plant roots.

Sodium is chiefly injurious because of its effect on the physical condition of the soil. The soil contains minerals and organic compounds that participate in reactions of base exchange. Reactions of this type are widely known and extensively utilized in many industries, particularly in zeolitic water softeners. In the soil, high concentrations of sodium in the solution tend to replace calcium and magnesium from the mineral and organic complexes with the result that the physical properties of the soil are impaired. The soil becomes gelatinous and relatively impermeable to the movement of water.

The rate and extent of the replacement of calcium and magnesium by sodium depend upon the relative concentrations of these constituents in the

soil solution. Thus the injurious effects of sodium occur only when its concentration is higher than that of the calcium and magnesium. This is the reason for emphasizing the relative concentration of sodium rather than its absolute concentration. By way of summary it may be stated that, so far as is known, sodium serves no useful purpose in irrigation water or in the soil solution. It becomes definitely harmful to the soil and thus indirectly to plants when its concentration becomes much higher than the sum of the calcium and magnesium.

It seems appropriate now to discuss briefly three other basic or cation constituents of irrigation water and the soil solution. These are calcium, magnesium, and potassium. They are all elements essential to plant growth. Of the three, potassium is the one most largely absorbed by plants. In fact, plants absorb it so avidly that it is seldom found in high concentrations even in very saline soil solutions or in drainage waters. Potassium deficiency is common, but potassium toxicity is very rare. Calcium and magnesium are often grouped together. They have some characteristics in common; for example, together they make up the hardness that is troublesome in waters intended for domestic and industrial uses. They differ in one important particular—namely, calcium combines with the anion, sulfate, to form a salt of low solubility (0.2%), whereas the corresponding salt of magnesium is one of the highest in solubility. This means that, whereas calcium seldom occurs in toxic concentrations even in very salty solutions, magnesium may do so, and may thus contribute to the high osmotic values in the soil solution that retard the absorption of water by plants.

The writer hesitates to take the space to discuss in detail the salt constituents of the anion group—namely, the carbonate-bicarbonate complex, the sulfate, chloride, and nitrate. There is much to be found in literature about the harmfulness of the carbonate-bicarbonate constituent, the so-called "black alkali." The injurious potentialities of this constituent are overrated. Most of the trouble allegedly due to black alkali is caused by sodium, and sodium is no less harmful when it occurs with sulfate, chloride, or nitrate than when it occurs with carbonate or bicarbonate. Furthermore, both carbonate and bicarbonate form salts of low solubility with calcium and may thus be made innocuous at small expense.

Of the other three anions, sulfate and nitrate at least are both essential to plant growth. In this respect there is some doubt about chloride. Nitrate is so largely absorbed by plants that it seldom occurs naturally in toxic concentrations. In artificial culture solutions it has been shown to be slightly more toxic than equivalent concentrations of chloride. Both of these constituents are more toxic than equivalent concentrations of sulfate.

In these remarks the writer has discussed four cation constituents and five anion constituents that occur in the salts of irrigation water. He has emphasized the constituents, rather than the salts they may form, because in water solutions, and in respect to their effects either on the soil or on the plants, they operate as constituents rather than as salts. Some of them operate independently, whereas the effects of others are modified by their associates. The

method of reporting and illustrating the characteristics of irrigation water as proposed by Mr. Hill is unquestionably of value in the direction of simplifying this complex problem. There can be no doubt that its complexity has been one of the reasons why its economic implications have not been more widely understood. However, one must not overlook the dangers of oversimplification. With this in mind the writer wishes to indorse particularly Mr. Hill's concluding sentence in which he states that much more study of these problems by irrigation engineers and agriculturists is needed.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

WIND STRESS ANALYSIS BY THE *K*-PERCENTAGE METHOD

Discussion

BY C. M. GOODRICH, M. AM. SOC. C. E.

C. M. GOODRICH,[12] M. AM. SOC. C. E.[12a]—Mr. Witmer's paper is a corollary to the Witmer Method in the Sixth Progress Report of the Committee on Wind Bracing in Steel Buildings,[2] where it was shown that, for cantilever action, multiplying each bay shear by the bay width gives a result proportional to the K-value of the girder treated. It supersedes clumsier methods, and gives a welcome program of calculations for one to follow.

In summing loads on the columns, the axial wind load is apparently omitted. In the design of outer columns C_O, first story, one might consider this computation:

$$\begin{array}{ll}
\text{Vertical load} & = 300 \text{ @ } 12 = 25 \text{ sq in.} \\
\text{Axial wind load} & = \ \ 95 \\
\text{Axial wind bending } 78 \times 12 \times \frac{c}{r^2} & = \underline{200} \\
\text{Top of column} & \ \ \ 595 \text{ @ } 18 \times \frac{4}{3} = 24.8 \text{ sq in.}
\end{array}$$

It is conceivable that in buildings of great height, and of relatively small lower story height, the axial load from wind would be of more importance than the wind bending.

Under the heading "Proposed Method of Design: Second Story," the average K-ratio of the column is found to be 3.87; the columns tentatively selected have a K-ratio of 3.77. The value of 2.50 given in the text refers to the relative areas of the tentative columns.

A little difference in the sequence of calculations might be considered desirable. If alongside Fig. 1 one places the total moments to points of contraflexure, then the axial wind loads may be calculated from them by one setting

NOTE.—This paper by F. P. Witmer, M. Am. Soc. C. E., was published in June, 1941, *Proceedings*.

[12] Cons. Engr., The Canadian Bridge Co., Ltd., Walkerville, Ont., Canada.

[12a] Received by the Secretary July 28, 1941.

[2] "Sixth Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, on Wind Bracing in Steel Buildings," *Proceedings*, Am. Soc. C. E., June, 1939, p. 975.

of the slide rule for C_O, dividing by 102.7; C_1 loads may be found from these, also by one setting. Moments in G_O may then be found by multiplying successive differences by the half girder length, and moments in G_1 taken in ratio to these.

The moment in G_O and C_O in the tenth floor may be approximated, on the high side, by taking the shear in C_O as $10 \times \frac{1}{6.18} = 1.62$, and multiplying by the half height or half bay 10, giving 16,200 ft-lb. Unfortunately such a procedure gives results increasingly too great as one goes down the column.

Among the "exact" methods is that of Georg Unold,[13] including wind and vertical loads in one equation per joint. It is an end-tangent method, of course, after the Mohr-Greene, elastic-weight, area-moment, graphical integration devices, as used sometime in the last century by the late Prof. C. E. Greene, M. Am. Soc. C. E., in an undated pamphlet entitled "Partially Braced Frames," given to the writer by his son about 1903. A line in this pamphlet indicates that Professor Greene regarded area moments as graphical integration. Professor Mohr introduced angle changes as unknowns in 1892.

Professor Unold suggests that in many cases the stiffness of the end walls may be such as to justify disregarding sway in interior bents, in view of floor stiffness. It is interesting to speculate as to whether it is better to make end walls adequate, or to make the interior bents adequate but capable of acting effectively only after the walls are shattered.

It would be of interest if Mr. Witmer would give an exposition of his favorite method of calculating sway in his closure.

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[13] "Die Praktische Berechnung der Stahlskelettrahmen," by Georg Unold, W. Ernst und Sohn, Berlin, 1933.

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DISCUSSIONS

PILE-DRIVING FORMULAS

PROGRESS REPORT OF THE COMMITTEE ON THE BEARING VALUE OF PILE FOUNDATIONS

Discussion

By Messrs. G. G. Greulich, C. O. Emerson and D. O. Northrup, Harry J. Engel, and John D. Watson

G. G. Greulich,[6] Assoc. M. Am. Soc. C. E.[6a]—With reference to Report *A*, observations for a period of more than twenty years indicate that it does not make much difference what formula is used so long as all data are observed carefully during the driving of test piles; and, after loading them to failure, the engineer then puts suitable factors in whatever formula is chosen so that it is correlated to the test results.

After a study of many driving records and tests, the writer stressed this principle early in 1935,[7] as follows:

> "Reliable values for a given site and total length of pile, as well as final penetration, can only be computed by first driving test piles and determining their actual capacities by means of loading tests. While the amount of penetration of a steel pile may be exactly determined, there are so many conditions that modify the rate of penetration and so many varying conditions of driving, and of soil, that it is virtually impossible to formulate any rule that can be considered entirely satisfactory for all of the essential conditions under which such piles are driven, unless, as stated before, the formulas are checked through actual load tests. This is also true with respect to any type of pile made of any kind of material."

A review of many additional tests during the six years from 1935 to 1941 has brought to light no information that would justify any change in the quoted statement.

The use of formulas, without a thorough knowledge of all factors at the site that might influence pile behavior and without check tests, may lead to

Note.—This Report was published in May, 1941, *Proceedings*.

[6] Sales Representative, Specialty Sales Div., Carnegie-Illinois Steel Corp., Pittsburgh, Pa.

[6a] Received by the Secretary July 14, 1941.

[7] "Steel Bearing Piles," Carnegie Steel Co., Pittsburgh, Pa., July 1, 1935; also, Illinois Steel Co., Chicago, Ill.

serious error—either by an unsafe or a very uneconomical and extravagant design.

For comparison, ultimate loads were computed by six formulas and compared with actual test loads on timber, concrete, and steel H-piles. Using the notation of the Report, the Hiley formula (Eq. 4), as simplified by the Joint Committee,[8] in Paragraph A-8 (but retaining the constant C and keeping constant C_1 in terms of R_d) is:

$$R_d = \frac{A\,E}{L}\left[-(s + C) \pm \sqrt{(s + C)^2 + \frac{2\,L}{A\,E}\,e\,w\,h \times \frac{W + n^2\,P}{W + P}}\right] \qquad (22)$$

in which the coefficient $C = 0.05$; $e = 0.75$ for a drop hammer and 0.9 for a steam hammer; and $n^2 = 0.25$ for steel and 0 for concrete and timber.

The "Engineering News" formula (Eq. 5)[9] is, for a steam hammer:

$$R = \frac{2\,W\,h}{s + 0.1} \qquad (23a)$$

and, for a drop hammer (see Eq. 6),

$$R = \frac{2\,W\,h}{s + 1} \qquad (23b)$$

A modification of the "Engineering News" formula is

$$R = \frac{2\,W\,h}{s + 0.1\,\frac{P}{W}} \qquad (24)$$

in which $\frac{P}{W} > 1$; F and W are in tons; h is in feet; and s is in inches. Ultimate loads are six times greater than the allowable loads given by Eqs. 23 and 24.

The Redtenbacher formula has been presented in the Report as Eq. 19. The so-called Pacific Coast formula developed by Fredrick J. Converse, Assoc. M. Am. Soc. C. E.,[10] and the one by Karl Terzaghi,[11] M. Am. Soc. C. E., reduce to the type of Eq. 19:

Pacific Coast Formula.—

$$R_d = \frac{A\,E}{2\,L}\left[-s \pm \sqrt{s^2 + 4\,W\,h\,\frac{W + n^2\,P}{W + P} \times \frac{L}{A\,E}}\right] \qquad (25)$$

in which $n = 0.5$ for steel piles and $\sqrt{0.1}$ for concrete and timber piles.

Terzaghi Formula.—

$$R_d = \frac{A\,E}{L}\left[-s \pm \sqrt{s^2 + 2\,W\,h\,\frac{W + P\,n^2}{W + P} \times \frac{L}{A\,E}}\right] \qquad (26)$$

in which n^2 may be 0.25 or 0.1.

[8] *Proceedings*, Am. Soc. C. E., May, 1941, p. 857.

[9] *Engineering News*, December 29, 1888.

[10] *Journal*, Boston Soc. of Civ. Engrs., Vol. XXIV, No. 2, April, 1937, pp. 102 and 103.

[11] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 281.

The comparisons shown in Table 2 were from selected records that had the most detailed test data to support them and in which, apparently, the greatest care was taken in making the tests. The selection was made at random, and no attempt was made to prove anything in particular; however, general study

TABLE 2.—ULTIMATE LOADS BY VARIOUS FORMULAS COMPARED WITH ACTUAL TEST LOADS AT FAILURE, IN TONS

Loads at failure[a]	Eq. 22		Eq. 23		Eq. 24		Eq. 19		Eq. 25		Eq. 26			
											$n^2 = 0.25$		$n^2 = 0.1$	
	Load	%[b]	Load	%[b]	Load	%[b]	Load	%[b]	Load	%[b]	Load	%[b]	Load	%[b]
(*a*) TIMBER PILES (SINGLE-ACTING HAMMER AND COHESIVE SOIL)														
93.0	67.0	72	270.0	290	....	..	108.0	116	82.5	89	113.0	122	109.0	117
81.0	131.5	162	169.8	210	....	..	104.0	128	85.7	106	109.0	135	106.0	131
70.0	37.0	53	75.0	107	....	..	43.5	62	40.0	57	50.3	72	46.0	66
66.5	58.0	87	106.0	159	....	..	62.0	93	60.0	90	73.0	109	69.0	103
96.5	41.0	42	88.0	91	....	..	47.0	49	44.4	46	54.3	56	50.0	52
100.0	68.5	68	180.0	180	....	..	73.0	73	60.8	61	81.8	82	77.5	77
110.0	29.0	26	56.0	51	....	..	33.5	30	32.8	30	38.4	35	35.5	32
70.0	37.0	53	75.0	107	....	..	43.5	62	40.0	57	50.3	72	46.0	66
(*b*) CONCRETE PILES (SINGLE-ACTING HAMMER AND COHESIVE SOIL)														
153.0	124.0	81	648.0	423	375.0	245	155.0	101	161.0	105	247.5	162	195.0	127
172.0	122.5	71	480.0	279	357.6	207	151.5	88	160.5	93	228.0	132	182.0	106
234.0	189.5	81	1,482.0	630	507.0	216	240.0	103	211.5	90	348.0	148	287.5	123
87.5	75.5	86	....	..	214.5	244	100.0	114	120.0	137	191.0	218	138.0	157
127.5	96.0	75	....	..	171.0	134	130.0	102	130.0	102	238.5	187	179.0	141
117.5	229.5	195	....	..	399.0	339	184.0	156	176.0	150	273.0	232	222.5	189
117.5	60.5	51	....	..	208.5	178	71.5	61	89.5	76	127.5	108	91.0	77
117.5	67.5	57	....	..	228.0	194	84.0	71	103.5	88	152.5	130	111.0	94
210.0	118.0	56	....	..	252.0	120	153.5	73	150.0	71	266.0	127	205.0	98
210.0+	153.0	73	....	..	330.0	157	200.0	95	183.0	87	315.0	150	252.5	120
210.0	145.0	69	....	..	306.0	146	188.5	90	175.0	83	304.0	145	241.0	115
(*c*) STEEL H-BEAM PILES (VARIOUS HAMMERS AND SOIL TYPES)														
203[c,d,e]	149.5	73	375.0	184	....	..	154.0	76	139.0	68	180.5	89	165.0	81
105[c,d]	81.0	77	222.0	211	192.0	183	70.5	67	69.5	66	88.0	84	76.5	73
100[c,d]	40.5	40	45.5	45	....	..	41.0	41	44.0	44	49.0	49	44.5	44
50[c,d]	43.5	87	69.6	139	....	..	45.6	91	48.0	96	51.6	103	48.3	97
150[c,d]	127.5	85	....	..	327.0	218	121.0	81	111.0	74	151.0	100	127.5	85
80+[c,f]	78.0	98	129.0	161	....	..	82.0	103	81.0	101	89.0	111	85.0	106
70[d,g]	44.0	63	85.8	122	79.2	113	41.5	59	48.3	69	54.0	77	46.8	67
50[d,g]	44.5	89	72.0	144	....	..	46.0	92	49.4	99	53.1	106	49.3	99
40[d,g]	78.5	196	127.2	318	....	..	78.0	195	76.5	191	89.0	222	83.0	207
81[f,h]	116.5	144	90.0	111	....	..	151.0	186	135.9	167	159.5	196	255.0	315
69[f,h]	110.5	160	166.8	241	....	..	241.5	350	236.0	341	255.0	370	246.5	357

[a] A load at failure is assumed to be the load that produced an increase in settlement disproportionate to the increase in pile load. [b] Percentage of test load at failure. [c] Single-acting hammer. [d] Cohesive soil. [e] Not the maximum load. [f] Non-cohesive soil. [g] Double-acting hammer. [h] Drop hammer.

of at least six times as many cases indicates equally erratic comparison between driving-formula values and test results. It may be asked: "What influences did the various kinds, layers, or strata and moisture content of the soils have on the test and formula results?" These influences are present and are important in each case; however, since there is nothing in the formulas to compensate for these factors except the factor C_2, which is only partly affected by

the elastic compression of the soil, detailed soil data have not been shown in Table 2. The Committee suggests neglecting factor C_2 in Paragraph A-8(c). In certain swampy or alluvial areas it is very important and the writer recommends its field determination, as discussed subsequently.

Reference to the percentage columns of Table 2 shows a wide range of formula values, as compared to test results. It should be obvious that a blind use of any formula would eventually lead to trouble, as very few of the formula loads are close to the desirable 100% of the test loads.

In examining Table 2, it should be borne in mind that the well-known "Engineering News" formulas (Eqs. 23 and 24) are used with a factor of safety of six, whereas other formulas are used with a factor of safety of three to four. For practical purposes, therefore, the ultimate values using Eqs. 23 and 24, although often far greater than test results, have proved satisfactory, due to the extremely high assumed factor of safety.

In Table 2 there are no examples of timber or concrete piles driven in non-cohesive soils, such as sands and gravels, because such piles very seldom penetrate these classes of soils by straight driving—jetting is usually required. Where straight driving is resorted to, the penetration per blow is generally so small that formula values ordinarily are not applicable. Only one example of a steel H-pile driven with a single-acting hammer, and two examples of driving with a drop hammer, are given for non-cohesive soils (Table 2(c)). Although straight driving of steel piles into non-cohesive soils is common practice, and there are numerous examples, the final penetration per blow is usually very small and formula values ordinarily are not applicable. Where any type of pile is driven to refusal on or into very hard soils, the ultimate load on the pile is naturally limited by its nominal capacity as a column.

Paragraph A-8(a).—It is not believed that the extreme case of end bearing should ever be used with a friction pile (see Paragraph A-8(a)). As stated elsewhere, the penetrations per blow on end-bearing piles are usually so small that it is questionable if any formula should be applied. The assumption for determining C_1 in Eq. 10 should be based on the pile having zero load at the point and the total at the top, so that the average intensity of strain is half of that of an end-bearing pile. The assumption then becomes

$$C_1 = \frac{3\,R\,L}{4\,E\,A} \qquad (27)$$

Instead of computing C_1, it can be determined easily by field recording, in combination with C_2.

Paragraph A-8(b).—The assumption that $C = 0$ should be made in the case of a hammer striking a steel anvil, placed directly on a steel pile.

Paragraph A-8(c).—The value of C_2 is easily determined in the field and should not be neglected. The value of $2\,C_1 + 2\,C_2$ can readily be obtained by moving a pencil or stylus across a piece of paper suitably attached to the pile as it is driven.[12]

[12] "The Resistance of Piles to Penetration," by Russell V. Allin, London, 1935, p. 14.

Curves are available[13] for a 12-in., 53-lb steel H-pile at a depth of 33.1 ft and a 16-in. octagonal pre-cast concrete pile weighing 244 lb per ft at a depth of 27.4 ft, both driven with a single-acting steam hammer delivering energy of 15 kip-ft per blow. The steel pile top moved down under a blow a total penetration of $\frac{7}{16}$ in.; then it rebounded $\frac{7}{32}$ in., leaving $\frac{7}{32}$ in. as the permanent penetration. The concrete pile had a total penetration of $\frac{13}{32}$ in., a rebound of $\frac{5}{32}$ in., and a permanent penetration of $\frac{1}{4}$ in. Inasmuch as the rebound is also a measure of the compression under the blow, twice the rebound approximates $2\,C_1 + 2\,C_2$, or the rebound equals $C_1 + C_2$.

Paragraph A-13.—A static formula is sometimes satisfactory in thick layers of uniform material. In stratified material the various layers may pick up loads of varying intensities in each layer and the intensities are further dependent on the amount of vertical movement or settlement of pile. In order for a friction pile to pick up load, there must be some relative movement between the pile and the soil. A certain movement in one soil may develop the maximum friction value, whereas in the adjoining layer the value may increase or decrease with the same amount of movement. For these reasons, the development of a general static formula for mixed deposits of soils is probably remote.

Paragraph A-14.—The data in Table 3 are intended to supplement Table 1.

TABLE 3.—VALUES OF f SUPPLEMENTING TABLE 1

Location	Soil	Penetration, in ft	Friction, f, in lb per sq ft
Monroe, La.	Sand and clay	49	525 to 681
Little Rock, Mo.[a]	Sand and clay	38	820 to 840
Little Rock, Mo.[a]	Sand and clay	53	690
Passaic River Bridge, N. J.	Cinders, swamp mud, soft clay and sand, and stiff red clay	52	494 to 575
Bonne Carre Spillway, La.	Mostly sticky stiff blue clay with thin layers of humus, sand, and shells	89 122	331 to 413 448

[a] Note decrease in f as depth increases from 38 ft to 53 ft.

Paragraphs A-20 and B-14.—Although pig iron is frequently used as a test load, it should not be used when other materials such as steel slabs, billets, sand, brick, concrete block, etc., are available. When stacked high, pig iron is likely to shift suddenly, and a mass of pigs moving in all directions is very dangerous to any one on or near the test platform.

General.—The writer would be opposed to the publication of any formula, unless the dangers and pitfalls of its use are made very clear.

It is suggested that paragraphs *A*-4 to *A*-14 inclusive be placed in an appendix and that the remainder of Report *A* be combined with, and added to, Report *B*. Any duplicate material, of course, should be eliminated. The appendix should also show other widely used formulas, together with brief statements as to their history, use, and limitations. If these discussions reveal additional suitable driving and test data, it may be possible to compile and in-

[13] "Report on Local Protection Project," War Dept., Corps of Engrs., U. S. Army Engr. Office, Cincinnati, Ohio, April, 1941, Plate 3.

clude a comprehensive tabulation showing comparisons between various formulas.

C. O. Emerson,[14] Esq., and D. O. Northrup,[15] Esq.[15a]—Report *A* of this Progress Report deserves comment, especially with reference to the several simplifications of the basic formula set forth.

In Eq. 11 R is an allowable load and is equal to $\frac{R_d}{3}$, compression in the driving cap is assumed to be 0.05 in., end bearing of the pile is assumed, and elastic compression in the soil is neglected. Because of extreme difficulty in determining compression in the soil accurately and the fact that pure end bearing is seldom, if ever, obtained, the latter two simplifying assumptions seem to be practicable. What their relative effect would be on the resulting calculated bearing value, the writers are not prepared to state. Constant $C = 0.05$ in. probably should be eliminated if cushion blocks are not used. Conceivably this constant could be much larger than 0.05 in. under hard driving. If it were eliminated, the resulting formula would be exactly the same as the Terzaghi formula,[16]

$$R_d\, s + \frac{R^2\, dL}{2\, A\, E} = W\, h \left(\frac{W + n^2\, P}{W + P} \right) \qquad \text{(28a)}$$

This reduces to

$$R_d = \frac{W\, h}{s + \frac{R\, dL}{2\, A\, E}} \times \frac{W + n^2\, P}{W + P} \qquad \text{(28b)}$$

If constant C is retained, and if constant C_1 is kept in terms of R_d, the quadratic equation

$$R_d = \frac{A\, E}{L} \left[-\,(s + C) \pm \sqrt{(s + C)^2 + e\, W\, h \times \frac{W + n^2\, P}{W + P} \times \frac{2\, L}{A\, E}} \right] \qquad \text{(29)}$$

results, in which R_d is computed directly. The writers believe that this form of equation probably will furnish the most convenient solution, although it is much too complicated (especially for the average engineer on small projects) to create much enthusiasm.

In Eqs. 7, 8, and 9, the coefficient of restitution is eliminated, and the writers agree that with timber and concrete piles this coefficient will be small enough so that its omission will make no practical difference. However, with steel piles, if $n = 0.5$ and $n^2 = 0.25$, this coefficient should be retained. The Redtenbacher formula (Eq. 19) is the same as the Terzaghi formula (Eq. 28), except that the coefficient of restitution is considered equal to zero and gives consistently lower values of pile capacity.

As stated in Paragraph *A-9*, constant h_0 is the greatest hammer drop that will result in zero penetration. Although this constant could be determined

[14] Specialty Sales Div., Carnegie-Illinois Steel Corp., Pittsburgh, Pa.

[15] Specialty Sales Div., Carnegie-Illinois Steel Corp., Pittsburgh, Pa.

[15a] Received by the Secretary July 14, 1941.

[16] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 281.

easily if a drop hammer is used, the writers do not see how it may be determined readily when either a single-acting or double-acting steam hammer is used. Obviously, it should be determined through the use of the same hammer with which the pile will be driven to final set, and this would be difficult on most of the important jobs where steam hammers are the rule rather than the exception. The writers believe that the construction of single-acting hammers is such that absolute control of the drop is impossible. When using a double-acting or differential-acting hammer, the desired control of energy might be effected through regulation of steam pressure if the pressure in the hammer could be gaged accurately. An interpolated energy equation based on number of blows per minute would not be sufficiently accurate unless the hammer were calibrated to a very slow speed. The manufacturers' catalogs tabulate calibrated energy outputs over only a small range of speeds very near to the rated speed of the hammer. Another factor that would influence results in any attempt to determine h_0 for a double-acting hammer is that rapidity of blows conceivably could have considerable effect on penetration of the pile. In some soils rapid blows might result in more penetration per blow by eliminating a tendency of the soil to set, whereas in other soils rapid blows might result in less penetration per blow, as in dense water-bearing sands.

Referring to Paragraph *A-10*, more than four piles should be tested, and in more than one location, before reaching the conclusion that Eq. 11 will be conservative in all cases. If more field determinations of h_0 have been made, they should be mentioned here. In applying Eq. 4 to various piles on which both driving records and load tests to failure are available, the writers have found that results are not always conservative; the k-constant used in Eq. 4 was that derived in Eq. 11.

TABLE 4.—COMPARATIVE VARIATIONS IN COMPUTED ULTIMATE LOADS (IN TONS), DUE TO CONSTANTS

Eq.	Loads at Failure									
	80	170	100	60	40	60	203	105	70	150
19	40	81	39	45	76	43	154	70	42	86
29[a]	43	73	40	44	73	43	150	81	44	77
30[b]	53	87	49	52	87	56	180	88	54	92

[a] For $C = 0.05$, $n = 0.5$, and $e = 0.9$. [b] For $n = 0.5$

Table 4 is offered to show variations in computed ultimate loads on bearing piles in the hope of demonstrating the influence of the various constants under discussion. The piles listed are all steel H-beam bearing piles, except two, which are fluted steel pile shells; all were driven by steam hammers to varying driving resistances and in various soils. All were then load-tested to failure. In order to draw the proper inferences from the results in Table 4, it must be kept in mind that the Terzaghi formula

$$R_d = \frac{A\,E}{L}\left[-s \pm \sqrt{s^2 + W\,h\,\frac{W + n^2 P}{W + P} \times \frac{2\,L}{A\,E}}\right] \quad\ldots\ldots (30)$$

differs from the Committee's proposed formula (Eq. 29) in that constant C is eliminated and the efficiency of the hammer is considered to be unity; the Redtenbacher formula (Eq. 19) further differs in that the coefficient of restitution is taken to be zero. Results given in Table 4 indicate to the writers that the influence of the constant C is much larger than that of the coefficient of restitution in most cases. Therefore, it will be important to drop this constant if no cushion blocks are to be used or to have some idea of its actual value in any case. These examples are scattered cases and were not chosen to show the superiority of any one pile formula.

HARRY J. ENGEL,[17] ASSOC. M. AM. SOC. C. E.[17a]—The Committee has commendably performed its task of assembling information from which to prepare a Manual of Engineering Practice, and has been careful to stress (as is proper) the risks involved in the use of any pile-driving formulas. The following comments will emphasize these risks in so far as they apply to "friction" piles, supported only by friction or adhesion of soil to their side surfaces.

The delta deposits of the State of Louisiana offer fertile ground for studies of such friction piles, because many structures in that state are necessarily supported on friction piles of timber embedded in deposits of clay and silt. Some of the approach footings of the New Orleans Bridge over the Mississippi River are founded on friction piles so embedded, and driving records of typical friction piles from these approaches are summarized in Table 5.

Friction piles supported in silty or clayey soils have a tendency to "freeze" fast during any lag in driving, clearly indicating, as the Committee states, that the bearing value determined by any dynamic formula is the value only at the time that the data were obtained. Such a phenomenon is indicated in the cases of test pile 42 of the East Approach and test pile 229 of the West Approach. In the first of these instances (Table 5(a)) a rest of 13 hr 25 min during driving increased the number of blows per foot from three before the rest to forty after the rest; and in the second instance (Table 5(b)) a rest of 16 hr 30 min increased the number of blows per foot from eight before the rest to twenty eight after the rest. Any dynamic formula would assign totally different allowable loads to these piles before and after their rest periods, and it would seem the wisest course, therefore, to use no dynamic formula for friction piles.

Nevertheless, Table 5 gives allowable loads R for the various piles based, first, on the Committee's version of the "Engineering News" formula, Eq. 6; and second, on the more usual version of the "Engineering News" formula for single-acting steam hammers

$$R = \frac{2\,W\,H}{S + 0.1} \qquad (31)$$

The Committee failed to differentiate between steam and drop hammers in presenting Eqs. 5 and 6. If the "Engineering News" formula is to be given at all, it should be given in both forms, Eqs. 6 and 31.

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[17a] Received by the Secretary July 17, 1941.

In connection with the formulas developed by the Committee (Eq. 16 and 17), there would seem to be no practical way of finding the value h_0, the minimum height of fall which is necessary to cause the pile to move, when single-acting steam hammers are employed. It would be inconvenient to use first a drop hammer to determine h_0 and then the steam hammer to conduct

TABLE 5.—Typical Driving Records of Timber Friction Piles; New Orleans (La.) Bridge Approaches

(Single-Acting Steam Hammer h = 3 Ft; W = 5,000 Lb; E = 1,600,000 Lb per Sq In.; and A = 95 Sq In.)[a]

Bent No.	Test Pile No.	Soil at the tip of the pile	L_s	A_s	F	N	P	s	Allowable Load	
									R_s	R_U
		(a) East Approach								
39	22	Clay and fine sand	80.4	232	87	12	2,250	1.00	7.5	13.6
41	31	Clay and fine sand	80.5	233	87	11	2,260	1.09	7.2	12.6
46	42[c]	....	77.8	225	84	16	2,180	0.75	8.6	17.6
51	52	Clay and fine sand	79.8	231	86	10	2,230	1.20	6.8	11.5
70	91	Clay	65.2	188	70	13	1,830	0.93	7.8	14.5
102	158	Fine sand, clay, and shells	62.6	181	68	7	1,760	1.71	5.5	8.3
...[b]	235	Gumbo and fine sand	65.4	189	71	9	1,835	1.33	6.4	10.5
		(b) West Approach								
1	3	....	56.8	164	61	6	1,590	2.00	5.0	7.15
129	187	Fine sand and clay	64.8	187	70	6	1,820	2.00	5.0	7.15
151	228	Clay, shells, and silt	79.0	228	85	8	2,210	1.50	6.0	9.35
152	229[d]	Clay, sand and shells	75.0	217	81	13	2,100	0.92	7.8	14.7
...[b]	276	Clay and streaks of sand	79.0	228	85	11	2,210	1.09	7.2	12.6

[a] In addition to the notation of the report: L_s = embedded length of pile, in feet; A_s = approximate side surface, in square feet; F = probable failure load by skin friction failure (tons); N = number of blows in last foot of penetration; R_s = allowable load, in tons, by Committee's version of "Engineering News" formula; and R_U = allowable load, in tons, by the usual "Engineering News" formula for single-acting steam hammers.

[b] Abutment.

[c] 3 blows per ft before 13 hr 25 min rest period; 40 blows per ft afterward.

[d] 8 blows per ft before 16 hr 30 min rest period; 28 blows per ft afterward.

the test. Because no value of h_0 is available from the New Orleans Bridge pile tests, no allowable load is shown in the table based on the Committee's simplification of Hiley's formula. None of the results of such dynamic formulas are believed to be applicable to these piles, however, as has been already stated.

Some form of static formula, on the other hand, seems more likely to evaluate the true carrying capacity of friction piles, particularly if it is based on static loading tests carried to failure. The Committee recommends such a static formula (Eq. 21) in which 3 is the factor of safety, A_s is the area of the friction surface, and f is the skin friction at failure determined by test. Although no loading tests to complete failure were made on the approach piles of the New Orleans Bridge, experience in that locality indicates that from 600 to 800 lb per sq ft is a fair value of skin friction at failure along the side surfaces of the piles. For comparison with the results of the dynamic formulas, Table 5

shows a probable failure load in skin friction for the single piles based on an assumed skin friction value at failure of 750 lb per sq ft.

Supposing that static tests of long duration were made to failure, on single piles, to determine the actual skin friction, it would be necessary to evaluate grouping action in some way to determine the failure load of a pile acting as part of a group because, in friction piles particularly, the effect of grouping in reducing strength is very marked. Caution should also be exercised in interchanging skin friction for tapered piles with such values for parallel-sided piles —experience has shown that tapered piles usually have a higher value of skin friction at failure than do parallel-sided piles.

If reasonably accurate failure loads could be determined for a single pile or pile group, it would remain necessary to assign some factor of safety to determine safe working loads. That some confusion exists on what should be the proper value of such a safety factor is evident from the fact that the "Engineering News" formula incorporates 6, and the Committee recommends 3 in its own formulas. A safety factor of 3 seems reasonable and safe if the failure loads per pile are known with sufficient accuracy and the effect of grouping has been taken into account. It is probably because of the wide approximations of the "Engineering News" formula that so large a safety factor is necessarily incorporated in it.

In testing piles, the Committee suggests the use of the rule that a total net settlement of 0.01 in. for each ton of applied load may be deemed satisfactory in some cases. This rule may give satisfactory results when applied to point-bearing piles where grouping and skin friction play a relatively small part; but with friction piles all load tests should be made to complete failure to determine the ultimate skin friction that may be used in the manner already suggested. Complete failure would be represented by "plunging" of the pile or group, or at least by a sharp increase in settlement which continues with time.

JOHN D. WATSON,[18] ASSOC. M. AM. SOC. C. E.[18a]—The writer wishes to commend those members of the Committee on the Bearing Value of Pile Foundations who prepared "Report *B*.—Pile Formulas and Pile Tests." Report *B* recommends nothing except load testing of piles to failure. This is as it should be, since no two foundations are ever sufficiently alike to warrant a handbook design. Engineers engaged in hydrological work have learned long ago that hydrological problems cannot be solved successfully with formulas, even though the formulas are bolstered up with all manner of limiting assumptions and empirical coefficients. It is high time that foundation engineers learned the same lesson. Rainfall and soils: Neither is man-made, and in its own way one is as diverse as the other.

The writer wishes further to deplore the moribund attitude that prompted other members of the Committee to prepare "Report *A*.—Pile Formulas." Although they may fervently wish to have a formula for the ready solution of

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[18a] Received by the Secretary July 24, 1941.

their problems, they should not ask the Society to fulfil their prayer by promulgating a Committee formula, unless they can prove their case in court. The most complete and comprehensive series of pile-loading tests ever performed was that made by the U. S. Engineer Office of Los Angeles, Calif., at the site of Sepulveda Dam in California. The report covering these tests is dated April, 1940, and it states categorically that "no dependable correlation has been found between driving resistances and static safe loads."

Correction: In line 6 following Eq. 18 change "Eq. 18" to "it" (meaning Eq. 11).

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DISCUSSIONS

EXPANSION OF CONCRETE THROUGH REACTION BETWEEN CEMENT AND AGGREGATE

Discussion

BY THOMAS E. STANTON, M. AM. SOC. C. E.

THOMAS E. STANTON,[27] M. AM. SOC. C. E. (by letter).[27a]—Considerable "water has gone over the dam" since this paper was first published in December, 1940. The presentation and data as of that date were based on laboratory tests and field studies conducted during a period of approximately eighteen months, an entirely too short time for the development of much of the data that have subsequently come to light (as of August, 1941). The subsequent developments, however, do not change the original conclusions in any material respect, but have served, rather, to confirm them and, in addition, indicate the possible answer to several as yet unsolved or at least only partly answered problems.

In closing the discussion on this important topic, the writer desires to express his appreciation to those who have so kindly participated, and likewise to repeat his grateful acknowledgment of the assistance of the members of his staff, without which most of the valuable data presented would have been unavailable. Many valuable suggestions have been made and it is hoped that the information presented in the paper and in the subsequent discussion will be of value in either solving some important problems relative to Portland cement concrete or pointing the way to further studies.

Parker Dam.—Interest in the subject has been accentuated by the findings of the U. S. Bureau of Reclamation that the severe cracking that developed in 1940 in parts of the Parker Dam on the Colorado River can be traced to a reaction between the cement and aggregate. The story of the Parker Dam is treated fully in the discussion by Mr. Blanks.[19,20] The Bureau has since

NOTE.—This paper by Thomas E. Stanton, M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by R. W. Carlson, Assoc. M. Am. Soc. C. E.; March, 1941, by Bailey Tremper, Esq.; April, 1941, by Messrs. Hubert Woods, and N. T. Stadtfeld; May, 1941, by Messrs. W. C. Hanna, J. C. Witt, and R. F. Blanks; and June, 1941, by Messrs. J. MacNeil Turnbull, and Robert A. Kinzie, Jr.

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[27a] Received by the Secretary August 11, 1941.

[19] "Concrete Deterioration at Parker Dam," by R. F. Blanks, *Engineering News-Record*, March 27, 1941, p. 46.

[20] "Cracking in Concrete Due to Expansive Reaction Between Aggregate and High-alkali Cement as Evidenced in Parker Dam," by H. S. Meissner, Assoc. M. Am. Soc. C. E., *Journal*, A. C. I., April, 1941.

started a very extensive and thorough nationwide investigation in cooperation with cement manufacturers.

A. S. T. M. Activities.—P. H. Bates, chairman of Committee C-1 of the American Society for Testing Materials (A. S. T. M.), has appointed a working committee on the "Effects of Alkalies in Portland Cement on the Durability of Concrete," of which Mr. Blanks is chairman and the writer a member.

Further Bradley Pavement Developments.—It will be recalled that the early (1.5 years) failure of the pavement north of Bradley, Monterey County, Calif., was confined to the sections in which the local (Oro Fino) sand had been used, and that the other sections in which Oro Fino coarse aggregate and imported (Coyote) sand had been used had, to the date of the original report, failed to show appreciable distress.

Two years later (spring, 1941) the Oro Fino coarse aggregate-Coyote fine aggregate sections had begun to show some distress, thereby indicating that under conditions similar to the California experience the reactive particles in the coarse aggregate may be expected to cause trouble ultimately, although apparently at a somewhat slower rate, and possibly to an ultimate less degree, than fine aggregate of a similar composition. This conclusion is checked by subsequent laboratory tests on coarse and fine aggregate containing reactive particles.

Reactive Properties of Opal.—The probable rôle of the hydrous silicates was fully discussed in the paper. It was pointed out that a sodium silicate gel was at the base of most, if not all, of the popouts and was also found throughout broken sections of concrete as well as exuding on the surface through pores and cracks.

However, in the early tests, specimens containing 10% of either opal or the highly opalized chert No. 28038 had failed to develop excessive expansion, whereas excess expansion was invariably encountered with the same percentage of the siliceous magnesian limestone No. 28039. This experience led to the effort to develop a formula based on a possible reaction between magnesium carbonate and sodium hydroxide.

The next step was to make chemical and petrographic analyses of the siliceous magnesian limestone No. 28039 to determine its mineralogical composition and then to make up expansion bars of each mineral component singly and in combination in the relative percentages found in nature. Table 11 shows the results of this study.

TABLE 11.—Mineral Composition and Reaction Characteristics of Different Minerals Similar to the Constituents of Rock No. 28039

Mineral	Approximate percentage in rock No. 28039	Percentage in bar	Age in months	Expansion units
Chalcedony	15	10	12	60
Opal Type 1	12	2.5	6	9,920
Opal Type 2	1	10.0	10	11,490
Limonite	1	5	10	30
Glauconite	1	5	6	10
Dolomite	10	10	6	10
Calcite	60	10	12	50
Combined[a]	100	10	12	7,470

[a] Sample including all minerals.

Although the combined synthetic sample was made up of the relative percentages found in No. 28039 and 10% of this combination added to a

neutral sand, the tests on each mineral separately were not at the relative percentages found in the original rock but in some cases in higher percentages, the main purpose of the tests being to reveal the probable reaction characteristics of each mineral when present in a sufficiently large percentage.

At the same time, however, excessive expansion has been observed even with much smaller percentages: 1% of Type 2 opal expanded 1,310 units in eight months and 5% of the same mineral expanded 10,250 units, or 1.025%, in the same period of time. From this study it was determined that in the California experience the opal constituent was at the base of most, if not all, of the trouble, and, therefore, the original study of the rôle of soluble silica in the form of opal was resumed.

It was then found that the reason little, if any, reaction was originally developed with the highly opalized material was that the mixture had been overdosed. If the entire coarse or fine aggregate is composed of opaline particles, there is no undue expansion, but when present in what might be called a "pessimum" amount, which appears to be substantially less than 10% or 15% of the total aggregate, a high expansion may be expected in the presence of a high-alkali cement. Table 12 shows the expansion results with a number of rocks, all of which contain opal in varying amounts.

TABLE 12.—RELATIVE DEGREE OF EXPANSION INDUCED BY 2½%, 5%, AND 10% OF DIFFERENT REACTIVE ROCKS ADDED TO A NEUTRAL SAND-HIGH-ALKALI CEMENT MORTAR

(1-In. by 1-In. by 10-In. Mortar Specimens; Mix 1 : 2; Age at Time of Measurement, Six Months)

Percentage added to a neutral sand	Rock No.									
	19374	22100[a]	23806	28038	28039-A	28039-B	28045	28056	28057	Opal[b]
	Expansion in Millionths of an Inch per Inch									
2.5	+220	+12,580	+6,310	+7,470	+3,850	+2,910	+320	+11,040	+4,850	+9,920
5.0	+1,150	....	+5,520	+5,460	+5,390	+4,160	+730	+8,490	+4,460	+8,820
10.0	+180	+110	+3,650	+210	+8,420	+7,470	+260	+130	+1,910	+1,740

[a] Only a small quantity of this rock available at time of fabrication of specimens. Analysis shows it to be similar to No. 28038. [b] Opal from dredger dumps on American River near Auburn, Calif.

In the case of the California aggregates the quantity of deleterious particles required for maximum expansion appears to be related both to the opal content and to the type of opal or hydrous silicate. There is definite evidence that opal with high water content is more soluble and reacts quicker than opal with low water content. Much work needs to be done along this line, however.

Further attention is called to the low percentage of opal required for maximum results, as well as the remarkably low percentage (in some cases less than 0.5%) which produces excessive expansion in a relatively short time (Table 13). This would appear to be conclusive evidence that, when expansion

does not occur in the field until after a year or more, the percentage of deleterious material may be as low as 0.75% and possibly even less.

Test results available by July, 1940, led to the conclusion that, since little, if any, measurable expansion was observed with deleterious particles below 80-mesh or 100-mesh size, the reason for the falling off in expansion as the

TABLE 13.—Showing Expansion Effect of Different Percentages of Two Highly Reactive Rocks [a]

(High-Alkali Cement *GS* Used Throughout; 1-In. by 1-In. by 10-In. Mortar Specimens; Mix 1 : 2; Age, Four Months)

Material	Percentage of Siliceous Material Added to a Neutral Sand									
	0.25%	0.5%	0.75%	1%	2%	3%	4%	5%	10%	15%
	Expansion in Millionths of an Inch per Inch									
Opal[b]	+70	+1,010	+3,400	+5,660	+9,400	+9,950	+7,750	+4,230	+390	+10
No. 28038[c]	+70	+ 110	+1,790	+2,380	+2,880	+5,080	+3,080	+ 240	+210	....

[a] Rock particles substituted in percentages shown for an equivalent percentage of neutral Russian River sand. [b] Opal secured from dredger dump on the American River near Sacramento, Calif. [c] Rock No. 28038 has been identified as one of the constituents of Oro Fino sand and was secured from ledge deposits found in Monterey County.

percentage of such particles was increased beyond a certain percentage might be an accelerated reaction between the cement and finely dispersed mineral particles.

Although the underlying cause may be that originally suspected, later tests show that neutralization also occurs in the presence of excess percentages of coarser particles from which all finer particles have been removed. From this it would appear that, for maximum reaction, there is a relation between the amount of reactive mineral in any unit area and the alkali in the cement, and that, when the area of such a mineral available for reaction with a given amount of alkali exceeds a certain amount, the magnitude of the reaction products at the locus of each particle may be insufficient to cause excessive expansion and rupture.

The power of the disruptive action is well illustrated in Fig. 16. In this case a mortar bar, 1 in. by 1 in. by 10 in., was fabricated from a high-alkali cement and an all limestone fine aggregate to determine the reaction, if any, between limestone and the cement. All specimens in this series showed no expansion except one which developed the cracks shown in Fig. 16. Upon breaking this specimen along the line of the transverse crack, it was found that a small piece of reactive mineral had inadvertently been included, and that, although this foreign mineral particle was only about 1.0% of the cross section of the 1-in. by 1-in. bar, the reaction had sufficient force to cause rupture.

Type of Mineral Aggregate.—In so far as the early reactions are concerned, the source of the California troubles was definitely defined as resulting from

some mineral in the 5% to 15% shale and chert portion of the aggregate, the bulk of the remaining 85% to 95% consisting of unreactive granitic rock, sandstone, quartzite, rhyolite, andesite, siltstone, and limestone.

The subsequent identification of the trouble with certain chert aggregates led to a suspicion in some quarters against all types of chert. It now appears

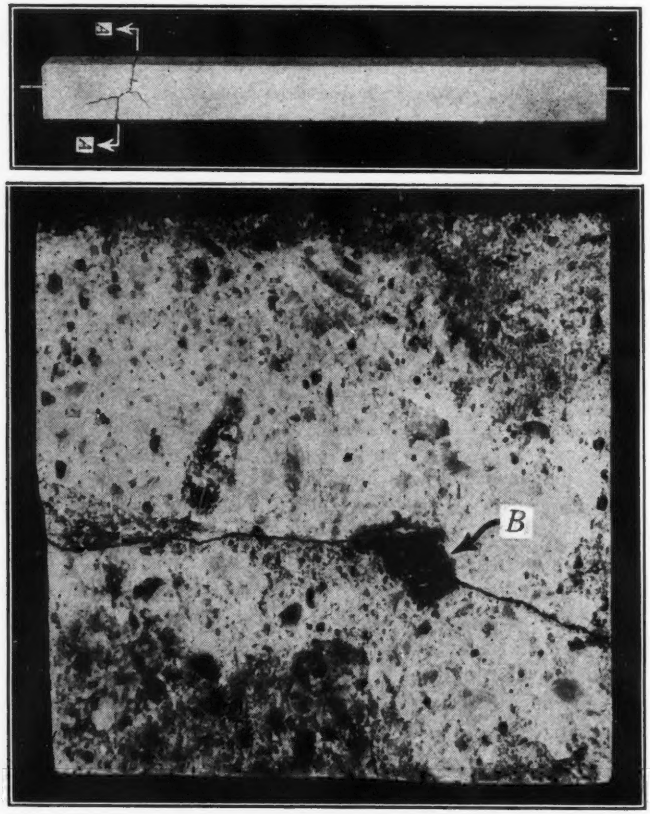


FIG. 16.—RUPTURING EFFECT OF A SMALL PARTICLE OF REACTIVE MINERAL ACCIDENTALLY INCLUDED IN A NEUTRAL LIMESTONE FINE AGGREGATE, HIGH-ALKALI CEMENT MORTAR (SECTION *A–A* SHOWS REACTIVE PARTICLE INCLUSION AT *B*)

probable that only those cherts which contain soluble silica of the opaline type are definitely reactive and that other cherts may be absolved of any suspicion in so far as any present evidence based on California experience is concerned. At the same time it has not yet been proved that all opaline cherts are equally reactive. It may be that only the highly soluble types are to be feared and then only when the percentage of solubility exceeds a certain amount. There is ample field for further investigation along this line.

Mr. Twenhoffel states[28] that "Opaline or amorphous silica is wanting or

[28] "Treatise on Sedimentation," by William Twenhoffel, 1932, p. 521.

extremely rare in Mesozoic and older flints and cherts, but may be found in those of the Tertiary and later."

Any type of aggregate that contains opal may be reactive. However, due to the prevalence of opaline cherts and cherty shales in the Tertiary formations west of the Rocky Mountains, the West is more likely to experience this difficulty than the East, so far as cherts in general are concerned.

It is evident, therefore, that all cherts, as such, should not be blacklisted, but that, before passing judgment against any particular chert, it is first in order to determine whether or not, and to what extent, opal is present. If an appreciable percentage of opal is observed, the mineral should be under suspicion until its reaction characteristics can be determined. The same procedure applies to any other rock that may be suspected of containing appreciable percentages of opal or any other form of readily soluble and, therefore, presumably reactive silica.

During the progress of this investigation the reaction characteristics of a number of minerals and rocks at 70° F were studied. A list of these minerals and rocks will be found in Table 14. Tests were made on mortar bars using the high-alkali cement, *GS*, combined with the percentages of mineral indicated in figures after the name. The material under investigation was added in the indicated percentages to a neutral sand.

The reactivity was determined by measuring the expansion characteristics of mortar bars of high-alkali cement, non-reactive sand, and variable percentages of materials under investigation. Test specimens were stored and tested at 70° F. As will be noted, chalcedony, as well as a

TABLE 14.—REACTIVE AND NON-REACTIVE MATERIALS

Identification	Percentages [a]	Fluorescence
(*a*) REACTIVE MINERALS		
Opal	25 to ¼, 2½ [b]	−
Selenite [c] (gypsum)	10	+
(*b*) REACTIVE ROCKS		
No. 19374 (Table 4)	10, 5, 2½ [b]	−
No. 28038 (Table 4)	10, 5, 2½ [b]	−
No. 28039 (Table 4)	100 to 0.1, 20 [b]	−
No. 28045 (Table 4)	10,[b] 5, 2½	−
No. 28046 (Table 4)	10,[b] 5, 2½	−
No. 30704 (Table 4)	25	−
No. 22100 (opaline chert)	10, 2½ [b]	+
No. 23806 (opaline chert)	10, 5,[b] 2½	−
No. 28056 (opaline chert)	10, 5, 2½ [b]	−
(*c*) NON-REACTIVE MINERALS		
Aragonite	10	+
Biotite	10, 5	−
Calcite	100, 10	+
Chalcedony	10, 5, 2½	+
Dolomite	10, 5, 2½	−
Gehlenite	10, 5, 2½	−
Glauconite	10, 5, 2½	−
Limonite	5, 2½	−
Magnesite	5	+
Natrolite	10	−
Pectolite	10	+
Prochlorite	10	−
Quartz	100	−
Sericite	10	−
Stellerite	100	−
Vermiculite	10	−
(*d*) NON-REACTIVE ROCKS		
Limestone	100	−
Quartz monzonite	100	−
Hornblende diorite	100	−
Amphibole schist	100	−
No. 19375, Table 4	10, 5, 2½	+
No. 22001, soft shale	10	−
No. 28042, soft shale	10, 5, 2½	−
No. 28044, soft shale	10, 5, 2½	−
No. 28053, sandstone	10	−
No. 28054, sandstone	10	−
No. 28055, sandstone	10	−
Serpentine	10, 5, 2½, 1¼	−

[a] Percentage substituted for an equivalent percentage of neutral sand.
[b] This percentage was found to be the most reactive.
[c] Expansion not believed to be due to cement-alkali reaction.

number of other minerals of the type tested, is not reactive, at least in a short period of time.

All fluorescence observations on minerals in Table 14 apply strictly to samples tested with equipment in the laboratory of the California Division of Highways. Other specimens of the same mineral species from different localities may show different fluorescence results. In all of the California series, storage and tests were at a normal laboratory temperature of approximately 70° F. Prof. Roy F. Carlson and others have been able to get expansion at 110° F and 150° F not obtainable at 70° F.

TABLE 15.—Expansion or Shrinkage of Concrete Specimens of Oro Fino and Coyote Aggregates Cured in Water, in Sealed Containers, and in Air

(Age, Twenty-one Months; Expansion in Millionths of an Inch per Inch)

Description	Cement							
	GS; 1.14% Alkali				*AS*; 0.45% Alkali			
	1	2	3	4	1	2	3	4
Aggregate:								
Fine	Coyote	Oro Fino	Coyote	Oro Fino	Coyote	Oro Fino	Coyote	Oro Fino
Coarse	Coyote	Oro Fino	Oro Fino	Coyote	Coyote	Oro Fino	Oro Fino	Coyote
6-In. by 6-In. by 34-In. Concrete Beams; Maximum Size of Coarse Aggregate, 1½ In.; 1 : 2 : 3.8 Mix								
Stored:								
In water	+100	+4,220	+67	+83	−8	0	−42	−37
Sealed	+80	+1,027	+72	+82	−63	−63	−92	−83
In air	−1,120	−1,027	−803	−923	−1,150	−927	−755	−877
2-In. by 2-In. by 11-In. Concrete Bars; Maximum Size of Coarse Aggregate, ¾ In.; 1 : 2.5 : 2.5 Mix								
Stored:								
In water	+100	+90	+120	+90	+10	+30	+20	+40
Sealed	+10	+2,320	+730	+2,280	−100	−50	−40	−60
In air	−1,570	−950	−1,130	−1,170	−1,340	−750	−1,100	−1,080

Autoclave Tests on Feldspars and Other Materials.—Inasmuch as several investigators have questioned feldspar as a mineral constituent, some studies were made to determine the effect on various unaltered feldspars and other petrographic types of aggregates when immersed in normal Na_2CO_3 and NaOH solutions after a method similar to that used by E. A. Stevenson.[29] The material tested included: (1) Three different types of opal, (2) Parker Dam andesite, (3) siliceous magnesian limestone No. 28039, (4) orthoclase, (5) microcline, (6) oligoclase, (7) andesine, and (8) labradorite. (Feldspars 4, 5, 6, 7, and 8 were obtained from Ward's Natural Science Establishment, Rochester, N. Y.)

The specimens submerged in the test solution were sealed in an autoclave at a pressure of approximately 300 lb per sq in., temperature 430° F, for six days. Results indicated the ready solubility of the opal and of parts of the

[29] *Journal of Geology*, 1916, Vol. 24, p. 180.

siliceous magnesian limestone No. 28039, but the feldspars and the Parker andesite were practically unaffected by this treatment.

Curing in Sealed Containers Not Necessary with Large Concrete Specimens.—The discovery that excessive expansion of mortar specimens occurred only when cured in sealed containers defined the test procedure requisite to a determination of the reactions between the fine aggregate and the cement. Hundreds of mortar-bar tests conducted during this study have confirmed the necessity for the sealed container or some equivalent method of storage for this size specimen.

Recent developments, however, demonstrate just as conclusively that, in the case of concrete specimens of 6-in. cross section, it is not necessary to cure in sealed containers, and that, in fact, the greatest reaction may be developed when such specimens are constantly immersed in water.

Tables 15 and 16 are illustrative of this development. It will be noted in Table 16 that some of the 6-in. by 12-in. cylinders and 6-in. by 6-in. by 34-in. beams composed of Oro Fino fine aggregate, stored continuously in water, developed excessive expansion and lowered strength in eighteen months, whereas the companion 2-in. by 2-in. by 10-in. concrete specimens, using aggregates from $\frac{3}{4}$-in. and smaller failed to develop expansion when stored in water twenty-one months, but have developed considerable expansion in sealed containers (Table 15). There would appear, therefore, to be a critical size of specimen below which the reaction will not take place unless the specimen is stored in a sealed container or in some other way in which the moisture can be controlled properly.

Concrete Tests.—Under the heading "Concrete Tests," reference was made in the paper to the 6-in. by 6-in. by 34-in. specimens of six-sack concrete, using different combinations of non-reactive and reactive coarse and fine aggregates, the specimens of each mix being cured under three conditions—(*a*) sealed, (*b*) in water, (*c*) exposed to air,—at laboratory temperature (approximately 70° F) and humidity. Similarly, 6-in. by 12-in. cylinder specimens were treated for compression strength tests.

The following mixes were used: (1) Oro Fino fine and coarse aggregate; (2) Oro Fino fine aggregate, Coyote coarse; (3) Coyote fine aggregate, Oro Fino coarse; and (4) Coyote fine and coarse aggregate.

Two sets of specimens were made—one with the high-alkali cement *GS*, and the other with the low-alkali cement *AS*. The test had been in progress too short a time when the report was closed to produce information of value. The writer now reports the situation as of one year later (1941).

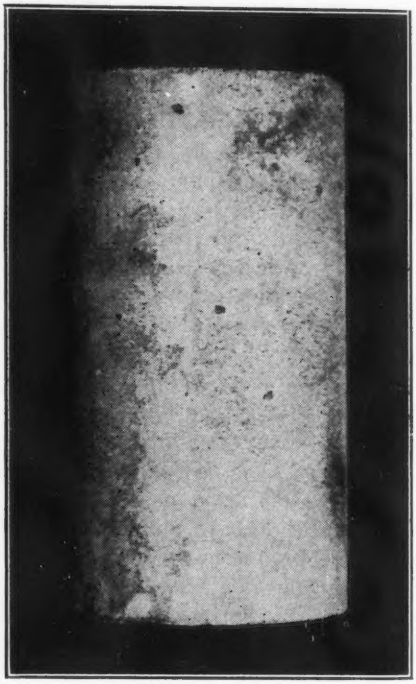
The specimens were fabricated in September, 1939. At one year, no appreciable expansion had taken place in any of the specimens, regardless of the method of cure. When next inspected, at eighteen months, a very marked expansion had taken place in the Oro Fino fine and coarse aggregate specimens stored in water, and some expansion in similar mixes covered with a watertight wrapping. Table 15 shows the expansion at twenty-one months for all concrete specimens in this series.

Attention is called particularly to the fact that little, if any, expansion and no cracking were noted at twelve months, whereas very marked expansion

TABLE 16.—CONDITION OF CONCRETE BEAM AND CYLINDER SPECIMENS AFTER STORAGE IN WATER FOR EIGHTEEN MONTHS

(*a*) 6-IN. BY 6-IN. BY 34-IN. BEAM

(*b*)



(*c*)

OBSERVATION PERIOD	6-IN. BY 6-IN. BY 34-IN. BEAM		6-IN. BY 12-IN. CYLINDERS	
	(*a*)		(*b*)	(*c*)
Cement	*GS*	*AS*	*GS*	*AS*
21 months[a]	4,220	0	....	....
28 days[b]	....	....	4,030	4,590
18 days[b]	....	....	2,440	5,800

[a] Expansion in millionths of an inch per inch. [b] Compressive strength, in pounds per square inch.

and deterioration took place in some specimens between twelve months and eighteen months, thus paralleling the Bradley job experience in which the same aggregates and cement were used, and where excessive expansion became apparent between twelve and eighteen months after construction.

Tests to Determine Aggregate Characteristics.—The only known suitable tests to determine the reactive characteristics of aggregates conclusively have been long-time expansion tests of mortar and concrete specimens, although petrographic analyses are of valuable assistance in furnishing rapid advance information relative to the presence of mineral inclusions that may be reactive.

Unfortunately, when the percentage of reactive mineral is small, several years may be required before the reaction becomes apparent. Attention has turned, therefore, to a study of accelerated methods of determining the probable reaction characteristics of an aggregate by chemical analysis and other means.

Chemical analyses of the white crust and efflorescences taken from the surface of disintegrated concrete usually indicate a rather high content of Na_2CO_3. Working on the assumption that the sodium is probably present originally in the concrete as hydroxide (later converted to the carbonate on exposure to air), studies of aggregate solubility in Na OH solutions were made in California studies as far back as 1936.

From 1936 to 1938 more than 500 specimens, representing various types of shale, chert, and other questionable aggregate particles were individually soaked in a 10% Na OH solution, in airtight containers, at ordinary laboratory temperature. Results of this early work showed the ready solubility of certain types of shales and cherts.

On further study, it was discovered that these shales, cherts, and particles of each solubility were high in opaline or hydrous silica content. It was felt, therefore, that this type of aggregate should be objectionable in the presence of any alkalis likely to be encountered in the concrete, whether contributed by ground water, mixing water, or the cement.

A further investigation was started in the early part of 1937 to determine the solvent action of Na OH solution on certain commercial concrete sands of the State of California. The purpose of this later investigation was to find, if possible, a better testing device than the present A. S. T. M. Sodium Sulfate Soundness Test, for those sands known to contain shale and chert of a deleterious nature.

After much experimentation, the method finally selected as showing the most promise consisted of grinding 50 g of each sand to pass the 100-mesh sieve, soaking in a 10% Na OH solution for fifteen hours at a temperature of 210° F, then determining the amount of dissolved carbonate by titration, and assuming the remainder as silicate. Results, however, showed generally as much solvent action on certain sands of apparent durability as on sands known to contain deleterious particles. Therefore, as the results were inconsistent and inconclusive, no further investigation along this line was made until June,1939.

In June, 1939, after the Bradley pavement failure investigation was started, chemical analyses were made on solutions from treating Oro Fino and Coyote d, respectively, in (*a*) tap water, (*b*) low-alkali cement leaching water, and

(*c*) high-alkali cement leaching water. Again it was noted that the non-reactive Coyote sand was nearly as soluble as the reactive Oro Fino..

When the more recent developments demonstrated that in all probability soluble silica of an opaline nature was "at the root" of the California trouble, a new series of solubility tests was started, using a weaker sodium hydroxide solution and a less severe procedure, on the theory that, as opal may be more soluble than other silica minerals, a procedure might be developed under which the opal could be dissolved more rapidly than other minerals, thereby enabling comparative determinations to be made.

The results of this later study have, to date (September, 1941) been disappointing and it is somewhat doubtful if the sodium hydroxide solubility test is sufficiently selective to be dependable in the case of aggregates containing quite small but at the same time objectionable percentages of reactive minerals. The study is being continued, however, in the hope that a satisfactory and dependable procedure can be developed.

Fluorescence.—Studies have been continued to ascertain, if possible, the nature and significance of the fluorescence observed and described in the paper, in the hope that an effective and reliable aid might be developed for detecting reactive aggregates. The results of this study have been none too encouraging, as the observations are too frequently inconsistent.

Efforts have been made to compare the amount of observed fluorescence of both reactive and non-reactive rock particles in concrete. Results indicate that, although the gel is fluorescent in most cases where expansion occurs through cement-aggregate reactions, there is likewise a noticeable fluorescence in samples where no reaction has taken place. In many cases fluorescent rims are observed surrounding durable aggregates. Many non-reactive mineral grains that fluoresced in the sand sample continue to fluoresce after the sand has been incorporated in concrete cylinders, so that it is difficult to distinguish between the non-reactive and the reactive particles. In addition, not all reactive particles show fluorescence after several months in the sealed curing test.

The observations in connection with this work were made with a standard commercial type of mercury-vapor lamp with built-in filter. Since the ultraviolet radiation characteristics of the many commercial lamps now available undoubtedly vary with the type and manufacturer, and since no other type of lamp was available for use in the experiments, it is not possible to say, at this time, what results would have been obtained from these same samples if different ultraviolet equipment, having different wave-length ranges and different filters, had been used.

It is possible that, for certain gels and reactive rims, as well as for the reactive aggregates themselves, different lamps may show decidedly different fluorescence effects. It is likewise very probable that many types of aggregates now known to be reactive, but which have not shown fluorescence under the mercury-vapor lamp used in the test, may fluoresce under a higher or lower wave length. Additional work needs to be done in this field (see Table 14 for fluorescent reactive and non-reactive materials).

Modulus of Elasticity by Means of Sonic Vibrations.—Considerable progress has been made during recent years in the development of equipment and procedure for determining modulus of elasticity by means of sonic vibrations.[30] Suggestions were made in several quarters that it might be possible to determine the effects of the expansive reaction between cements and aggregates more readily and at an earlier date by means of sonic vibrations than through expansion measurements.

A number of investigators are now studying this phase, including the Research Department of the Portland Cement Association, the Bureau of Reclamation, the Washington State Highway Department, the Riverside Cement Company, and undoubtedly many others, as well as the California Division of Highways.

TABLE 17.—SONIC MODULUS OF ELASTICITY

(1 : 2 Mortar; Specimens, 1-In. by 1-In. by 10-In.; *GS* Cement with Russian River Sand; and Varying Amounts of 30-Mesh to 80-Mesh Opal[a])

Material	Age of Specimen in Weeks[b]									
	1[b]	7[b]	2	3	4	5	6	7	8	10
	Modulus of Elasticity $\times 10^{-6}$									
Russian River sand	1.48	3.31	3.34	3.36	3.48	3.54	3.63	3.67	3.68	3.69
+2½% opal	1.59	3.67	3.85	3.79	3.55	3.18	2.39	1.57	1.40	1.25
+5% opal	1.47	3.70	3.86	3.75	3.73	3.73	3.74	3.70	3.68	3.12
+10% opal	1.39	3.36	3.55	3.46	3.43	3.52	3.52	3.52	3.53	3.48
Expansion in Millionths of an Inch per Inch										
Russian River sand	....	....	....	....	−60	....	−120	....	−120	−110
+2½% opal	....	....	....	....	+60	....	+720	....	+1,650	+2,910
+5% opal	....	....	....	....	+50	....	+270	....	+520	+1,090
+10% opal	....	....	....	....	+80	....	+150	....	+190	+220

[a] Opal from dredger dumps near Roseville, Calif. [b] The first two columns (1 and 7) are in days.

The results of some of the California studies are shown in Table 17. The measurements shown were made first on the specimens immediately upon removal from the molds and thereafter at weekly intervals. Each value is the average of measurements made on four sides of three bars in each set, or a total of twelve determinations.

All of the tests were made on specimens without end gage plugs. A duplicate set of specimens was made with end plugs for expansion determinations. Modulus tests on the bars with end plugs were somewhat erratic and therefore are not shown in this table.

The control specimens were made from a neutral sand and high-alkali cement. These control specimens have shown a steady increase in modulus, whereas all specimens containing variable added percentages of reactive

[30] "Measuring Young's Modulus of Elasticity by Means of Sonic Vibrations," by T. C. Powers, *Proceedings*, A. S. T. M., Vol. 38, Pt. II, 1938, p. 460.

mineral in the form of opal reached a peak modulus in not more than three weeks and then a retrogression in direct relation to the expansion characteristics. However, although the modulus decreases as expansion develops, the decrease at early ages is not appreciable except in the case of a high expansion. The expansion measurements are more marked, and it appears doubtful, therefore, if the sonic method will be found more satisfactory than direct expansion measurements.

Also, as most commercial fine aggregates contain substantially less deleterious particles than the synthetic sands reported in Table 17, a longer time before the reaction becomes detectable by the sonic method may be anticipated with reactive commercial sands. Commercial sands have not been tested long enough to produce information of any value.

Correctives.—The study of correctives has been continued with little practical success to date. At first it was thought that if the sodium hydrate could be converted to a chloride the sodium (or potassium) chloride might be harmless. However, such apparently is not the case. Calcium, ferric, and aluminum chloride have all been added to the mixing water, but none have proved effective in reducing the expansion. In fact, in most cases the expansion has been considerably increased.

Other possible correctives have included diatomaceous earth, resins, and sodium alginate but none of these materials has proved sufficiently effective in reducing expansion to warrant their use. Likewise, magnesium carbonate was added to react with the sodium hydroxide, but without success.

The writer's attention was called to work done in which sodium sulfate was added to cement to increase its alkali (sulfate) resistance.[31] However, a 4% addition of anhydrous sodium sulfate had the effect of increasing instead of decreasing the expansion, even with the low-alkali cement *AS*, to such an extent that, although this cement is non-reactive under normal conditions, the expansion in twenty-eight days in the presence of the sodium sulfate was even greater than in the case of the high-alkali cement *GS*.

In this short period the *AS* cement specimens expanded 3,800 units and the *GS* cement specimens 2,640 units per unit. The specimens were of a 1 : 2 mix made with Russian River sand to which had been added 2.5% opal.

Aluminum powder has proved effective in stopping the expansion. When 0.05% by weight of the cement was added, the high-alkali cement bars have shown no expansion up to four months, but the mortar was so fluffed by the expulsion of gas that this method is not considered suitable for construction purposes.

Silica or Puzzolanic Admixtures.—As reported in the paper, the high-silica or puzzolanic type cement HP (Fig. 4) had developed very little expansion in twelve months although the cement contained 0.78% alkali. At the end of twenty-eight months the same specimens show even a slight shrinkage, whereas specimens containing normal cement CL with 0.77% alkali have continued to expand, reaching 3,600 units or 0.36% at twenty-eight months.

This fact gave encouragement to the enlargement of this series to include all California commercial puzzolanic type cements and likewise to include a

[31] *Industrial and Engineering Chemistry*, Industrial Ed., Vol. 33, No. 5, May, 1941, p. 692.

series of blends of the high-alkali cement *GS* to which had been added, or for a part of which had been substituted, a percentage of certain silica admixtures.

These studies have not progressed sufficiently to be considered conclusive, but they have developed one interesting fact: When the siliceous material is added without reducing the cement content, little benefit is noticed. The expansion is reduced effectively only when the siliceous material is substituted for an equivalent amount of cement. This is revealed in Table 18.

TABLE 18.—Effect on Cement-Aggregate Expansion Reaction, of Ground[a] Mineral Additions to, and Substitutions for, Equivalent Percentages of Cement; in Millionths of an Inch per Inch

(Sand, 90% Russian River plus 10% No. 28039-B; 1 : 2 Mortar Specimens)

Type of admixture[a]	Age, in months	Percentage of Siliceous Material:							
		0	5	10	15	20	30	40	50
(*a*) Siliceous Material Added to Cement									
Monterey shale	4	+5,370	+6,670	+6,070	+4,980	+4,050	+1,240	....	....
Pumicite	4	+4,900	+5,380	+5,930	+4,920	+4,580	+3,540	....	....
Opalite[b]	4	+5,640	+5,940	+6,250	+5,970	+6,290	+6,430	....	....
No. 28039-A	4	+3,430	+3,980	+3,800	+3,630	+3,360	+2,410	....	....
(*b*) Siliceous Material Substituted for Cement									
Monterey shale	4	+3,860	....	+1,140	....	+30	−10	−40	−120
Pumicite	4	+4,650	....	+3,250	....	+1,180	+140	+100	+40
No. 28039-A	4	+5,070	....	+4,740	....	+2,610	+140	+50	+30
Opalite[b]	4	+4,560	....	+4,150	....	....	+2,710	....	....
Ground Ottawa sand	4	+4,550	....	+4,660	....	+4,280	+2,340	+1,030	+150
Opal[c]	4	+4,550	....	+410	....	+20	−50	....	....
No. 28038	4	+4,560	....	+180	....	+60	+10	....	....

[a] All mineral additions ground to pass 325–mesh. [b] From Arkansas reported to be a disintegrated chert of nearly pure opaline silica. [c] From dredger dump near Roseville, Calif.

Comments on Discussions.—Having brought all test data up to date, it is possible to comment on the discussions of the original paper.

Professor Carlson propounded several questions. The answers to some of these are at least partly known and some are definitely in the process of determination. The solution of others may be long deferred but offer a definite goal for the interested researcher.

The trouble is not due entirely, if at all, to the formation of an alkali carbonate but undoubtedly also in major part, if not entirely, to the formation of an alkali silicate. No further evidence has been developed either to sustain or to disprove the original suggestion that an alkali carbonate might be one of the contributing factors.

Such tests as are available indicate that the alkalis of sodium and potassium are equally objectionable, but this phase is to be explored thoroughly as a part of the program instigated and being conducted by the Reclamation Bureau.

Although it may be that a combination of moderate-alkali cements with susceptible aggregates may produce expansion in correspondingly longer times,

the results of at least twenty-four-month tests with high percentages of No. 28039, and shorter periods in the case of other reactive minerals, indicate that this is highly improbable, at least for any commercial aggregates studied where the percentage of offending mineral is as low as that present in the California aggregates.

It was found, however, that, if a high percentage of a relatively low-alkali cement is used with a highly reactive aggregate, expansion may occur, probably due to the presence of a high total alkali content. In illustration, cement *AS* with 0.45% alkali, which showed no expansion in eighteen months in the 1 : 3 and 1 : 2 mortars, shows a definite expansion (2,210 units) in eighteen months in the 1 : 1 mix with no expansion of the same mix at eight months.

Tests up to thirty-three months do not indicate that certain combinations of high-alkali cement and non-reactive aggregate will ever cause trouble, although it is admitted that longer tests may upset this conclusion.

There is no answer at present to the inquiry as to the possibility of the formation of alkali carbonate through the carbonation of the alkalis in aggregates, especially those like crushed feldspar containing a considerable amount of alkali; nor to the last two questions raised by Mr. Carlson. This is a fertile field for study.

Mr. Woods has well stated the problems facing the cement manufacturers in reducing the alkalis in cement. It would undoubtedly be an ideal solution if some inexpensive addition to the cement to circumvent the expansive reaction could be found. Such a solution may be found in the nature of a silica admixture that will not only nullify the expansive reaction, but likewise result in a less expensive product. In this respect the data shown in Table 18 are of interest.

Mr. Stadtfeld makes a plea for placing restrictions on water-soluble alkalis rather than on total alkalis and a further plea for the Merriman free alkali test and a limit of 3.5% free alkali under this test. As stated in the paper, under the heading "Merriman Alkalinity and Free Alkali Test," it was found that in the case of the California cements this test is not always consistent with results and, therefore, is not a suitable test as a substitute for total alkalis as determined by chemical analyses. This phase of the subject was discussed with Mr. Merriman when he visited the Laboratory of the California Division of Highways in July, 1939, just prior to his untimely death. After leaving Sacramento, Mr. Merriman wrote the writer as follows:

> "Just a word to say how much I enjoyed my visit of Saturday and how instructive it was. You are on the track and close to the goal of demonstrating that all cements do not always act alike. To me it was inspiring to see what you have done. Far too few ever look beyond the obvious because they are blind even before they start.
>
> "The literature is full of results attained by the use of unknown materials which are assumed to act alike under all conditions and in all locations. Their history and origin are never known! Is that engineering?"

It may be that, as Mr. Stadtfeld states, the Merriman alkalinity and free alkali tests are suitable quality tests under average conditions, but it is evident that they do not fully satisfy the situation covered by this paper and, on the other hand, may be too severe for universal application.

It is possible that, under certain circumstances, the destructive action taking place in concrete may be one of base exchange involving the calcium in solution, the alkali, alumina, and the silica, somewhat in the manner suggested by Mr. Tremper. If this should be the case, the alkali would be replaced and liberated by the calcium and then could combine with the silica.

Search of available literature yielded no information regarding the expansion, if any, which occurs when the reaction $Na_2OH + Si\,O_2$ takes place. Probably the lack of data is due to the large number of possible combinations of alkali and silica, the majority of which are noncrystalline or colloidal.

Although it is true that large amounts of calcium hydroxide result from the hydration of cement, only a small part of it is in solution and available for chemical reaction.

The solubility of calcium hydroxide at several temperatures is as follows:

1 g of calcium hydroxide is soluble in—

(cc of water)	(Degrees centigrade)
540	0
630	25
1,300	100

As there is not enough water present in concrete to dissolve all of the calcium hydroxide formed, it is not necessary that sufficient sodium carbonate be present to combine with all of the lime hydrate to form the carbonate before the sodium carbonate can crystallize as stated in Mr. Tremper's discussion.

It must be understood that the possibility of the reactions—magnesium carbonate plus sodium hydroxide yielding magnesium hydroxide plus sodium carbonate ($Mg\,CO_3 + 2\,Na\,OH \rightarrow Mg(OH)_2 + Na_2CO_3$); sodium hydroxide plus silica ($Na_2O + Si\,O_2$); or any other chemical activity—is visualized as taking place only at the points of contact of the reactive aggregate and the sodium hydroxide from the cement and not throughout the entire mass of the concrete. This localizes the reaction and further reduces the amount of calcium hydroxide available for the calcium carbonate reaction to which reference has been made.

Mr. Tremper's suggestion that the Na OH acts in a roving commission rather than becoming fixed as a silicate is interesting and a possible situation, but would be more probable if the $Ca(OH)_2$ were more soluble or if there were more free water available.

Mr. Hanna, in his comments, differentiates between the sodium and potassium hydroxide. It was found, in a limited series of tests in which Na OH and K OH were added to the mixing water, that the addition of either accelerated and increased the expansion of the specimens up to a certain percentage addition. Additions above this amount retarded and decreased the action. The writer's work was done on a synthetic sand, made by adding No. 28039 to a neutral sand. Data published by the Bureau of Reclamation show that an increase of Na OH beyond the amount added by the writer accelerates the action with Parker andesite. It is probable that there is a different "pessimum"

amount of alkali for various minerals, just as there is a different "pessimum" amount of different minerals for a given alkali content.

The same study indicated that there is very little difference in the effect of the K OH and the Na OH when they are used in equimolar amounts. The molecular weight of sodium hydroxide is 40, whereas that of potassium hydroxide is 56. If the bases are used in that proportion, or equimolar, they react very similarly.

Mr. Blanks lists many materials which he states are known to be reactive. It should not be concluded that all materials of the types listed, such as andesite, rhyolite, felsite, and granite are, of necessity, reactive in all cases, or even at any time reactive just because they are of the types noted, but rather that when reactive it may be because they contain a sufficient percentage of reactive soluble silica of the opaline type.

It is unfortunate if the paper was so worded as to justify the inference of Mr. Blanks that only the sand in the California aggregates carried alkali-reactive materials, as such, of course, is not the case. The rôle of the coarse aggregates has been fully covered in the earlier part of this discussion and need not be repeated. Mr. Blanks has submitted a very valuable contribution to the discussion on this subject.

It is gratifying to note the extent to which Mr. Kinzie, reproduces the results secured in the California studies described in this paper. As previously stated, it is apparent that one highly potential corrective procedure is through the use of suitable puzzolanic admixtures, and this phase merits thorough exploration.

Summary.—In summarizing developments since the paper was published in *Proceedings*, it is interesting to note the extent to which different investigators are now tracing some of their hitherto unexplained concrete failures to a reaction between alkali in the cement and the aggregate.

There is still considerable to be learned relative to the mineral ingredient or ingredients at the base of the trouble, but in the California case, at least, the evidence points conclusively to the opaline types of rocks as contributing at least a major part. Experiences of others, however, indicate rather definitely that opal may not be the only reactive mineral. It is possible that any form of silica that is readily soluble in Na OH may in time contribute to undue expansion, although the writer's tests to date, at normal temperature, do not indicate any excessive expansion in the case of non-opaline rocks.

The maximum amount of alkali that can be tolerated is still a matter for future determination. The California tests indicate that with the California aggregates and California standard cements little concern need be felt if the alkali content is kept below 0.6%, but whether this is universally true, or even locally over a greater period of years than covered by the present series of tests, can only be determined by future developments. At the same time, however, there is considerable ground for hope that some corrective will be found that will afford ample protection in any doubtful cases.

The nationwide investigations now being made by the Bureau of Reclamation will be a most valuable contribution to the study and it is hoped that the Bureau will make the data collected available to the profession at large.